I-B-VP-0: The holonomy-odd vacuum current of the charged Friedrichs ground state

A local observable on the prescribed conical Enter spacetime

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Abstract

We construct and evaluate the renormalized $U(1)$ current of the massless charged Friedrichs spectral ground state on the regular conical spacetime prescribed in I-B. For angular equivariance $\Phi(\varphi + 2\pi) = e^{2\pi i\beta}\Phi(\varphi)$, opening parameter $0 < \alpha < 1$, and the convention $J_\mu = \mathrm{i}\{\Phi^\dagger D_\mu \Phi - (D_\mu \Phi)^\dagger \Phi\}$, the only nonzero covariant component is

$$\langle J_\varphi \rangle_{\mathrm{ren}} = \frac{\beta(1-\beta)(2\beta-1)}{12\pi^2\alpha^2 r^2}, \qquad 0 \le \beta < 1.$$

The physical orthonormal component is $\langle J_{\hat{\varphi}} \rangle_{\mathrm{ren}} = \langle J_\varphi \rangle_{\mathrm{ren}}/(\alpha r)$.

The principal contribution is a convention-complete, extension-dependent derivation that remains valid at every regular point, including cone angles at which an image boundary meets the diagonal. We retain the complex, holonomy-odd first jet of the Hadamard remainder; replacing it by the real part used for the stress tensor would incorrectly erase the current. A finite-character quotient calculation gives an independent exact sign and normalization check. The result agrees with the established cosmic-string current literature and is not claimed as a new cosmic-string coefficient.

Unlike the Wick square and stress tensor, the current distinguishes charge-conjugate holonomy sectors. Within the fixed-background Friedrichs family, the Wick square together with this current determines the fractional holonomy. This provides a useful orientation-sensitive diagnostic, not general state tomography. The current is

conserved and independent of the curvature coupling on the regular host. No apex extension of the observable, electromagnetic backreaction, Einstein dynamics, interacting algebra, or downstream programme closure is asserted.

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Book expansion: conventions and object label

toprule Convention Fixed choice for this sheet

midrule Charge / holonomy As in the charged Friedrichs construction below

Meridian orientation Declared with the current component before formulae

Parallel vs periodic gauge Stated beside the mode-sum and finite-image derivations

Lower vs orthonormal current Kept distinct; do not silently convert

Point-splitting sign Fixed before coincidence; unchanged by reporting

Object class **regular one-point current** on the smooth host

Not claimed Apex extension / distributional completion across $\mathscr{W}$

bottomrule Keep the mode-sum and finite-image derivations adjacent so the normalization check remains auditable. This chapter does *not* construct an apex extension.

Purpose, background, and precise scope

The expectation value of a charged scalar current around a flux-carrying cosmic string is an established vacuum-polarization effect. Explicit calculations include Sriramkumar [1], Sitenko–Vlasii [2], and Bezerra de Mello et al. [3]. Our purpose is not to claim priority for that effect or its massless coefficient. It is to complete one specified observable calculation for the extension and state already fixed in Cheung's I-B-G and I-B-VP, with all signs, bundle conventions, and regularity qualifications explicit [13, 14].

I-B-G constructs intrinsic scalar wave operators after an apex extension has been supplied. I-B-VP-v2 constructs the distinguished Friedrichs spectral ground state and evaluates its Wick square and stress tensor. Its item V4 leaves the twisted current unevaluated. The present manuscript evaluates that current and supplies an exact finite-image cross-check. It does not resolve the other extension, state, apex, or interacting questions listed there.

Enter and its philosophy. Cheung's *The Philosophy of Enter* distinguishes destructive readout from quantum recovery, a classical record from a decoder, a channel from a thermal machine, and an analogy from an identification [11]. These distinctions constrain the interpretation here: a finite report of a current coefficient is not

a continuum field, a state-reconstruction map, or a source law. The essay is an introductory philosophical map, not a premise of the analysis.

The spacetime Enter. I-B pairs finite clock and transport operations with a *prescribed* regular conical product [12]. Its finite destructive instrument does not select a scalar field, an apex extension, or a vacuum. Those are separate inputs in this paper. In particular, our expectation value is not extracted from a conditional finite-register density matrix.

What is new in this companion. The mathematical contribution within the supplied companion framework is:

1. a renormalized current derived from the full complex regular covariance, rather than from its real part;

2. an all-angle near-diagonal justification, avoiding unjustified differentiation of changing image sectors;

3. an independent finite-character image identity fixing the current's sign and normalization;

4. a precise charge-conjugation discrimination statement within the declared state family. The last item concerns identifiability in a specified mathematical family. It is not a physical acquisition or universal reconstruction theorem.

Symmetry being used. The current below is the Noether current of the internal transformation $\Phi \mapsto e^{i\varepsilon}\Phi$. It is not the angular-momentum current associated with spacetime rotations. Changing the holonomy parameter changes the background bundle data; it is not itself a continuous symmetry operation within one fixed bundle.

Source conventions. We cite the supplied I-B-G-v2 and I-B-VP-v2 by source labels rather than unstable theorem numbers. The analytical ingredients needed for the current are also stated and proved below. Other companion comparisons retain their own domains and non-transfer boundaries.

Prescribed geometry, bundle, and spectral state

Assumption (Fixed background and charged field). Fix

$$0 < \mu < 2\pi, \qquad \alpha = 1 - \frac{\mu}{2\pi} \in (0, 1).$$

The regular spacetime is

$$M = \mathbb{R}_t \times (0,\infty)_r \times (\mathbb{R}/2\pi\mathbb{Z})_\varphi \times \mathbb{R}_z$$

with

$$g = -\mathrm{d}t^2 + \mathrm{d}r^2 + \alpha^2 r^2 \mathrm{d}\varphi^2 + \mathrm{d}z^2.$$

The completed apex worldsheet $\mathscr{W} = \{r = 0\}$ is not part of M. We use $\hbar = c = 1$.

For $\beta \in [0, 1)$, the charged complex scalar is represented in a lifted parallel frame by

$$\Phi(r, \varphi + 2\pi, z) = e^{2\pi i\beta}\Phi(r, \varphi, z).$$

This equation fixes the meridian and character conventions. The massless equation is

$$(-D^\mu D_\mu + \xi R)\Phi = 0.$$

On M, the metric and bundle curvatures vanish, so in a parallel frame the equation is $\mathrm{Box}_g \Phi = 0$. No distributional ξR interaction at $\mathscr{W}$ is modeled.

The spatial operator is the Friedrichs extension

$$B_\beta = A_{F,\beta} \geq 0$$

of the covariant expression $-\Delta_q$, where

$$q = \mathrm{d}r^2 + \alpha^2 r^2 \mathrm{d}\varphi^2 + \mathrm{d}z^2, \qquad \mathrm{d}V = \alpha r\, \mathrm{d}r\, \mathrm{d}\varphi\, \mathrm{d}z.$$

Its Hilbert space is $\mathscr{H}_\beta = L^2_\beta(N, \mathrm{d}V)$. The angular functions and radial Bessel orders are

$$\frac{e^{\mathrm{i}(n+\beta)\varphi}}{\sqrt{2\pi\alpha}}, \qquad \nu_n = \frac{|n + \beta|}{\alpha}, \qquad n \in \mathbb{Z}.$$

The Friedrichs form is the closure of $\int_N |Du|^2\, \mathrm{d}V$ on compact smooth sections. The extension is an input, not a consequence of the regular metric [13, 6].

Signature and developing frame. With

$$\mathrm{dev}(t, r, \varphi, z) = (t, r\cos(\alpha\varphi), r\sin(\alpha\varphi), z)$$

and completion metric $\eta = \mathrm{d}t^2 - \mathrm{d}X^2 - \mathrm{d}Y^2 - \mathrm{d}z^2$,

$$\mathrm{dev}^{*}\eta = -g, \qquad -\mathrm{Box}_g = \mathrm{dev}^{*}\mathrm{Box}_\eta.$$

This is the equation-operator ledger of I-B-G and the ST sheets [13, 15, 16]. An inverse of an action Hessian $-P$ is $-G_{\pm}$, not $G_{\pm}$. The current is defined directly from its Noether expression below, without importing a Hessian sign.

Massless infrared finiteness

Lemma (Heat kernel and compact-test covariance). *The spatial Friedrichs heat kernel, relative to* $\mathrm{d}V$*, is*

$$K_\beta(s; x, x') = \frac{e^{-\Delta z^2/(4s)}}{\sqrt{4\pi s}} \frac{e^{-(r^2 + r'^2)/(4s)}}{4\pi\alpha s} \times \sum_{n \in \mathbb{Z}} e^{\mathrm{i}(n+\beta)(\varphi - \varphi')} I_{\nu_n}\left(\frac{rr'}{2s}\right).$$

On every compact regular spatial set K*,*

$$\sup_{x, x' \in K} |K_\beta(s; x, x')| \le C_K s^{-3/2 - \nu_*}, \qquad s \ge 1, \qquad \nu_* = \frac{\min(\beta, 1-\beta)}{\alpha}.$$

Every compact smooth spatial section v *belongs to* $\mathrm{dom}(B_\beta^{-1/4})$*, continuously in the fixed-support test topology.*

Proof. The radial Friedrichs transform uses $J_{\nu_n}(kr)$. Weber's integral

$$\int_0^\infty e^{-sk^2} J_\nu(kr) J_\nu(kr')\, k\, \mathrm{d}k = \frac{e^{-(r^2 + r'^2)/(4s)}}{2s} I_\nu\left(\frac{rr'}{2s}\right)$$

gives after the angular normalization and longitudinal heat kernel are included [7, 13]. For bounded r, r' and $s \ge 1$, the Bessel power series gives

$$I_\nu\left(\frac{rr'}{2s}\right) \le C_K \frac{(C_K/s)^\nu}{\Gamma(\nu+1)}.$$

The orders grow linearly in $|n|$, so their sum is bounded by a constant times $s^{-\nu_*}$. This proves .

The spectral identity

$$\mathrm{lVertB_}\beta^{-1/4}v\|^2 = \frac{1}{\sqrt{\pi}}\int_0^\infty s^{-1/2}\langle v, e^{-sB_\beta}v\rangle\,\mathrm{d}s$$

is finite: on $0 < s < 1$ use the contraction bound $\langle v, e^{-sB_\beta}v\rangle \le \|v\|^2$; on $s \ge 1$ use and $\|v\|_{L^1}^2$. The large-s integrand is $O(s^{-2-\nu_*})$. These bounds also prove continuity. There is no L^2 zero eigenfunction, since zero form energy would give a parallel section, which cannot be square integrable on N.

Definition (Distinguished charged Friedrichs ground state). The state ω_F is the zero-mean, gauge-invariant quasifree spectral Fock state with particle and antiparticle sectors on L_β and its conjugate bundle. Its positive-frequency kernel is

$$W_\beta(t-t';x,x') = \left[\frac{e^{-\mathrm{i}(t-t')\sqrt{B_\beta}}}{2\sqrt{B_\beta}}\right](x,x') = \omega_F(\Phi(X)\Phi^\dagger(X')).$$

Indeed, on spacetime tests its positive form is the Gram form of

$$f\mathrm{longmapsto}\int_{\mathbb{R}} e^{\mathrm{i}t\sqrt{B_\beta}}(2\sqrt{B_\beta})^{-1/2}f(t)\,\mathrm{d}t,$$

which is finite by Lemma . The standard Fock construction has nonnegative one-particle Hamiltonian $\sqrt{B_\beta}$ [10]. With the conventions of I-B-G,

$$[\Phi(X),\Phi^\dagger(X')] = -\mathrm{i}E_\beta(X,X'), \qquad E_\beta(t) = \frac{\sin(t\sqrt{B_\beta})}{\sqrt{B_\beta}}.$$

This is the state of I-B-VP-v2 def:state, not a claim of uniqueness among all algebraic ground states [14].

Euclidean kernel and analytic remainder

For Euclidean points $X = (\tau, r, \varphi, z)$, set

$$\cosh\eta = \frac{r^2 + r'\wedge 2 + \Delta\tau^2 + \Delta z^2}{2rr'}, \qquad \theta = \alpha(\varphi - \varphi').$$

Proposition (Mode kernel and regular diagonal). *For $\eta > 0$, the Euclidean covariance is*

$$G_{E,\beta}(X,X') = \frac{1}{8\pi^2\alpha rr'\sinh\eta}\sum_{n\in\mathbb{Z}} e^{-|n+\beta|\eta/\alpha}e^{\mathrm{i}(n+\beta)(\varphi-\varphi')}.$$

On a sufficiently small developing neighbourhood of every regular point, in a local parallel frame,

$$H_{E,\beta}(X,X') = G_{E,\beta}(X,X') - \frac{1}{4\pi^2\sigma_E^2(X,X')},$$

where

$$\sigma_E^2 = r^2 + r'\wedge 2 - 2rr'\cos\theta + \Delta\tau^2 + \Delta z^2,$$

is jointly real-analytic, including the diagonal. It is a bisolution and is Hermitian under exchange of endpoints. This holds for every $0 < \alpha < 1$, *including image-boundary parameter values.*

Proof. Multiply by $(4\pi s)^{-1/2}e^{-\Delta\tau^2/(4s)}$ and integrate in s. The large-s integral converges on compact regular sets by ; the small-s integral defines a distribution. Substitute $\zeta = rr'/(2s)$ and use

$$\int_0^\infty e^{-\zeta\cosh\eta} I_\nu(\zeta)\,\mathrm{d}\zeta = \frac{e^{-\nu\eta}}{\sinh\eta}.$$

Absolute convergence for $\eta > 0$ gives [7]. This also derives the mode kernel used in I-B-VP-v2 prop:euclidean [14].

The heat-integrated kernel is a fundamental distribution for $-\Delta_E$ in each endpoint. In a parallel Cartesian chart, the direct term in has the same delta source. Their difference therefore satisfies both homogeneous equations across the diagonal. It is annihilated by the elliptic operator

$$\Delta_{E,X} + \Delta_{E,X'}$$

on the product chart. Elliptic and analytic elliptic regularity make the difference jointly real-analytic [8, 9]. Hermiticity follows from the self-adjoint heat kernel.

This argument applies to the total kernel, not separately to moving image-sector indicators.

The spectral continuation

$$\frac{e^{-a\sqrt{B_\beta}}}{2\sqrt{B_\beta}}, \qquad a > 0, \qquad a \to \varepsilon + \mathrm{i}(t - t')$$

gives . Continuing the analytic remainder in Proposition gives, on a sufficiently small Lorentzian neighbourhood,

$$W_\beta(X,X') = \frac{1}{4\pi^2(|\mathbf{X}-\mathbf{X}'|^2 - (t-t'-\mathrm{i}0)^2)} + H_\beta(X,X'),$$

with smooth H_β. This is the local Hadamard property proved in I-B-VP-v2 thm:hadamard [14]. We use only this local statement. A global microlocal-spectrum characterization and behaviour at $\mathscr{W}$ are not established here.

Noether normalization and renormalized current

The classical normalization

Use the local charged action

$$S[\Phi] = -\int_M \Big((D_\mu \Phi)^\dagger D^\mu \Phi + \xi R \Phi^\dagger \Phi \Big) \mathrm{dvol}_g.$$

The global phase transformation $\delta\Phi = \mathrm{i}\varepsilon\Phi$ has Noether current

$$J_\mu = \mathrm{i}\Big(\Phi^\dagger D_\mu \Phi - (D_\mu \Phi)^\dagger \Phi \Big).$$

The normalization is unit $U(1)$ charge. An electric current with charge e is eJ_μ when the same connection and orientation conventions are retained.

Locally,

$$\Phi = \frac{\varphi_1 + \mathrm{i}\varphi_2}{\sqrt{2}}, \qquad J_\mu = \varphi_2 \partial_\mu \varphi_1 - \varphi_1 \partial_\mu \varphi_2$$

in a parallel frame. This is compatible with $T^{\mathrm{complex}} = T[\varphi_1] + T[\varphi_2]$. There is no additional factor of two to append to .

Point splitting and the complex first jet

Definition (Renormalized one-point current). In a local parallel frame, define

$$\mathscr{J}_\mu := \langle J_\mu \rangle_{\mathrm{ren}} = \mathrm{i}\Big[\big(\partial_\mu - \partial_{\mu'}\big) H_\beta(X,X') \Big],$$

where brackets mean coincidence and primed covectors are parallel transported before identification.

Equivalently, one may use the locally regularized symmetrized two-point function with coefficient i/2. For spatial components the same formula is obtained from $H_{E,\beta}$ at equal Euclidean times.

At spacelike-separated endpoints the fields commute, so

$$\mathrm{i}(\partial_\mu - \partial_{\mu'})\omega_F(\Phi(X)\Phi^\dagger(X'))$$

has exactly the ordering in after coincidence. The Hadamard subtraction extends this to the local definition . The difference of endpoint derivatives is essential.

Remark (Do not replace the kernel by its real part). The stress tensor in I-B-VP uses $2\mathrm{Re}\,H_\beta$. The current does not. Its information is in the imaginary, endpoint-antisymmetric first jet of the *complex* kernel. Hermiticity gives

$$[(\partial_\mu - \partial_{\mu'})H_\beta] = 2\mathrm{i}\,\mathrm{Im}\,[\partial_\mu H_\beta],$$

and consequently

$$\mathscr{J}_\mu = -2\mathrm{Im}\,[\partial_\mu H_\beta].$$

Replacing H_β by its real part would incorrectly set this current to zero. Symmetrizing operator order is not averaging the two charge-conjugate bundle sectors.

The charged Hadamard current prescription and its finite renormalization freedom are treated in [4, 5].

Assumption (Renormalization class). The current has the fixed Noether normalization . Renormalization is local and gauge covariant, state independent, compatible with the standard engineering dimension and charge conjugation, and normalized to zero in the ordinary Minkowski vacuum with trivial connection. Only the usual local background counterterms are allowed.

No preferred radial vector, angular Killing field, global Wilson loop, or apex-supported distribution is admitted as an additional local subtraction datum.

Proposition (Well-definedness, gauge invariance, and conservation). *Definition gives a real, smooth, gauge-invariant one-form on M. In the class of Assumption , no nonzero local background ambiguity survives on*

this regular flat host. Moreover,

$$\nabla_{\mu}\mathscr{J}^{\mu} = 0$$

as an identity of smooth functions, and hence of distributions on M.

Proof. Smoothness and independence of the direction of coincidence follow from Proposition and . Hermiticity proves reality. Using covariant derivatives in both endpoints makes the coincidence expression gauge invariant.

The standard charged-current ambiguity includes a multiple of $\nabla^{\nu}F_{\nu\mu}$, which vanishes here [4, 5]. More directly, a locally covariant, state-independent background covector on this host can be evaluated in a flat parallel chart. The curvature jets vanish, and a flat connection is locally gauge equivalent to zero. There is no nonzero Lorentz-invariant covector built from these local data. Thus the allowed local ambiguity is zero. This reasoning does not classify nonlocal additions or distributions on the removed worldsheet.

Finally, in a parallel Cartesian chart, differentiation of a coincidence limit acts as $\partial + \partial'$. Therefore

$$\partial^{\mu}\mathscr{J}_{\mu} = \mathrm{i}\Big[(\partial^{\mu} + \partial^{\mu'})(\partial_{\mu} - \partial_{\mu'})H_{\beta}\Big] = \mathrm{i}[(\mathrm{Box}_{X} - \mathrm{Box}_{X'})H_{\beta}] = 0.$$

The mixed derivatives cancel, and the two remaining terms vanish by the bisolution equations. The calculation is covariant under the local identifications used above.

Remark (A current-specific ambiguity statement). The preceding argument is not inferred from the stress-tensor ambiguity theorem. The two observables have different renormalization freedoms. Analyticity makes the selected point-split expression finite; the declared covariance class controls its allowed finite changes. Neither statement supplies apex counterterms or a contact classification for a current–current product.

Parallel and periodic gauges

Write the connection as $D = \partial + \mathrm{i}a$. In the lifted parallel representation $a = 0$. The periodic field

$$v = e^{-\mathrm{i}\beta\varphi}\Phi$$

has

$$D_{\varphi} v = (\partial_{\varphi} + \mathrm{i}\beta)v, \qquad H_{\mathrm{per}}(X, X') = e^{-\mathrm{i}\beta(\varphi - \varphi')} H_{\mathrm{par}}(X, X').$$

Thus

$$\mathscr{J}_{\varphi} = \mathrm{i}\left[\left(\partial_{\varphi} - \partial_{\varphi'} + 2\mathrm{i}\beta\right) H_{\mathrm{per}}\right] = \mathrm{i}[(\partial_{\varphi} - \partial_{\varphi'}) H_{\mathrm{par}}].$$

The endpoint phase must also be included in the subtracted parametrix in periodic gauge. Using ordinary derivatives on H_{per} alone would produce the spurious shift $2\beta[H_{\mathrm{par}}]$.

Changing angular lifts multiplies a bundle kernel by the corresponding character. The local fiber identification in the coincidence operation cancels that change. The final current is a global ordinary one-form.

Exact evaluation

Define the third Bernoulli polynomial

$$\mathrm{B}_3(\beta) = \beta^3 - \frac{3}{2}\beta^2 + \frac{1}{2}\beta = \frac{1}{2}\beta(1 - \beta)(1 - 2\beta).$$

Lemma (The signed angular sum). *For $a > 0$,*

$$\mathscr{A}_{\beta}(a) := \sum_{n \in \mathbb{Z}} (n + \beta) e^{-|n + \beta| a} = -\frac{\mathrm{d}}{\mathrm{d}a} \left\{ \frac{\sinh\left(\left(\frac{1}{2} - \beta\right)a\right)}{\sinh\left(a/2\right)} \right\}.$$

*As a*downarrow 0,

$$\mathscr{A}_{\beta}(a) = \frac{2}{3}\mathrm{B}_3(\beta)a + O(a^3).$$

Proof. Separate $n \geq 0$ from $n \leq -1$. The difference of the corresponding unweighted geometric sums is

$$\sum_{j=0}^{\infty} \left(e^{-(j + \beta)a} - e^{-(j + 1 - \beta)a}\right) = \frac{\sinh\left(\left(\frac{1}{2} - \beta\right)a\right)}{\sinh\left(a/2\right)}.$$

Its negative derivative is . All series and derivatives converge absolutely for $a > 0$.

If $t = 1 - 2\beta$, then

$$\frac{\sinh(ta/2)}{\sinh(a/2)} = t + \frac{t^3 - t}{24}a^2 + O(a^4) = (1 - 2\beta) - \frac{\mathrm{B}_3(\beta)}{3}a^2 + O(a^4).$$

Differentiate to obtain .

Theorem (Charged Friedrichs vacuum current). *For Assumptions and , the current of Definition is*

$$\text{boxed}\,\mathscr{J} = -\frac{\mathrm{B}_3(\beta)}{6\pi^2\alpha^2 r^2}\,\mathrm{d}\varphi = \frac{\beta(1-\beta)(2\beta-1)}{12\pi^2\alpha^2 r^2}\,\mathrm{d}\varphi.$$

In particular,

$$\mathscr{J}_t = \mathscr{J}_r = \mathscr{J}_z = 0.$$

The formula is valid for every $0 < \alpha < 1$ *and every* $0 \le \beta < 1$*, including image-boundary parameters.*

Proof. Use a longitudinal spacelike split:

$$t = t', \qquad r = r', \qquad \varphi = \varphi', \qquad z - z' = s \neq 0.$$

Equivalently on the Euclidean section, $\Delta\tau = 0$ and

$$\cosh\eta = 1 + \frac{s^2}{2r^2}.$$

This split requires no convention for a Euclidean time component of the current.

At fixed $s \neq 0$, differentiate in the angular variables before setting their separation to zero. Absolute convergence holds even after multiplication by $n + \beta$. The angular derivative of the direct flat term is zero at $\varphi = \varphi'$. Hence

$$\mathrm{i}(\partial_\varphi - \partial_{\varphi'})H_{E,\beta}\big|_{\Delta\varphi = 0}$$

$$= -\frac{1}{4\pi^2\alpha r^2 \sinh\eta}\,\mathscr{A}_\beta(\eta/\alpha).$$

By Lemma ,

$$\frac{\mathscr{A}_\beta(\eta/\alpha)}{\sinh\eta} \to \frac{2\mathrm{B}_3(\beta)}{3\alpha}.$$

This proves . Analyticity of the regular remainder makes the result the coincidence limit, independently of the

chosen split.

The Euclidean kernel is even in $\Delta\tau$ and Δz, so its regular remainder has vanishing first difference derivatives in those variables at coincidence. It is also symmetric in r, r' at fixed angular difference, so the radial difference derivative vanishes. Continuation gives the stated Lorentzian components.

Remark (The limits are taken in the required order). The series $\sum_n (n+\beta)$ is not assigned a value by formal coincidence. We first differentiate the convergent kernel for $s \neq 0$, sum , and only then take the regular coincidence limit. This is ordinary point splitting, not a replacement of the proof by a summability prescription for a divergent numerical series.

Properties and information carried by the current

Components, scaling, and charge conjugation

The positive angular orthonormal coframe is $e^{\hat{\varphi}} = \alpha r\, \mathrm{d}\varphi$. Thus

$$\mathscr{J}_{\varphi} = \frac{\beta(1-\beta)(2\beta-1)}{12\pi^2\alpha^2 r^2},$$

$$\mathscr{J}_{\hat{\varphi}} = \frac{\beta(1-\beta)(2\beta-1)}{12\pi^2\alpha^3 r^3},$$

$$\mathscr{J}^{\varphi} = \frac{\beta(1-\beta)(2\beta-1)}{12\pi^2\alpha^4 r^4}.$$

The r^{-2} statement concerns the covariant component in a dimensionless angular coordinate. The physical orthonormal current has the expected r^{-3} scaling.

Under

$$\delta_\lambda(t, r, \varphi, z) = (\lambda t, \lambda r, \varphi, \lambda z),$$

the one-form satisfies

$$\delta_\lambda^{*}\mathscr{J} = \lambda^{-2}\mathscr{J}.$$

Directly,

$$\nabla_\mu \mathscr{J}^\mu = \frac{1}{\alpha r}\partial_\varphi(\alpha r\, \mathscr{J}^\varphi) = 0,$$

in agreement with Proposition .

Corollary (Holonomy parity and zeros). *Extend the result periodically in β. Then*

$$\mathscr{J}_{\beta+k} = \mathscr{J}_\beta \quad (k \in \mathbb{Z}), \qquad \mathscr{J}_{1-\beta} = -\mathscr{J}_\beta.$$

On $0 \le \beta < 1$, its only zeros are $\beta = 0$ and $\beta = 1/2$. It is negative in the positive covariant angular component for $0 < \beta < 1/2$, and positive for $1/2 < \beta < 1$.

Proof. Integer shifts reindex the angular modes. Equation gives $B_3(1-\beta) = -B_3(\beta)$. The factorization in gives the remaining claims.

The relevant ``symmetry under $\beta \leftrightarrow 1-\beta$'' is therefore *oddness*, not evenness. Reversing the meridian or the current's charge convention reverses the corresponding sign.

Flat-space Aharonov–Bohm limit

At fixed $r > 0, \beta$,

$$\mathscr{J}_\varphi \to \frac{\beta(1-\beta)(2\beta-1)}{12\pi^2 r^2} \qquad (\alpha\text{uparrow } 1).$$

This is the punctured flat-space flux result, not the ordinary untwisted Minkowski vacuum. For $\beta = 0$, the current vanishes for every cone angle. The geometric deficit alone does not supply an independent charged-scalar character.

Compatibility with the even observables

For reference, set

$$P_\beta = \alpha^{-2}(1 - 6\beta(1-\beta)) - 1, \qquad C_\beta = \alpha^{-4}(1 - 30\beta^2(1-\beta)^2) - 1.$$

With the same complex-field normalization, I-B-VP-v2 proves [14, thm:phi2, thm:T-complex]

$$F_\beta := \langle \Phi^\dagger \Phi \rangle_{\mathrm{ren}} = \frac{P_\beta}{48\pi^2 r^2},$$

$$\langle T^\mu{}_\nu \rangle_{\mathrm{ren}} = \frac{C_\beta}{720\pi^2 r^4}\mathrm{diag}\,(1,1,-3,1) + 2\left(\xi - \frac{1}{6}\right)\frac{P_\beta}{48\pi^2 r^4}\mathrm{diag}\,(4,-2,6,4).$$

Their mixed-component order is (t, r, φ, z). Both are even under $\beta \leftrightarrow 1-\beta$. The current is odd. Its ξ-independence is consistent with –: on the regular host, changing ξ changes the stress-tensor improvement but neither the chosen equation, the spectral state, nor the internal Noether current.

Corollary (Conditional identification of fractional holonomy). *Fix α, r, the orientation, and the state family of Definition . The pair*

$$(F_\beta, \mathscr{J}_{\varphi,\beta})$$

determines $\beta \in [0,1)$. In contrast, F_β and $\langle T^\mu{}_\nu \rangle_{\mathrm{ren}}$ together do not distinguish β from $1-\beta$ when these are distinct.

Proof. Equation determines

$$u = \beta(1-\beta) = \frac{1 - \alpha^2(1 + 48\pi^2 r^2 F_\beta)}{6}.$$

For $0 < u < 1/4$, the two possible values are

$$\beta_\pm = \frac{1 \pm \sqrt{1-4u}}{2}.$$

The sign of $\mathscr{J}_\varphi$ selects the upper or lower root by Corollary . If $u = 0$, the representative is $\beta = 0$; if $u = 1/4$, it is $\beta = 1/2$. The evenness of – proves the last assertion.

This corollary assumes the entire fixed-background, fixed-extension spectral family in advance. It does not determine an arbitrary state, select the extension, or provide a finite measurement procedure acquiring these continuum expectation values.

An independent exact quotient check

For

$$\alpha = \frac{1}{N}, \qquad \beta = \frac{p}{N}, \qquad N \geq 2, \quad p \in \{0, \ldots, N-1\},$$

the character is compatible with the cyclic quotient. The Friedrichs Euclidean kernel is the finite character-weighted image kernel [13, cor:structure]. In a developing parallel frame, its regular part is

$$H_{E,p/N}(X, X') = \sum_{k=1}^{N-1} \frac{e^{2\pi i p k/N}}{4\pi^2 |\mathbf{X}_E - \mathscr{R}_k \mathbf{X}'_E|^2},$$

where $\mathscr{R}_k$ rotates the transverse plane by $2\pi k/N$ and fixes τ, z. The sum contains the full quotient images, also for even N.

Lemma (Finite character identity). *For $p = 0, \ldots, N$,*

$$\sum_{k=1}^{N-1} \sin\left(\frac{2\pi p k}{N}\right) \cot\left(\frac{\pi k}{N}\right) \csc^2\left(\frac{\pi k}{N}\right) = \frac{2}{3} p(N-p)(N-2p).$$

Proof. Call the left side S_p, and put $x_k = \pi k/N$. Then $S_0 = S_N = 0$, while

$$S_{p+1} - 2S_p + S_{p-1} = -4 \sum_{k=1}^{N-1} \sin(2p x_k) \cot x_k.$$

For $1 \leq p \leq N-1$,

$$\sin(2px) \cot x = 1 + \cos(2px) + 2 \sum_{j=1}^{p-1} \cos(2jx).$$

Summing over k, and using the finite root-of-unity sums, gives

$$\sum_{k=1}^{N-1} \sin(2p x_k) \cot x_k = N - 2p.$$

Thus the second difference of S_p is $8p - 4N$. The polynomial

$$f_p = \frac{2}{3} p(N-p)(N-2p)$$

has the same second differences and endpoint values. Their difference has zero second difference and zero values

at $0, N$, so it vanishes identically.

Proposition (Finite-image current and sign). *Direct differentiation of gives*

$$\mathcal{J}_{\varphi} = -\frac{1}{8\pi^{2}Nr^{2}}\sum_{k=1}^{N-1}\sin\left(\frac{2\pi pk}{N}\right)\cot\left(\frac{\pi k}{N}\right)\csc^{2}\left(\frac{\pi k}{N}\right).$$

It equals Theorem at $(\alpha, \beta) = (1/N, p/N)$.

Proof. The image denominator is

$$D_{k}^{2} = r^{2} + r'\text{^}2 - 2rr'\cos\left(\frac{\varphi - \varphi'}{N} - \frac{2\pi k}{N}\right) + \Delta\tau^{2} + \Delta z^{2}.$$

At coincidence $D_{k}^{2} = 4r^{2}\sin^{2}(\pi k/N)$. Applying $\mathrm{i}(\partial_{\varphi} - \partial_{\varphi'})$ to , and pairing conjugate images, gives . Lemma then yields

$$\mathcal{J}_{\varphi} = \frac{p(N-p)(2p-N)}{12\pi^{2}Nr^{2}},$$

which is precisely . This calculation does not use the small-separation Bernoulli expansion.

For example, the following are exact rational identities, not numerical execution reports: toprule $N\ p\ \pi^{2}r^{2}\mathcal{J}_{\varphi}$

Evidence

midrule 2 1 0 ANALYTIC

3 1 −1/18 ANALYTIC

3 2 1/18 ANALYTIC

4 1 −1/8 ANALYTIC

4 3 1/8 ANALYTIC

bottomrule

A strict geometric image set can omit boundary representatives when N is even. Equation avoids that ambiguity by using the full quotient sum. One must not use a boundary omission to redefine the underlying covariance.

Literature comparison

A direct comparison is available in Sriramkumar [1], equations (2.10), (2.12), and (2.13). Its covariant angular component is

$$\langle \hat{J}_{\varphi} \rangle = -\frac{e\,\gamma(1-\gamma)(1-2\gamma)}{12\pi^2\alpha^2\rho^2}.$$

Set $\rho = r$, $\gamma = \beta$, and divide by e. Its periodic-gauge convention $D_{\varphi} = \partial_{\varphi} + \mathrm{i}eA_{\varphi}$, with $eA_{\varphi} = \gamma$, is . Its symmetrized covariance uses coefficient $\mathrm{i}e/2$; our positive-frequency convention uses i. The formulas therefore agree, including normalization and sign. Its opposite metric signature does not change this comparison of the explicitly defined lower-index component.

This checks an established coefficient, not the wider fluctuation calculations of that paper. The current literature also includes massive and finite-core models [2, 3]. We import neither their core physics nor their Maxwell response. A comprehensive audit of all conventions and extensions in that literature is not claimed.

Companion interfaces and reporting discipline

Supported Green replacement does not replace the state

In an angularly small chart

$$U \subset \{r > r_U,\ \varphi \in I\}, \qquad r_U > 0, \qquad |I| < \pi,$$

I-B-G defines an additional-arrival threshold $T_{\mathrm{arr}}(x, x')$ and proves

$$T_{\mathrm{arr}}(x, x') \geq 2r_U \sin(\pi\alpha/2).$$

Replacing an intrinsic retarded or advanced scalar operation by a completion operation requires its signed pair window, a local parallel frame, and the independent retained-shadow and compact-reserve conditions [13, thm:transfer, ass:transfer-contract]. The ST scalar sheets retain these support obligations [15, 16, 17].

This theorem does not transfer a symmetric covariance or a vacuum state. In particular, it does not imply that the

current must vanish in a short laboratory window. The current is a local observable of a globally specified spectral state, not a retarded signal with a newly assigned emission event.

XI-E's named polynomial Hadamard domain [18, def:domain, thm:star] has its own flat-background multiplet and product. It does not select the present positive state or construct a conical interacting algebra. Conversely, evaluating the one-point current does not transfer that multiplet or establish interacting gauge closure.

Finite reports are not continuum acquisition

A finite report may contain the inputs (α, β, r), the current convention, a formula identifier, an approximation, and its error status. The I-B/En09-A destructive supplied-report interface is

$$\mathscr{M}(X) = \sum_j \mathrm{lvertj}, w_j\rangle\mathrm{langlej}, w_j|_R \otimes D_j X D_j^\dagger, \qquad D_j = \langle\tau_j|_C \otimes I_A.$$

It is completely positive and trace preserving because

$$\sum_j D_j^\dagger D_j = I_{CA}.$$

Expanding X in the $\{\tau_j\}$ basis gives

$$\mathrm{Tr}_R \mathscr{M}(X) = \mathrm{Tr}_C X,$$

also after an untouched reference is adjoined [12, 20, 19]. The output contains the surviving A, not the consumed quantum clock C. It reports installed words; it does not evaluate an unknown vacuum current by measurement.

The finite-test coverage distinction of Oai282-A [21] also applies at the logical level. Finitely many linear evaluations cannot determine a general distribution in an infinite-dimensional class. For currents, an explicit infinite-dimensional test class consists of

$$j^\mu = \partial_\nu A^{\nu\mu}, \qquad A^{\nu\mu} = -A^{\mu\nu} \in C_c^\infty$$

in a regular flat chart. These are conserved, and disjoint compact supports give arbitrarily large linearly independent families. Finite evaluation maps have nontrivial kernels there. This example is not asserted to be a

difference of two Friedrichs-state currents. It shows why finite tests alone cannot prove a general continuum identity.

Evidence gate

We use the following evidence labels:

\texttt{ANALYTIC} Established by the displayed proof, with cited analytical inputs identified.

\texttt{SUPPLIED} An inherited computational report, not independently executed.

\texttt{UNCERTIFIED} A computational enclosure or verification claim whose required artifacts or total error bounds have not been supplied.

No numerical current runs, raw current data, or execution logs were supplied for this companion, and none are claimed. In particular, the numerical discrepancies reported for the Wick square and stress tensor in I-B-VP-v2 are not relabeled as checks of the current.

Following XVI-O1-ST's residual-retention discipline [19], a future VERIFY report must state its actual scope and retain at least:

(source/version, current sign and component,
parameter encodings, frame and lift, raw paired values,
precision, cutoffs, tail and input bounds, residual, status).

Agreement at shared rounded inputs is not an input-error bound. For the closed form, input propagation can use the exact derivatives

$$\partial_\alpha \mathscr{J}_\varphi = -\frac{2}{\alpha}\mathscr{J}_\varphi, \qquad \partial_r \mathscr{J}_\varphi = -\frac{2}{r}\mathscr{J}_\varphi,$$

$$\partial_\beta \mathscr{J}_\varphi = \frac{-6\beta^2 + 6\beta - 1}{12\pi^2\alpha^2 r^2}.$$

Quadrature or mode-tail errors require separate certificates. At a zero of the current, relative residuals may be inapplicable.

A successful test that preserves a known discrepancy does not make the discrepancy vanish. No numerical

execution, independent rebuild, formal verification, or successful compiler run is asserted in this manuscript.

Ledger update and remaining questions

begingroup 1.1 toprule Extension/state Current status Boundary

midrule endfirsthead toprule Extension/state Current status Boundary

midrule endhead bottomrule endfoot

Friedrichs, charged distinguished spectral ground state Constructed and evaluated Theorem ; all $0 < \alpha < 1$, $0 \le \beta < 1$, on $r > 0$.

Friedrichs, untwisted complex spectral ground state Zero current The covariance, Wick square, and stress tensor need not vanish.

Twisted non-Friedrichs E_0, E_λ **Open** No state or current evaluation is supplied here. Nonnegativity alone does not settle infrared finiteness.

Massless untwisted logarithmic L_L Outside the nonnegative square-root prescription Its transverse negative eigenvalue generates a negative longitudinal spectral band.

Mass-shifted logarithmic or negative-λ families **Open** The positivity and threshold infrared qualifications of I-B-G/I-B-VP remain.

Thermal states, spinors, constrained gauge systems **Open** No such state or constrained apex domain is constructed.

endgroup

The update to I-B-VP-v2 is restricted to the current-evaluation part of V4. A direct comparison with one current reference has also been supplied in Section ; a broader literature audit is still outside the result. The remaining scopes are unchanged:

1. Open: prescribed distributional extensions of the local observables across $\mathscr{W}$, including apex-supported counterterms. No such extension of the current is selected here.

2. Open: a complete global microlocal-spectrum analysis on the regular non-globally-hyperbolic host.

3. Open: the non-Friedrichs state and local-observable families, including massive logarithmic thresholds.

4. Evaluated here: the charged Friedrichs spectral ground-state current. A comprehensive current-literature audit remains a separate task.

5. Open: an apex curvature-coupling model selecting or modifying the extension.

6. Open: the corresponding massive-field companion construction and observable calculations.

7. Open: spinors and constrained gauge systems.

8. Open: thermal and other non-ground states.

Here **Open** means unresolved in this manuscript or the stated companion scope, not unknown throughout the literature.

Behaviour at the apex. The current is a smooth one-form and therefore a distribution on M. Its orthonormal component behaves as r^{-3} when the coefficient is nonzero. It does not define an ordinary locally integrable vector field across the completed transverse origin. This does not preclude distributional extensions; it means an extension prescription and its ambiguities have not been chosen. Regular-part conservation alone does not choose them.

Non-identifications and conclusion

The geometry, character, extension, and state prescription are inputs. Conservation of this current does not derive the cone or impose a metric field equation. The stress-tensor non-sourcing separations in I-B prop:not-source and X-A-ST prop:noeinstein remain applicable in their stated scopes [12, 15].

A nonzero current could enter a separately declared semiclassical Maxwell model, but no Maxwell equation is imposed or solved here. In particular, the flat background connection is not asserted to solve a self-consistent sourced Maxwell equation with the computed expectation value. There is no electromagnetic backreaction or

Einstein backreaction claim.

The result is a free-field one-point expectation value, not a current–current product, a dynamical response hierarchy, an interacting Ward theorem, or a conical BRST construction. No external OpenAI catalogue family or documentary separation is used as a mathematical premise. No corpus-wide nonexistence claim is made.

No standing downstream status is updated. O.1 remains wherever it was not independently closed; XVI-N is not discharged, XV-B's quantum-admission exit is not opened, and is not promoted by this calculation [19, 20]. These programme qualifications do not deny the explicit free-field current proved here.

Conclusion. The complex regular covariance contains an odd first angular jet that the Wick square and stress tensor do not retain. Its point-split evaluation gives the exact current . Analytic regularity controls coincidence for all cone angles, and finite-character images independently check the sign. Together with I-B-VP's even observables, this completes the three specified local one-point functions for the charged Friedrichs spectral ground state. It does not complete the state's higher products, its global microlocal analysis, or an apex or dynamical completion.

appendix

A differentiated image–diffraction identity

This appendix records a second representation, with its differentiation restriction explicit. It is not needed for Theorem .

For a chosen angular lift define

$$\mathcal{K}(\theta) = \{k \in \mathbb{Z} : |\theta - 2\pi\alpha k| < \pi\},$$

$$R_k^2 = r^2 + r'^2 - 2rr'\cos(\theta - 2\pi\alpha k) + \Delta z^2, \qquad \rho(u)^2 = r^2 + r'^2 + 2rr'\cosh u + \Delta z^2.$$

The diffraction weight is

$$\Phi_\beta(u,\theta) = \sum_{n \in \mathbb{Z}} \sin(\pi\nu_n) e^{-\nu_n u} e^{i(n+\beta)\theta/\alpha}.$$

I-B-G's angular summation and the Euclidean heat integral give [13, thm:closed] [14, prop:contour]

$$G_{E,\beta} = \sum_{k \in \mathcal{K}(\theta)} \frac{e^{2\pi i \beta k}}{4\pi^2 (R_k^2 + \Delta\tau^2)} - \frac{1}{\pi\alpha} \int_0^\infty \frac{\Phi_\beta(u,\theta)}{4\pi^2 (\rho(u)^2 + \Delta\tau^2)} \, \mathrm{d}u.$$

Assume here that $\frac{1}{2\alpha} \notin \mathbb{N}$, so zero angular separation is not an image boundary. Define the real weight

$$\Xi_\beta(u) = -i\alpha \, \partial_\theta \Phi_\beta(u,0) = \sum_n (n+\beta) \sin(\pi \nu_n) e^{-\nu_n u}, \qquad u > 0.$$

Then

$$\mathcal{J}_\varphi = -\frac{\alpha}{4\pi^2 r^2} \sum_{0 < k < 1/(2\alpha)} \frac{\sin(2\pi\beta k)\cos(\pi\alpha k)}{\sin^3(\pi\alpha k)} + \frac{1}{8\pi^3 \alpha r^2} \int_0^\infty \frac{\Xi_\beta(u)}{\cosh^2(u/2)} \, \mathrm{d}u.$$

Its value is the polynomial in .

To justify , choose a small angular neighbourhood of zero containing no image boundary. The image set is constant there. The closed geometric-series expression for Φ_β in I-B-G lem:Phi has denominators bounded away from zero at $u = 0$, also after one angular derivative. At infinity, the differentiated series decays exponentially. The diffraction denominator grows as e^u, uniformly on compact regular spatial sets. Differentiation under the integral is therefore justified on this neighbourhood. Apply $2i\alpha\partial_\theta$, set coincidence, pair the $k, -k$ image terms, and use $\rho(u)^2 = 4r^2\cosh^2(u/2)$.

When $\alpha > 1/2$, the image sum is empty, so the full current is represented by the differentiated diffraction term. This is a representation of a stationary spectral observable, not a claim about an emitted retarded signal.

Why joint integrability alone is insufficient. At an image boundary, local integrability of $\Phi_\beta(u,\theta)$ does not by itself justify pointwise differentiation under its integral. Angular derivatives can worsen the boundary singularity, and changing image indicators also contribute distributionally. For such parameters, use the differentiated *total* distribution in and its analytic remainder from Proposition , or use the mode proof. No separate pointwise

boundary prescription is introduced.

References

[1] L. Sriramkumar, Fluctuations in the current and energy densities around a magnetic-flux-carrying cosmic string, *Classical and Quantum Gravity* **18** (2001), 1015–1025. DOI: 10.1088/0264-9381/18/6/304. https://arxiv.org/abs/gr-qc/0011074. Used for the explicit current comparison: equations (2.10), (2.12), (2.13), and (A2).

[2] Yu. A. Sitenko and N. D. Vlasii, Induced vacuum current and magnetic field in the background of a cosmic string, *Classical and Quantum Gravity* **26** (2009), 195009. DOI: 10.1088/0264-9381/26/19/195009. https://arxiv.org/abs/0909.0405. Historical and analytical context; its Maxwell response is not imported.

[3] E. R. Bezerra de Mello, V. B. Bezerra, A. A. Saharian, and H. H. Harutyunyan, Vacuum currents induced by a magnetic flux around a cosmic string with finite core, *Physical Review D* **91** (2015), 064034. DOI: 10.1103/PhysRevD.91.064034. https://arxiv.org/abs/1411.1258. Cited for the distinction between idealized and finite-core current problems.

[4] V. Balakumar and E. Winstanley, Hadamard renormalization for a charged scalar field, *Classical and Quantum Gravity* **37** (2020), 065004. DOI: 10.1088/1361-6382/ab6b6e. https://arxiv.org/abs/1910.03666. Used for charged point splitting and the current renormalization ambiguity, especially equation (5.6).

[5] C. Klein and J. Zahn, Renormalized charged scalar current in the Reissner–Nordström–de Sitter spacetime, *Physical Review D* **104** (2021), 025009. DOI: 10.1103/PhysRevD.104.025009. https://arxiv.org/abs/2104.06005. Cited for local gauge-covariant current point splitting and its conserved-current ambiguity, not for its black-hole state.

[6] B. S. Kay and U. M. Studer, Boundary conditions for quantum mechanics on cones and fields around cosmic strings, *Communications in Mathematical Physics* **139** (1991), 103–139. DOI: 10.1007/BF02102731.

[7] G. N. Watson, *A Treatise on the Theory of Bessel Functions*, 2nd edition, Cambridge University Press, 1944.

[8] L. Hörmander, *The Analysis of Linear Partial Differential Operators I: Distribution Theory and Fourier Analysis*, 2nd edition, Springer, 1990. Used for elliptic regularity of distributional solutions.

[9] C. B. Morrey, Jr., and L. Nirenberg, On the analyticity of the solutions of linear elliptic systems of partial differential equations, *Communications on Pure and Applied Mathematics* **10** (1957), 271–290. DOI: 10.1002/cpa.3160100204.

[10] R. M. Wald, *Quantum Field Theory in Curved Spacetime and Black Hole Thermodynamics*, University of Chicago Press, 1994. Cited for the spectral one-particle and quasifree construction; no global-hyperbolicity theorem is applied to the entire regular cone.

Cheung companion records. The following unpublished manuscripts and supplied excerpts are cited by identifier, version, and source label. Documentary use does not assert independent verification of an entire record, publication, compiler execution, or formal proof checking.

[11] N. Cheung, *The Philosophy of Enter: Destructive Readout, Quantum Recovery, and the Refusal of Identification*, Cheung Ngo Medical Limited (2026). Framing essay; not a theorem premise. Programme note: https://enter.grok.me/enter.

[12] N. Cheung, *I-B-v1: A Finite Enter Spacetime: Calibrated Destructive Time Readout on a Prescribed Conical Product*, version 1, Enter Calculus Companion. Used: the prescribed host, thm:instrument, and prop:not-source. https://enter.grok.me/browse/I-B. Companion record; not a Scholar-indexed journal article.

[13] N. Cheung, *I-B-G: Intrinsic and holonomy-twisted Green operators on a prescribed conical spacetime: Self-adjoint extensions, diffraction, finite propagation, and supported transfer from flat-completion laboratories*, version 2, October 2026. Supplied source I-B-G-v2.tex, Enter Calculus Companion. Used: ass:host, ass:product, thm:positivity, thm:closed, lem:Phi, cor:structure, thm:transfer, and ass:transfer-contract. https://enter.grok.me/browse/I-B-G. Companion record; not a Scholar-indexed journal article.

[14] N. Cheung, *I-B-VP: Vacuum polarization of the Friedrichs ground state on a prescribed conical spacetime:*

Local Hadamard subtraction, holonomy, and the renormalized stress tensor, version 2, October 2026. Supplied source I-B-VP-v2.tex, Enter Calculus Companion. Used: lem:infrared, prop:euclidean, def:state, thm:analytic, thm:hadamard, thm:phi2, thm:T-complex, and prop:contour. Its current-evaluation item V4 is the specific successor target addressed here. https://enter.grok.me/browse/I-B-VP. Companion record; not a Scholar-indexed journal article.

[15] N. Cheung, *X-A-ST*, version 1, Enter Calculus Companion. Used: prop:signature-ledger, prop:noeinstein, and the supported-completion scope. https://enter.grok.me/browse/X-A-ST. Companion record; not a Scholar-indexed journal article.

[16] N. Cheung, *X-B-ST: One scalar retarded map on the X-A-ST host*, version 1, Enter Calculus Companion. Used: ass:host, def:supported, and the equation/Hessian normalization and retained-shadow distinctions. https://enter.grok.me/browse/X-B-ST. https://enter.grok.me/browse/X-A-ST. Companion record; not a Scholar-indexed journal article.

[17] N. Cheung, *X-C-ST: Second-order scalar completion on the inherited X-A-ST/X-B-ST host*, version 1, Enter Calculus Companion. Cited for compact intermediate-source support obligations and unchanged product boundaries. https://enter.grok.me/browse/X-C-ST. https://enter.grok.me/browse/X-A-ST. https://enter.grok.me/browse/X-B-ST. Companion record; not a Scholar-indexed journal article.

[18] N. Cheung, *XI-E*, version 1, Enter Calculus Companion. Supplied excerpts: def:domain and thm:star. Its declared polynomial Hadamard product domain is not a positive-state selection or a conical interacting construction. https://enter.grok.me/browse/XI-E. Companion record; not a Scholar-indexed journal article.

[19] N. Cheung, *XVI-O1-ST*, version 5, XVI-O1-ST-v5.tex, Enter Calculus Companion. Used for the complete supplied-report interface and the integrity-aware distinction between proof, retained discrepancy, and execution evidence. No unrelated closure status is imported. https://enter.grok.me/browse/XVI-O1-ST. Companion record; not a Scholar-indexed journal article.

[20] N. Cheung, *En09-A*, version 1, Enter Calculus Companion. Supplied source excerpts concerning destructive

finite reporting, record–residual separation, and standing non-identifications.

https://enter.grok.me/browse/En09-A. Companion record; not a Scholar-indexed journal article.

[21] N. Cheung, *Oai282-A*, version 1, Enter Calculus Companion. Used for the finite-test/coverage distinction identified by prop:coverage and prop:invisible in the supplied companion material. No external catalogue theorem is used as a premise. https://enter.grok.me/browse/Oai282-A. Companion record; not a Scholar-indexed journal article.

[22] N. Cheung, *Black-Hole Information as an Access Problem: Enter, Quantum Recovery, and Complete Residuals*, figshare (2026). DOI: 10.6084/m9.figshare.34335024.v1.

https://doi.org/10.6084/m9.figshare.34335024.v1.

[23] N. Cheung, *Operational Wormhole Certificates—Existence, Dimension-Dependent Scaling, Operational Functions, and Composition*, figshare (2026). DOI: 10.6084/m9.figshare.34549575.v1.

https://doi.org/10.6084/m9.figshare.34549575.v1.

[24] N. Cheung, *Enter, Quantum Magic, and Quantum Gravity: Readout, Recovery, and the Limits of Geometric Response*, figshare (2026). DOI: 10.6084/m9.figshare.34335582.v1.

https://doi.org/10.6084/m9.figshare.34335582.v1.

[25] N. Cheung, *Operational Consistency Does Not Select Lorentzian Geometry*, figshare (2026). DOI: 10.6084/m9.figshare.34441758.v1. https://doi.org/10.6084/m9.figshare.34441758.v1.

[26] N. Cheung, *Realization of Relational Time by Destructive Enter on a Clock Register*, figshare (2026). DOI: 10.6084/m9.figshare.34442907.v1. https://doi.org/10.6084/m9.figshare.34442907.v1.