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I-B-VP-1-Ex1 Worldsheet domains, core diagnostics, and first-order apex dependence

A proof-level extension and corrective audit of prescribed conical one-point finite parts

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Abstract

We strengthen and qualify the apex constructions of I-B-VP-1 on a prescribed conical product, without changing its background, state-selection inputs, or standing interaction boundaries. The principal results concern domains rather than formal apex values. For the Friedrichs scalar field, we construct the timelike-worldsheet restriction as a strong limit on the finite-particle domain, identify its one-particle range and exact two-point function, and prove that the restriction is zero in nontrivial holonomy sectors. For deficient modes, weighted boundary values have an explicit positive matrix Källén–Lehmann representation; their Wick squares exist as worldsheet-smeared operators, but are not themselves generalized free fields. A second result supplies an analytic $-1/(4\pi)$ double-logarithmic coefficient for an explicitly normalized exterior core diagnostic, and proves negative-coupling instability directly on the core geometry, avoiding an unjustified transfer of a Euclidean weak-binding theorem. At first order in $\lambda\varphi^4$, two disjoint local subtractions define the tadpole, its finite constant is absorbed by an explicit contact-coordinate change, and the off-apex response to the free contact is calculated exactly. Finally, a Mellin criterion makes the $U(2)$ resonance exclusion precise, an explicit boundary-chart example develops a logarithm, and a supported tensor counterexample shows why the previously stated broad Killing–homothety uniqueness conclusion requires an additional normalization condition. The potential significance is a sharper distinction between bulk-derived boundary operators, observable contact limits, and spatial operator limits. No interacting

apex algebra, universal core selection, resonant classification, or all-orders interaction is claimed.

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Book expansion: superseded → corrected

toprule Superseded claim (documentary) Corrected statement (this sheet)

midrule Unqualified boundary-field transfer Domain- and Gaussianity-qualified estimates

Physical-core closure implied Physical-core stop retained beside the claim

KH confirmation as theorem KH stops remain UNCERTIFIED/OPEN

bottomrule Emphasize **operator domain** and **Gaussianity** before any boundary-field reading. Keep physical-core and KH stops adjacent to the claims they qualify.

Purpose, attribution, and the prescribed Enter background

What this extension does

The supplied source I-B-VP-1-v4.tex constructs one-point finite parts on a declared smooth completion, repairs their Ward defects, and classifies their contact freedom in a specified class [18]. The present companion follows the requested priority order:

1. worldsheet restriction and deficient-mode boundary fields;

2. core flow and curvature coupling;

3. the first-order scalar tadpole;

4. non-diagonal $U(2)$ asymptotics and resonance;

5. the optional Dirac comparison;

6. an explicit reliability and stopping ledger. A stage is completed here by a proved statement on a declared domain or by an explicit stop. An explicit stop is not described as a solution of the corresponding open problem.

Two corrections are essential. First, existence of a boundary linear field does not make its Wick square Gaussian. Second, noninteger leading powers alone do not prove uniqueness under the Killing–homothety Ward prescription. Section gives a concrete supported counterexample to that inference.

The analysis below does not take I-B-VP-AB or I-B-VP-SAE as analytical premises. They remain comparison records [23, 24]. The spectral kernels and finite-dimensional boundary matrices needed here are specified and calculated explicitly.

Enter: an interface discipline, not a source equation

Cheung's philosophy of Enter distinguishes destructive readout, quantum return, installed descriptions, and physical identification [14]. We use that distinction only as interpretive background. It is not an analytical premise.

In the finite interface of I-B, a clock basis $\{|\tau_j\rangle\}$ and a surviving finite register A give

$$D_j = \langle \tau_j|_C \otimes I_A, \qquad \mathscr{M}(X) = \sum_j \mathrm{proj}\, j_R \otimes D_j X D_j^\dagger.$$

Kraus completeness gives

$$\sum_j D_j^\dagger D_j = I_{CA}, \qquad \mathrm{Tr}_R \circ \mathscr{M}(X) = \mathrm{Tr}_C X,$$

also after adjoining an untouched reference. The quantum clock names are not returned [15, thm:instrument].

This finite channel neither acquires a continuum worldsheet field nor selects a field domain, state, or apex normalization. The geometric time coordinate is not consumed. No metric update or Einstein equation follows from the channel. The distinction between a supplied finite report and field acquisition is retained throughout.

Host, smooth completion, and density

Fix

$$0 < \alpha < 1, \qquad 0 \le \beta < 1.$$

The regular host is

$$M = \mathbb{R}_t \times (0,\infty)_r \times S^1_\varphi \times \mathbb{R}_z, \qquad g = -\mathrm{d}t^2 + \mathrm{d}r^2 + \alpha^2 r^2 \mathrm{d}\varphi^2 + \mathrm{d}z^2.$$

Its completed defect is the worldsheet

$$\mathscr{W} = \mathbb{R}_t \times \{0\} \times \mathbb{R}_z.$$

This is the prescribed conical product of I-B, not a geometry derived from a finite record [15].

As in I-B-VP-1, declare

$$\overline{M} = \mathbb{R}^{1+1}_{y} \times \mathbb{R}^{2}_{x}, \qquad y = (t, z), \qquad x = (r\cos\varphi, r\sin\varphi),$$

with auxiliary smooth metric

$$\bar{g} = -\mathrm{d}t^2 + \mathrm{d}z^2 + (\mathrm{d}x^1)^2 + (\mathrm{d}x^2)^2$$

and pairing density

$$\mathrm{d}\Omega = \alpha\, \mathrm{d}t\, \mathrm{d}z\, \mathrm{d}^2x.$$

Write

$$\langle \delta_{\mathscr{W}}, f \rangle = \int_{\mathbb{R}^2} f(y, 0)\, \mathrm{d}^2y, \qquad \sigma_{\alpha} = 2\pi\alpha.$$

Thus $\delta_{\mathscr{W}} = \alpha^{-1}\delta^{(2)}(x)$ relative to $\mathrm{d}\Omega$.

These smooth coordinates are not developing coordinates. The conical metric is not assigned a smooth value at $x = 0$. No product of that metric with a delta derivative is used.

Holonomy and the choice of frame

A charged scalar in the parallel lifted frame obeys

$$\Phi(\varphi + 2\pi) = e^{2\pi\mathrm{i}\beta}\Phi(\varphi).$$

For transverse Sobolev statements we use the periodic gauge

$$\widetilde{\Phi} = e^{-\mathrm{i}\beta\varphi}\Phi.$$

Its angular modes are $e^{\mathrm{i}n\varphi}$, and the radial orders are

$$q_n = n + \beta, \qquad \nu_n = \frac{|q_n|}{\alpha}.$$

The periodic gauge is a declared singular gauge on the punctured plane. We do not assert that nontrivial flat holonomy extends as a smooth flat bundle through the apex. The zero restriction proved below is independent of

multiplication by a bounded phase.

The retained one-point prescription

Put

$$\Delta = \partial_1^2 + \partial_2^2, \qquad E = x^i \partial_i, \qquad K_{ij} = \partial_i \partial_j - \delta_{ij} \Delta, \qquad \varepsilon = \begin{pmatrix} 0 & -1 \\ 1 & 0 \end{pmatrix}.$$

Definition (Generating finite part). For $L > 0$ define

$$\operatorname{langle} U_L, f \rangle = \lim_{\varepsilon \operatorname{downarrow} 0} \left[\int_{r > \varepsilon} \frac{f}{r^2} \, \mathrm{d}\Omega + \sigma_\alpha \log(\varepsilon / L) \int_{\mathbb{R}^2} f(y, 0) \, \mathrm{d}^2 y \right].$$

Lemma (Existence and residue). *The limit exists strongly in $\mathcal{D}'(\overline{M})$. On each fixed compact test support its sharp-cutoff error is $O(\varepsilon^2 p_2(f))$. Moreover,*

$$(E + 2) U_L = \sigma_\alpha \delta_{\mathscr{W}}, \qquad U_{L'} - U_L = -\sigma_\alpha \log(L'/L) \delta_{\mathscr{W}}.$$

Proof. Angular averaging gives

$$\frac{1}{2\pi} \int_0^{2\pi} f(y, r\omega) \, \mathrm{d}\varphi = f(y, 0) + O(r^2 p_2(f)).$$

Subtracting the constant term leaves an integrable radial integrand with measure $\sigma_\alpha \, \mathrm{d}r/r$. Its omitted integral below ε is $O(\varepsilon^2 p_2(f))$, uniformly on bounded sets of tests. This proves strong convergence. For the Euler identity, $\langle (E + 2) U_L, f \rangle = -\operatorname{langle} U_L, Ef \rangle$; radial integration gives $\sigma_\alpha \int f(y, 0) \mathrm{d}^2 y$. The scale difference follows directly from the subtraction.

The regular scalar, stress, and current coefficients may be written

$$F^\circ = \frac{s}{r^2}, \quad T^\circ = r^{-4} \operatorname{diag}(a, b, -3b, a), \quad J^\circ_\varphi = \frac{j}{r^2},$$

where

$$q = \xi - \frac{1}{6}, \qquad a = h_0 + 4nqs, \qquad b = h_0 - 2nqs.$$

Here n is the real-component stress multiplicity. The coefficients are prescribed regular data. The Friedrichs expressions are compared with I-B-VP [17, thm:friedrichs].

The derivative-compatible extensions are

$$\widehat{F}_{L} = sU_{L},$$

$$\widehat{T}^{A}{}_{B} = \frac{a}{4}\delta^{A}_{B}\Delta U_{L}, \qquad \widehat{T}^{i}{}_{j} = \frac{b}{2}K_{ij}U_{L},$$

$$\widehat{J}^{i} = -\frac{j}{2\alpha^{2}}\varepsilon^{i}{}_{k}\partial_{k}U_{L}.$$

Mixed blocks and longitudinal current components vanish.

Define

$$\mathscr{I}[G]^{A}{}_{B} = \delta^{A}_{B}\Delta G, \qquad \mathscr{I}[G]^{i}{}_{j} = -K_{ij}G,$$

and

$$\mathscr{H}^{A}{}_{B} = \delta^{A}_{B}\Delta\delta_{\mathscr{W}}, \qquad \mathscr{H}^{i}{}_{j} = 2K_{ij}\delta_{\mathscr{W}}.$$

Proposition (Retained contact coordinates and no-go). *In the rotation-covariant, longitudinally translation- and boost-invariant, reference-symmetric, almost-homogeneous one-point class of I-B-VP-1, with its reflection and charge-conjugation conditions, the permitted differences are*

$$\Delta F = c\delta_{\mathscr{W}}, \qquad \Delta T = \tau\mathscr{H} + nqc\,\mathscr{I}[\delta_{\mathscr{W}}], \qquad \Delta J^{i} = w\varepsilon^{i}{}_{k}\partial_{k}\delta_{\mathscr{W}}.$$

The generic charged dimension is three, and the neutral dimension is two. Charge conjugation can remove w in an invariant self-conjugate state.

Weighted transverse dilations translate the coordinates by

$$(c,\tau,w)\text{longmapsto}\,(c,\tau,w) + \sigma_{\alpha}\log\lambda\left(s, \frac{h_{0}}{4}, -\frac{j}{2\alpha^{2}}\right).$$

If this direction is nonzero there is no dilation-fixed point in this class.

Proof. Supported differences are finite normal delta jets. The scaling bounds and almost-homogeneity retain orders $0, 2, 1$ for F, T, J, respectively. Rotation covariance, reference symmetry, and the stated discrete symmetries leave

$$\Delta T^{A}{}_{B} = u\delta^{A}_{B}\Delta\delta_{\mathscr{W}}, \qquad \Delta T^{i}{}_{j} = vK_{ij}\delta_{\mathscr{W}}.$$

Conservation has already been imposed in the second expression. The trace relation is

$$2u - v = 3nqc.$$

Writing $u = \tau + nqc$ gives $v = 2\tau - nqc$. The conserved current difference is the rotated first derivative in . This proves the retained count.

Equation gives the dilation shift. The scalar coordinate is s and the longitudinal stress coordinate satisfies $\tau + nqs = a/4$, hence $\tau = h_{0}/4$. A nonzero translation has no fixed point.

Remark (No enlargement of this classification). Proposition is deliberately restricted. It is not a contact count for arbitrary non-diagonal data, arbitrary worldsheet dependence, or the broader Killing–homothety prescription. Section examines a different class without altering this proposition.

Stage A: the Friedrichs worldsheet restriction

Spectral convention and test space

Let $\omega_{\lambda,k} = (\lambda^{2} + k^{2})^{1/2}$. We normalize the Friedrichs two-point function by

$$W_{\beta}(X, X') = \frac{1}{8\pi^{2}\alpha} \sum_{n \in \mathbb{Z}} e^{\mathrm{i}q_{n}(\varphi - \varphi')} \int_{\mathbb{R}} \mathrm{d}k \int_{0}^{\infty} \frac{\lambda\, \mathrm{d}\lambda}{\omega_{\lambda,k}} e^{-\mathrm{i}\omega_{\lambda,k}(t - t' - \mathrm{i}0) + \mathrm{i}k(z - z')} J_{\nu_{n}}(\lambda r) J_{\nu_{n}}(\lambda r').$$

This formula specifies the state used in this section. Its normalization is the ordinary scalar normalization at $\alpha = 1, \beta = 0$.

For $h \in \mathcal{S}(\mathbb{R}^{1+1})$, use

$$\widehat{h}(\omega, k) = \int_{\mathbb{R}^{2}} e^{\mathrm{i}\omega t - \mathrm{i}kz} h(t, z)\, \mathrm{d}t\, \mathrm{d}z.$$

Compact smooth tests are included.

Let $\mathscr{H}_1$ be the one-particle completion defined by , and let $\mathscr{F}_s(\mathscr{H}_1)$ be its bosonic Fock space. The finite-particle domain is

$$\mathcal{D}_{\mathrm{fin}} = \bigcup_{N \geq 0} \mathrm{bigoplus}_{m=0}^{N} \mathscr{H}_1^{\otimes_s m}.$$

For a one-particle vector v,

$$\|(a(v) + a^*(v))\Psi\| \leq 2\|v\| \, \|(N+1)^{1/2}\Psi\|.$$

The Sobolev scale

For a Hilbert space B, $H^s(\mathbb{R}^2_x; B)$ denotes the Bessel-potential space with norm

$$\|u\|^2_{H^s(\mathbb{R}^2; B)} = \int_{\mathbb{R}^2} (1 + |\zeta|^2)^s \left\| \widehat{u}(\zeta) \right\|_B^2 \mathrm{d}^2\zeta.$$

Local versions are defined with smooth compact cutoffs.

Lemma (The trace used here). *For $s > 1$, point evaluation extends continuously to*

$$\gamma_0 : H^s(\mathbb{R}^2_x; B) \to B.$$

More generally, for every $a \in \mathbb{R}$,

$$\gamma_0 : H^a(\mathbb{R}^2_y; H^s(\mathbb{R}^2_x; B)) \to H^a(\mathbb{R}^2_y; B)$$

is bounded and onto. For a cutoff transverse monomial $r^\gamma e^{\mathrm{i}m\varphi}$, $\gamma > 0$, one has

$$\chi(r) r^\gamma e^{\mathrm{i}m\varphi} \in H^s(\mathbb{R}^2) \quad \text{for} s < 1 + \gamma.$$

No converse is asserted when the expression is smooth.

Proof. Fourier inversion and Cauchy–Schwarz give

$$\mathrm{lVertu}(0)\|_B \leq C \Big(\int_{\mathbb{R}^2} (1 + |\zeta|^2)^{-s} \mathrm{d}^2\zeta \Big)^{1/2} \|u\|_{H^s},$$

and the integral is finite exactly when $s > 1$. A fixed χ with $\chi(0) = 1$ gives the bounded right inverse $b \mapsto \chi b$. Applying the same argument after Fourier transformation in y proves the mixed-space statement.

For , use a dyadic partition near zero. A piece on $r$asymp 2^{-j} has H^s size bounded by $C2^{-j(\gamma+1-s)}$; the square sum converges for $s < 1 + \gamma$. Smooth polynomial cases can have better regularity, which is why an ``if and only if'' statement without exceptions is incorrect.

For fixed h, let $v_{h,x} \in \mathscr{H}_1$ be the creation vector of the field smeared by h in (t, z) and evaluated transversely at $x \neq 0$.

Lemma (Worldsheet smoothing gives a transverse trace domain). *In the periodic gauge, $x \mapsto v_{h,x}$ has a local $\mathscr{H}_1$-valued H^s extension near zero for*

$$1 < s < \min\{2, 1 + \nu_*\}, \qquad \nu_* = \frac{\min(\beta, 1-\beta)}{\alpha}, \quad 0 < \beta < 1.$$

For $\beta = 0$, it has such an extension for every $1 < s < 2$. In the latter case the zero angular mode is smooth at zero.

Proof. For $\lambda r \le 1$, the Bessel series gives

$$J_{\nu_n}(\lambda r) = \frac{(\lambda r/2)^{\nu_n}}{\Gamma(1+\nu_n)}\bigl[1 + O((\lambda r)^2)\bigr].$$

The periodic angular factor is $e^{in\varphi}$. Lemma applies to each positive-order leading monomial. For $\beta = 0$, the $n = 0$ expansion is a power series in r^2; all other orders are at least $1/\alpha > 1$.

The angular sum of the series estimates is locally convergent: the gamma denominators dominate every fixed polynomial in ν_n. On $\lambda r > 1$, the usual oscillatory Bessel estimates and their fixed-order derivatives have at most polynomial growth in spectral energy and angular order. The rapid decay of $\widehat{h}(\omega_{\lambda,k}, k)$ absorbs these polynomials. One may therefore first truncate the angular and spectral sums, apply the dyadic estimate in the preceding proof, and pass to the limit in the asserted H^s norm. All low-energy integrals are finite because $\lambda\, d\lambda\, dk/\omega_{\lambda,k}$ is locally integrable. Classical Bessel series and estimates may be found in [9, Chapter 10].

Remark (The domain is not an unsmeared Sobolev assertion). Lemma concerns the one-particle-valued function after smearing in the worldsheet variables. It does not assert that the unsmeared two-point distribution is an ordinary Sobolev function jointly in all variables. That distinction avoids using a transverse regularity slogan as a substitute for a field-domain proof.

Restriction theorem and its range

Theorem (Friedrichs worldsheet restriction). *For $h \in \mathcal{S}(\mathbb{R}^{1+1})$, the limit*

$$\Phi_{\mathscr{W},\beta}(h) = \lim_{x \to 0} \Phi_{\beta}(h, x)$$

exists strongly on $\mathcal{D}_{\mathrm{fin}}$ in the periodic gauge.

For $\beta = 0$ it is a nonzero Gaussian generalized free field with

$$W_{\mathscr{W}}(t, z) = \frac{1}{4\pi^2 \alpha} \frac{1}{z^2 - (t - \mathrm{i}0)^2}$$

and

$$\| \Phi_{\mathscr{W},0}(h)\Omega \|^2 = \frac{1}{8\pi^2 \alpha} \int_{\mathbb{R}} \mathrm{d}k \int_{|k|}^{\infty} |\widehat{h}(\omega, k)|^2 \, \mathrm{d}\omega.$$

The map $h \mapsto v_{\mathscr{W},h}$ extends isometrically from the completion of this seminorm onto the zero-angular-mode one-particle subspace of the bulk Fock representation.

For $0 < \beta < 1$,

$$\Phi_{\mathscr{W},\beta}(h) = 0.$$

Precisely, $\mathrm{lVertv_}h, x\| \to 0$ *and $\Phi_{\beta}(h, x)\Psi \to 0$ for every $\Psi \in \mathcal{D}_{\mathrm{fin}}$. The same conclusions hold for normalized transverse approximate identities supported in shrinking disks.*

Proof. For each fixed λ, $J_0(\lambda r) \to 1$, whereas $J_{\nu_n}(\lambda r) \to 0$ for every $\nu_n > 0$. The spectral estimates in Lemma justify dominated convergence for the two-endpoint inner products $\mathrm{langlev_}h, x, v_{h,x'}\rangle$. They give convergence in $\mathscr{H}_1$, not only convergence of a scalar covariance. Equation then proves strong convergence on $\mathcal{D}_{\mathrm{fin}}$.

At $\beta = 0$, only $n = 0$ remains. Since $\lambda\,\mathrm{d}\lambda/\omega = \mathrm{d}\omega$ at fixed k, gives . Its inverse Fourier boundary value is . Gaussianity is inherited from the bulk quasifree representation.

The map $(\lambda, k) \mapsto (\omega_{\lambda,k}, k)$ identifies the zero-mode measure with Lebesgue measure on $\{\omega > |k|\}$, up to the constant in . Restrictions of Schwartz Fourier transforms are dense in this L^2 space. The completed range is therefore the entire zero-mode subspace.

For nontrivial holonomy every v_n is positive, so the same argument gives the zero vector. Averaging a norm-continuous one-particle limit against an approximate identity preserves the limit.

The Euclidean mixed endpoint formula is

$$G_F((y,x),(y',0)) = \frac{1}{4\pi^2\alpha}\frac{1}{r^2 + |y-y'|^2}.$$

It follows either from the surviving zero mode or from

$$\int_0^\infty \lambda J_0(\lambda r) K_0(\lambda D)\,\mathrm{d}\lambda = \frac{1}{r^2 + D^2}.$$

Corollary (No independent linear degree of freedom is added). *The field in Theorem is realized in the original bulk Fock space and uses its zero angular channel. It does not introduce an independent tensor factor. It does not select* (c, τ, w).

Proof. The isometric range statement identifies its creation vectors with a closed subspace of the original one-particle space. Changing the c-number contacts in changes none of these vectors or their covariance.

Remark (Form domain versus trace domain). The untwisted Friedrichs spatial form domain is comparable to $H^1(\mathbb{R}^3)$ in the Cartesian identification. An axis has zero capacity for this energy: a logarithmic radial cutoff has energy $O(1/|\log\varepsilon|)$. Thus the form domain itself has no bounded point trace in the two transverse variables. Theorem uses worldsheet smearing and spectral decay to enter the stronger transverse scale $s > 1$. It is not a consequence of form positivity alone. This separates the present result from I-B-G thm:positivity and from the spatial operator selection in I-B-VP-5 [16, 22].

Stage A continued: deficient-mode boundary fields

Boundary relation and resolvent normalization

Let $0 < \nu_j < 1$ be the finitely many deficient orders. In the two-channel case $j = 0, -1$. A radial function has boundary expansion

$$f_j(r) = a_j r^{-\nu_j} + b_j r^{\nu_j} + o(r^{\nu_j}).$$

We normalize the product boundary relation by

$$a \in V, \qquad P_V \mathrm{diag}\,(\nu_j) b = H_V a,$$

where $V \subseteq \mathbb{C}^d$, P_V is its orthogonal projection, and H_V is Hermitian and nonnegative on V. $V = \{0\}$ is Friedrichs. For the full chart $V = \mathbb{C}^d$, write $H = H_V$.

Set

$$A_j = 2^{\nu_j - 1}\Gamma(\nu_j), \qquad S_j = \frac{\pi}{2\sin(\pi\nu_j)}, \qquad d_j = \frac{S_j}{A_j^2} > 0.$$

For Euclidean longitudinal momentum $p > 0$, the correction to the Friedrichs radial resolvent has matrix

$$C(p) = (S + 2E(p)HE(p))^{-1}, \qquad E(p) = \mathrm{diag}\,(A_j p^{-\nu_j}),$$

in the full chart. Boundary charts are the corresponding limits of the inverse, or equivalently the compressed construction below.

Lemma (Direct boundary normalization check). *Formula follows from and the radial Green kernel*

$$I_{\nu_j}(pr_<)K_{\nu_j}(pr_>) + K_{\nu_j}(pr)C_{jk}(p)K_{\nu_k}(pr').$$

No companion Krein theorem is required for this calculation.

Proof. The small-argument coefficients are

$$I_\nu(pr) \sim I^0_\nu p^\nu r^\nu, \quad I^0_\nu = \frac{2^{-\nu}}{\Gamma(1+\nu)},$$

$$K_\nu(pr) \sim A_\nu p^{-\nu} r^{-\nu} + B_\nu p^\nu r^\nu, \quad B_\nu = 2^{-\nu-1}\Gamma(-\nu).$$

The gamma reflection identity gives

$$I^0_\nu S_\nu + B_\nu = 0, \qquad 2\nu I^0_\nu A_\nu = 1.$$

Substitution into $\operatorname{diag}(\nu) b = Ha$ yields $C^{-1} = S + 2EHE$. The Wronskian of I_ν, K_ν supplies the unit radial Green normalization. Compression to V is obtained by sending the boundary energy on $V^\perp$ to infinity. The Bessel-extension framework is also discussed in [4].

Positive matrix spectral measure

For $z \in \mathbb{C} \setminus (-\infty, 0]$, use the principal powers z^{ν_j} and define on V

$$B_V(z) = 2H_V + P_V \operatorname{diag}(d_j z^{\nu_j})|_V.$$

Let $\iota_V : V \hookrightarrow \mathbb{C}^d$ be the inclusion.

Theorem (Boundary covariance and positivity). *The weighted modal boundary values*

$$a_j(h) = \lim_{r \downarrow 0} r^{\nu_j} \frac{1}{2\pi} \int_0^{2\pi} e^{-\mathrm{i} q_j \varphi} \Phi(h, r, \varphi)\, \mathrm{d}\varphi$$

exist on a dense finite-particle spectral domain in the Fock representation of . Their Euclidean momentum covariance is

$$\widetilde{W}_a(z) = \frac{1}{2\pi\alpha}\, \iota_V B_V(z)^{-1} \iota_V^\dagger.$$

It has the matrix Stieltjes representation

$$\widetilde{W}_a(z) = \int_0^\infty \frac{\rho_a(t)}{z+t}\, \mathrm{d}t,$$

where

$$\rho_a(t) = \frac{1}{2\pi^2 \alpha}\, \iota_V B_V(-t + \mathrm{i}0)^{-1} \big[P_V \mathrm{diag}\,(d_j \sin(\pi \nu_j) t^{\nu_j})|_V \big] B_V(-t + \mathrm{i}0)^{-*} \iota_V^{\dagger} \geq 0.$$

Consequently the boundary fields are Gaussian generalized free fields on the test space $\mathcal{S}(\mathbb{R}^{1+1}; \mathbb{C}^d)$, *with Wightman covariance*

$$W_a(y) = \int_0^{\infty} \rho_a(t)\, \Delta_+^{(1+1)}(y; t)\, \mathrm{d}t.$$

Proof. Multiplying the two radial endpoints of the correction by $r^{\nu_j} (r')^{\nu_k}$ leaves $A_j A_k p^{-\nu_j - \nu_k} C_{jk}(p)$. The Friedrichs part vanishes under this weighted limit. In the full chart,

$$\mathrm{diag}\,(A_j p^{-\nu_j}) C(p) \mathrm{diag}\,(A_j p^{-\nu_j}) = (2H + \mathrm{diag}\,(d_j p^{2\nu_j}))^{-1}.$$

The two-dimensional Fourier convention converts the radial prefactor to $1/(2\pi\alpha)$. Compression gives .

For $\mathrm{Im}\, z > 0$, every $\mathrm{Im}\, z^{\nu_j}$ is positive. Thus $\mathrm{Im}\, B_V(z) > 0$ and $\mathrm{Im}\, B_V(z)^{-1} < 0$. The matrix is analytic off the negative axis, positive for $z > 0$, and vanishes at infinity. A keyhole contour applied entrywise gives ; its small circle vanishes because $\nu_{\max} < 1$. The identity

$$-\mathrm{Im}\, B^{-1} = B^{-1} (\mathrm{Im}\, B) B^{-*}$$

gives . This establishes spectral positivity, rather than inferring it merely from positivity at positive Euclidean momentum.

For completeness, the boundary limits are limits in the original Fock representation, not only an abstract reconstruction. First restrict spectral energy to a compact interval separated from zero. The Bessel expansions used in Lemma then converge uniformly and give convergence of the creation vectors in . For the omitted spectral ends,

$$\|\rho_a(t)\| \leq \begin{cases} C t^{-\nu_{\max}}, & 0 < t \leq 1, \\ C t^{-\nu_{\min}}, & t \geq 1. \end{cases}$$

Indeed the imaginary part of $B_V(-t + \mathrm{i}0)$ is bounded below by the minimum of $d_j \sin(\pi \nu_j) t^{\nu_j}$ times the identity. The inverse bound and $\|\mathrm{Im}\, B^{-1}\| \leq$ lVertB\^{}−1‖ give the estimate.

The corresponding scaled bulk Bessel eigenfunctions obey the same threshold bound up to fixed polynomial energy factors, uniformly for $0 < r < r_0$: use the two small-argument series when $\lambda r \le 1$ and the oscillatory estimates when $\lambda r > 1$. At threshold the remaining longitudinal integral grows at most as $1 + |\log t|$. Hence

$$\int_0^1 t^{-\nu_{\max}}(1 + |\log t|)\,\mathrm{d}t < \infty.$$

At high energy the Schwartz decay of $\widehat{h}$ absorbs the polynomial factors. Dominated convergence therefore removes both spectral truncations and proves convergence of the creation vectors. The creation-annihilation estimate proves the asserted operator limits on the dense finite-particle spectral domain. Gaussianity is inherited from the bulk quasifree state.

Remark (Meaning of generalized free field). Here the phrase means a Gaussian Wightman field whose higher vacuum functions are given by Wick pairing of . The two-point function and test class are part of the definition. The general spectral viewpoint is discussed in [3]; no AdS or brane interpretation is imported.

Corollary (Scale-invariant field and crossover). *For one channel with* $H = 0$,

$$W_{a,E}(y) = \frac{|y|^{2\nu-2}}{4\pi^2\alpha}, \qquad 0 < \nu < 1.$$

Its scaling dimension is $1 - \nu$.

For $H > 0$, *put* $M^{2\nu} = 2H/d_\nu$. *Then*

$$\widetilde{W}_a(p^2) = \frac{1}{2\pi\alpha d_\nu}\frac{1}{(p^2)^\nu + M^{2\nu}}.$$

The separated-point covariance has short-distance dimension $1 - \nu$ *and large-distance leading power of dimension* $1 + \nu$. *The constant term of its small-momentum expansion is a contact distribution and is not a separated-point long-distance term.*

Proof. For $H = 0$, use the two-dimensional Riesz transform

$$\int_{\mathbb{R}^2} \frac{\mathrm{d}^2p}{(2\pi)^2} e^{\mathrm{i}p\cdot y}(p^2)^{-\nu} = \frac{\Gamma(1-\nu)}{4^\nu\pi\Gamma(\nu)}|y|^{2\nu-2}$$

and the definition of d_ν. For $H > 0$, the ultraviolet asymptotic is proportional to $p^{-2\nu}$. At small momentum the first noncontact term is $-p^{2\nu}/M^{4\nu}$, whose Fourier transform away from zero is proportional to $|y|^{-2-2\nu}$. The positivity and meaning of both expansions follow from the exact spectral measure, not from a formal negative spectral density.

Proposition (Why the unweighted singular restriction fails). *If $a_j(h)$ has nonzero creation vector, the unweighted j-th modal field has no finite one-particle limit as r*downarrow 0.

Proof. The weighted creation vectors converge to a nonzero vector by Theorem . The unweighted norm therefore grows as $r^{-\nu_j}$. Failure follows from norm divergence, not merely from failure of a sufficient Sobolev trace criterion.

Stage A concluded: Wick squares and the c-number obstruction

Theorem (Worldsheet Wick square). *For the neutral Friedrichs field, the normal-ordered square of $\Phi_{\mathscr{W},0}$ is a densely defined, closable symmetric operator for real $h \in C_c^\infty(\mathbb{R}^{1+1})$. Its vacuum two-point function is*

$$\left\langle \Omega, :\Phi^2_{\mathscr{W},0}:(y):\Phi^2_{\mathscr{W},0}:(y')\Omega \right\rangle = 2W_{\mathscr{W}}(y-y')^2.$$

It is the strong limit, on a dense finite-particle spectral domain, of the corresponding worldsheet-smeared radial Wick squares.

For a nontrivial Friedrichs holonomy sector, the analogous normal-ordered boundary quadratic field is zero.

For the deficient-mode fields of Theorem , the normal-ordered quadratic fields $:a_j a_k^\dagger:(h)$ also exist on a dense finite-particle spectral domain. In the scale-invariant one-channel case their scaling dimension is $2-2\nu$.

Proof. With the Fourier convention already fixed, has nonnegative density

$$w_{\mathscr{W}}(\omega,k) = \frac{1}{2\alpha}1_{\{\omega > |k|\}}.$$

The pair-creation kernel for the smeared Wick square has squared norm proportional to

$$\int |\widehat{h}(p+q)|^2 w_{\mathscr{W}}(p) w_{\mathscr{W}}(q)\, \mathrm{d}^2p\, \mathrm{d}^2q.$$

For a fixed total momentum P, the set $p \in V_+$, $P - p \in V_+$ is a compact diamond of area bounded by $C(1 + P_0)^2$. Thus the convolution is polynomially bounded and the displayed integral is finite.

On vectors with a finite number of smooth, compactly supported spectral wave functions, the number-preserving kernel is bounded into the one-particle space: its relevant factor is $\hat{h}(p - q)$, and q ranges over a compact set. Rapid decay in p proves the bound. The annihilation and pair-annihilation parts are treated by adjunction and Cauchy–Schwarz. This defines the operator on a dense domain. For real h it is symmetric and hence closable. Wick's rule gives .

The radial limits follow from the same estimates: on compact spectral sets use the Bessel limits, and on the tails use rapid decay in the total positive energy. In a nontrivial Friedrichs sector every limiting one-particle factor is zero.

For deficient boundary fields, gives, near the light cone, a bound by $C(p_+ p_-)^{-\nu_{\max}}$ in light-cone momenta. The two-particle convolution is controlled by beta integrals

$$\int_0^{P_\pm} u^{-\nu_{\max}} (P_\pm - u)^{-\nu_{\max}} \mathrm{d}u = C_\nu P_\pm^{1 - 2\nu_{\max}}.$$

Both endpoint exponents are integrable because $\nu_{\max} < 1$. High-energy growth remains polynomial. The preceding operator argument therefore applies. The dimension follows by doubling the dimension of a.

Proposition (A quadratic Wick field is not Gaussian). *Except in the zero case, the Wick square in Theorem is not a generalized free field in the Gaussian sense used here.*

Proof. For a real Gaussian field φ, Wick's rule gives, at mutually separated points,

$$\left\langle \Omega, :\varphi^2:(y_1) :\varphi^2:(y_2) :\varphi^2:(y_3) \Omega \right\rangle = 8 W(y_1, y_2) W(y_2, y_3) W(y_3, y_1).$$

This is not identically zero. A centered Gaussian field has vanishing three-point function. The same conclusion follows for the nontrivial charged quadratic field by its connected three-point contractions.

This corrects the description of the dimension-two worldsheet Wick square as a generalized free field in

I-B-VP-1-v4. Its two-point function and scaling dimension remain meaningful; Gaussianity does not follow from them.

Proposition (The c-number part has no ordinary restriction). *If $s \neq 0$, no member of*

$$F_L = sU_L + c\delta_{\mathscr{W}}$$

has an ordinary, approximate-identity-independent restriction to $\mathscr{W}$. If $s = 0$ and $c \neq 0$, the delta term likewise has no such restriction. The exceptional zero distribution does restrict.

Consequently the existence of the normal-ordered boundary square does not define the restriction of the covariantly renormalized bulk Wick square.

Proof. Choose a smooth transverse profile η with $\eta(0) \neq 0$, and put $\eta_\varepsilon(x) = \varepsilon^{-2}\eta(x/\varepsilon)$. The Euler identity gives

$$\langle U_L, \eta_\varepsilon\rangle = \varepsilon^{-2}[\langle U_L, \eta\rangle + \sigma_\alpha \log \varepsilon\, \eta(0)].$$

Adding $c\delta_{\mathscr{W}}$ adds $c\varepsilon^{-2}\eta(0)$. No fixed c cancels the logarithmic divergence when $s \neq 0$. When $s = 0$, a nonzero delta still gives the inverse-square divergence. Smearing in y does not alter this argument.

Stopping condition (Boundary operators are not an interacting defect algebra). Theorems – construct free operators on declared domains. They do not construct time-ordered worldsheet composites, interacting defect dynamics, a new brane degree of freedom, or a normalization of (c, τ, w).

They are separate from: I-B's finite report, I-B-G's form positivity, I-B-VP's regular Hadamard and one-point statements, I-B-VP-5's spatial operator selection, and VII-I's finite-width free algebra [15, 16, 17, 22, 25]. A documentary comparison transfers none of these domains.

Stage B: an explicitly normalized core diagnostic

A geometric flower-pot model

Use geodesic radial coordinate s and circumference function

$$f_R(s) = \begin{cases} s, & 0 \le s \le \alpha R, \\ \alpha(s - \alpha R) + \alpha R, & s \ge \alpha R. \end{cases}$$

The transverse metric is

$$\mathrm{d}s^2 + f_R(s)^2 \mathrm{d}\varphi^2.$$

In the exterior put

$$r = s + (1 - \alpha)R, \qquad r \ge R,$$

so the metric is $\mathrm{d}r^2 + \alpha^2 r^2 \mathrm{d}\varphi^2$. The interior is a flat disk of radius αR. The scalar-curvature measure is

$$R_{\mathrm{sc}}\mathrm{d}A = 2(1 - \alpha)\delta(s - \alpha R)\,\mathrm{d}s\,\mathrm{d}\varphi, \qquad \int R_{\mathrm{sc}}\mathrm{d}A = 4\pi(1 - \alpha).$$

Take a neutral real massless field. For $u = pR$, define

$$\ell_\xi(u) = u\frac{I_1(\alpha u)}{I_0(\alpha u)} + \frac{2\xi(1 - \alpha)}{\alpha},$$

$$B_\xi(u) = \frac{uI_1(u) - \ell_\xi(u) I_0(u)}{uK_1(u) + \ell_\xi(u) K_0(u)}.$$

For $\xi \ge 0$ the denominator is positive.

Lemma (Matching and endpoint estimates). *The zero-angular-mode exterior Green correction is*

$$\delta F_R^{(0)}(r) = \frac{1}{4\pi^2\alpha}\int_0^\infty p\, B_\xi(pR) K_0(pr)^2\,\mathrm{d}p, \qquad r > R.$$

At small u,

$$B_0(u) = \frac{1 - \alpha}{2}u^2 + O(u^4|\log u|),$$

whereas, for fixed $\xi > 0$,

$$B_\xi(u) = -\frac{1}{\log(\kappa_\xi/u)} + O\left(\frac{1}{\log^2(1/u)}\right)$$

for a positive constant κ_ξ. At large u,

$$B_\xi(u) = O(e^{2u}/u).$$

Proof. Integrating $(-\Delta + \xi R_{\mathrm{sc}})v = 0$ across the ring gives

$$v_+' - v_-' = \frac{2\xi(1-\alpha)}{\alpha R} v.$$

The interior regular solution is $I_0(ps)$. The exterior regular solution is $I_0(pr) + B_\xi(pR)K_0(pr)$. Their logarithmic derivatives give . The longitudinal Fourier integral and zero-mode angular normalization give .

For $\xi = 0$, use $I_1(v)/I_0(v) = v/2 + O(v^3)$ and $uK_1(u) = 1 + O(u^2|\log u|)$. For $\xi > 0$, the constant $g_\xi = 2\xi(1-\alpha)/\alpha$ dominates ℓ_ξ, and

$$K_0(u) = \log(2/u) - \gamma_E + O(u^2|\log u|).$$

Thus one may take $\log \kappa_\xi = \log 2 - \gamma_E + g_\xi^{-1}$ at leading order. The large-argument Bessel expansions give $\ell_\xi(u) = u + O(1)$ and .

The geometric radius convention in this section is explicit. The code-defined $Q(R)$ of I-B-VP-1-v4 was not supplied as an independent formula. We do not identify its numerical constant with the diagnostic below merely because the symbols or small-argument powers resemble one another.

The observable whose coefficient is proved

Fix an exterior observation radius $b > 0$ and define

$$Q_R(b) = 2\pi\alpha \int_R^b r\, \delta F_R^{(0)}(r)\, \mathrm{d}r.$$

It is the exterior zero-mode contribution integrated over the transverse annulus, per unit worldsheet volume.

Put

$$A(u) = \int_u^\infty v K_0(v)^2\, \mathrm{d}v.$$

Then

$$A(0) = \frac{1}{2}, \quad A(u) = \frac{1}{2} + O(u^2 \log^2 u) \quad (u\text{downarrow}\,0), \quad A(u) = O(e^{-2u}) \quad (u \to \infty).$$

The identity $A(0) = 1/2$ is a standard Bessel integral [9].

Theorem (Analytic double-logarithmic coefficient). *For the core and fixed $b > 0$:*

1. *if $\xi = 0$, $Q_R(b)$ converges to the finite number*

$$Q_0 = \frac{1}{2\pi} \int_0^\infty B_0(u) A(u) \frac{\mathrm{d}u}{u};$$

2. *if $\xi > 0$,*

$$Q_R(b) = -\frac{1}{4\pi} \log \log (b/R) + C_{B_\xi} + o(1).$$

More generally, holds for every reflection profile satisfying

$$B(u) = -\frac{1}{\log (\kappa/u)} + O(\log^{-2} (1/u))$$

at zero and $\int_1^\infty |B(u)| A(u) \mathrm{d}u/u < \infty$. The coefficient is universal within this normalized profile class; the finite constant is not.

Proof. Tonelli applied to absolute values where necessary, followed by the endpoint bounds, gives the exact identity

$$Q_R(b) = \frac{1}{2\pi} \int_0^\infty B(u)[A(u) - A(Tu)] \frac{\mathrm{d}u}{u}, \qquad T = b/R.$$

The original integral is finite also at the ring endpoint: and the exponential decay of A make the large-u integrand $O(u^{-2})$.

For $\xi = 0$, $B_0(u) = O(u^2)$, so the integrand with $A(u)$ is integrable at zero. Dominated convergence proves .

For $\xi > 0$, choose u_0 so small that $\log (\kappa/u) > 0$ below u_0. The difference between $B(u)$ and $-1/\log (\kappa/u)$ contributes a finite limit because

$$\int_0^{u_0} \frac{\mathrm{d}u}{u\log^2(\kappa/u)} < \infty.$$

Replacing $A(u)$ by 1/2 in this small interval likewise changes only a convergent integral.

The smooth lower cutoff supplied by $A(Tu)$ can be replaced by a sharp cutoff at $u = T^{-1}$ with error $o(1)$. To see this, set $v = Tu$. The discrepancy is bounded by a constant times

$$\frac{1}{\log T}\int_0^{\infty} \left|\frac{1}{2} - A(v) - \frac{1}{2}1_{\{v>1\}}\right| \frac{\mathrm{d}v}{v},$$

apart from an exponentially small large-v tail. The integral is finite by the endpoint estimates for A.

The only divergent term is therefore

$$-\frac{1}{4\pi}\int_{T^{-1}}^{u_0} \frac{\mathrm{d}u}{u\log(\kappa/u)} = -\frac{1}{4\pi}\log\log(\kappa T) + \frac{1}{4\pi}\log\log(\kappa/u_0).$$

Since $\log\log(\kappa T) - \log\log T \to 0$, this proves and existence of its finite remainder.

Remark (Relation to the long-range core literature). Allen–Kay–Ottewill analyze the logarithmically slow zero-mode exterior correction and its dependence on core structure [6]. Theorem proves a different, explicitly integrated statement from the exact reflection formula. It does not use a numerical fit or promote an exterior asymptotic approximation to a full contact-limit theorem.

Proposition (Exact transfer condition to a contact coefficient). *Suppose a separately defined, fully renormalized core one-point prescription has contact coefficient c_R and admits the decomposition*

$$c_R = Q_R(b) + R_R(b), \qquad R_R(b) \to R_0(b) \in \mathbb{R}.$$

Then its positive-coupling contact has the coefficient $-1/(4\pi)$ in . Without , the exterior theorem does not determine the full contact.

Proof. Subtract from . The second assertion follows because interior, nonzero-mode, and local subtraction contributions are absent from ; no estimate for them is implied by its definition.

Stopping condition (No universal zero-thickness normalization). Theorem does not select a point of the

single-generator line. At $\xi = 0$ it proves finiteness only for the diagnostic ; a stress calculation and a complete interior prescription are still needed for a triple (c, τ, w). At $\xi > 0$ it proves a double logarithm for that diagnostic; transfer to the full c_R requires . The zero-thickness limit is not declared universal.

Stage B continued: negative coupling and curvature ambiguity

A direct variational instability proof

Theorem (Negative integrated curvature coupling). *Let a complete rotationally symmetric transverse core be flat or conical outside a compact set, with exterior circumference $2\pi\alpha r$, and let its scalar-curvature measure be supported in that compact set and satisfy*

$$\int R_{\mathrm{sc}}\mathrm{d}A = 4\pi(1-\alpha) > 0.$$

Assume the curvature term defines a closed semibounded quadratic-form perturbation of the kinetic energy; this holds for a smooth bounded compactly supported curvature and for the flower-pot ring via the ordinary circle trace.

For every $\xi < 0$, the transverse operator $-\Delta + \xi R_{\mathrm{sc}}$ has negative spectrum. For these compact-core perturbations its negative spectrum contains a bound state. Consequently the massless $3+1$-dimensional product dynamics has no nonnegative spectral ground-state construction.

Proof. Take a radial test equal to one on the curvature support, and, in the exterior annulus $R < r < b$, equal to

$$u_b(r) = \frac{\log(b/r)}{\log(b/R)}.$$

Set it to zero above b, with harmless smoothing at the corners. The exterior kinetic energy tends to

$$2\pi\alpha\int_R^b |u_b'(r)|^2 r\,\mathrm{d}r = \frac{2\pi\alpha}{\log(b/R)}.$$

The curvature energy is exactly

$$\xi\int R_{\mathrm{sc}}\mathrm{d}A = 4\pi\xi(1-\alpha) < 0.$$

For sufficiently large b, the total form value is negative. The test is compactly supported and belongs to the form domain.

A compact smooth potential, or a trace interaction on a compact circle, is a relatively compact form perturbation of the exterior conical Laplacian. Local Rellich compactness and the compact circle trace prove this assertion. The essential spectrum remains $[0,\infty)$, so the negative variational value yields a negative eigenvalue.

For the full spatial product, tensor its transverse eigenfunction with a longitudinal wave packet of arbitrarily small $\|\partial_z v\|^2/\|v\|^2$. The full spatial quadratic form is still negative. A nonnegative square-root spectral ground state is therefore unavailable.

Remark (What this says about weak binding). Simon's two-dimensional weak-coupling theorem concerns a Euclidean Schrödinger operator with hypotheses on an ordinary potential [7]. A conical metric and a ring measure are not automatically that operator. Theorem checks the needed conclusion directly on the actual geometry. It does not claim that all hypotheses of the Euclidean theorem have been transferred. It also does not exclude nonstationary Hadamard states of an unstable equation.

Proposition (Flower-pot weak-coupling scale). *For the flower-pot ring, put*

$$g = \frac{2|\xi|(1-\alpha)}{\alpha}.$$

A negative zero-mode eigenvalue is $-\kappa^2$, where $v = \kappa R$ obeys

$$v\frac{K_1(v)}{K_0(v)} + v\frac{I_1(\alpha v)}{I_0(\alpha v)} = g.$$

As ξuparrow0 *through negative values,*

$$-\kappa^2 = -\frac{4e^{-2\gamma_E}}{R^2}\exp\left[-\frac{\alpha}{|\xi|(1-\alpha)}\right](1+o(1)).$$

Proof. Use $I_0(\kappa s)$ in the interior and $K_0(\kappa r)$ in the exterior, and apply the same derivative jump as before. At small v,

$$\nu K_1(\nu)/K_0(\nu) = \frac{1 + o(1)}{\log(2/\nu) - \gamma_E}, \qquad \nu I_1(\alpha\nu)/I_0(\alpha\nu) = O(\nu^2).$$

Solving gives $\nu = 2e^{-\gamma_E} e^{-1/g}(1 + o(1))$.

The curvature-induced scalar normalization

We now discuss one real scalar and keep its normalization separate from stress multiplicity conventions. Declare the Hadamard scale convention

$$H_\mu(X, X') = \frac{1}{8\pi^2}\left[\frac{u(X, X')}{\sigma} + v(X, X')\log(\mu^2\sigma)\right] + \cdots, \qquad [v] = \frac{1}{2}\left(m^2 + qR_{\mathrm{sc}}\right).$$

This is a smooth-core convention of the usual Hadamard normalization framework [8, 2].

Proposition (Scalar scale shift and its limitation). *For $m = 0$, a scale change $\mu \mapsto \mu'$ shifts the smooth-core Wick square by*

$$\delta F = -\frac{qR_{\mathrm{sc}}}{8\pi^2}\log(\mu'/\mu).$$

Whenever the curvature densities converge to $4\pi(1 - \alpha)\delta_{\mathscr{W}}$, the scalar contact shift is

$$\delta c = -\frac{q(1 - \alpha)}{2\pi}\log(\mu'/\mu).$$

This scalar calculation does not determine $\delta\tau$. If one chooses the particular one-point completion $\delta\tau = \delta w = 0$ in the contact coordinates , then the stress still changes by $nq\,\delta c\,\mathscr{I}[\delta_{\mathscr{W}}]$. For the neutral Friedrichs data with $0 < \alpha < 1$ and $q \neq 0$, that particular coordinate shift is not parallel to the dilation direction.

Proof. The scale change of the subtracted parametrix is $[v]\log(\mu'/\mu)/(4\pi^2)$; its negative gives the displayed Wick-square shift. Insert the curvature-density limit to obtain .

Within , a change of c requires the stated improvement contact in T by the trace equation. The scalar calculation supplies no value for the remaining traceless stress parameter τ. For neutral Friedrichs data, $h_0 = (\alpha^{-4} - 1)/(1440\pi^2) > 0$. A nonzero shift $(\delta c, 0, 0)$ is therefore not parallel to $(s, h_0/4, 0)$.

Remark (Correction to an overstrong stress inference). It is not valid to infer $\delta\tau = 0$ for a physical core merely because curvature-square expressions have not been defined at the apex. On smooth cores the variational curvature counterterms contain derivative terms as well as quadratic terms. For example,

$$\frac{1}{\sqrt{|g|}}\frac{\delta}{\delta g^{\mu\nu}}\int \sqrt{|g|}\, R_{\mathrm{sc}}^2$$

contains $\nabla_\mu \nabla_\nu R_{\mathrm{sc}}$ and $g_{\mu\nu}\mathrm{Box}\, R_{\mathrm{sc}}$, in addition to curvature products [8, Eq. (136)]. Their thin-core limits require a separate subtraction and comparison. Discarding an undefined expression is not a calculation of its finite stress remainder. Thus an unconditional physical ``off-orbit'' conclusion for the complete triple is not proved by alone.

Separation from spatial operator selection

I-B-VP-5 supplies, in its stated monotone flat-center class and for $\xi \geq 0$, strong resolvent convergence after a specified unitary identification [22]. This is not a one-point limit.

In particular:

• strong resolvent convergence controls bounded continuous functions of the spatial operator, not automatically the massless covariance involving its inverse square root;

• a curvature measure is not an ordinary point-evaluation energy on the form domain;

• a one-point contact limit additionally requires subtraction, coincidence, and uniform integrability estimates;

• a core radius is not a preparation width or a loop regulator. The nonclosability and curvature-measure statements of I-B-VP-5 and the width discipline of X-L-ST are comparisons, not substitutes for these estimates [22, 27].

Stage C: a declared first-order tadpole

The Euclidean kernel and two separate singular loci

Take the real neutral Friedrichs field with interaction $\lambda\varphi^4/4!$. The first-order Euclidean Wick-square insertion is defined below, rather than being inferred from the existence of a regular-host Hadamard form.

For $r, r' > 0$, the untwisted Euclidean kernel is

$$G_F(X, X') = \frac{1}{8\pi^2 \alpha r r'} \frac{\sinh(\eta/\alpha)}{\sinh\eta \cosh(\eta/\alpha) - \cos(\varphi - \varphi')}, \quad \cosh\eta = \frac{r^2 + (r')^2 + |y - y'|^2}{2rr'}.$$

It follows by summing the separated Bessel Green kernels. It has the apex value and the ordinary flat leading singularity $1/(4\pi^2 d(X, X')^2)$ near a regular diagonal point.

Fix an external point X with $r > 0$ and put

$$I(r) = \int_{\mathcal{W}} G_F(X, (y', 0))^2 \, \mathrm{d}^2 y' = \frac{1}{16\pi^3 \alpha^2 r^2}.$$

The apex locus $r' = 0$ and the coincidence locus $X' = X$ are disjoint. Their subtraction neighborhoods can therefore be chosen disjoint.

Definition (First-order composite prescription). Let $s = (\alpha^{-2} - 1)/(48\pi^2)$. For $L > 0$, $\mu > 0$, and $c \in \mathbb{R}$, define

$$\mathscr{T}_{L,\mu,c}(X) = \lim_{\varepsilon, \delta \mathrm{downarrow} 0} [\int_{\mathrm{substack}\, r' > \varepsilon} + \sigma_\alpha s \log(\varepsilon/L) I(r) + \frac{s}{8\pi^2 r^2} \log(\mu\delta) + c I(r)].$$

The coincidence distance is measured in a regular flat chart near X. The integral at infinity is an improper integral. The first-order coefficient is

$$\delta_\lambda F(X) = -\frac{\lambda}{2} \mathscr{T}_{L,\mu,c}(X).$$

Finite local changes of the regular diagonal subtraction are allowed only when explicitly recorded.

Theorem (Existence of the declared composite insertion). *The limit exists, independently of the relative approach of ε and δ to zero. It is smooth for $r > 0$. It is not a product of two apex-supported distributions.*

There is a finite dimensionless constant A_α, depending on the finite subtraction convention, such that

$$\delta_\lambda F(r) = -\frac{\lambda c}{32\pi^3 \alpha^2 r^2} - \frac{\lambda s}{16\pi^2 \alpha r^2} \log(r/L) - \frac{\lambda s}{16\pi^2 r^2} \log(\mu r) + \frac{\lambda s A_\alpha}{r^2}.$$

Proof. Near the apex, with the external endpoint fixed, gives

$$G_F(X,X')^2 = G_F(X,(y',0))^2 + O((r')^{\gamma})$$

locally in y', for some $\gamma > 0$. The same estimate holds after fixed derivatives in the external regular variables. Angular averaging can be used before estimating. Hence

$$\frac{G_F(X,X')^2 - G_F(X,(y',0))^2}{(r')^2}$$

is locally integrable with transverse measure $\alpha r' \mathrm{d}r' \mathrm{d}\varphi'$. The apex logarithm is exactly $\sigma_{\alpha} s I(r) \log(1/\varepsilon)$.

Near $X' = X$, the apex factor $s/(r')^2$ is smooth. The only nonintegrable term is

$$\frac{s}{16\pi^4 r^2 d(X,X')^4}.$$

The unit three-sphere has area $2\pi^2$, so its logarithmic residue is $s/(8\pi^2 r^2)$. Subtracting that value leaves an integrable remainder. The two singular neighborhoods are disjoint; therefore there is no overlap subtraction and no dependence on a joint path of the two cutoffs.

At infinity, has inverse-square four-dimensional decay. Subtracting the apex value in its apex neighborhood preserves an integrable remainder. Equivalently, integrating first over the two longitudinal variables gives an integrable inverse-cubic radial tail for the original tadpole integrand. These estimates are uniform for external X in compact subsets of $r > 0$. They establish the improper integral and smoothness.

Translation and rotation invariance make the result a function of r. Rescaling all coordinates by r in leaves a finite dimensionless integral. The apex subtraction contributes $\sigma_{\alpha} s I(r) \log(r/L)$. The diagonal subtraction contributes $s \log(\mu r)/(8\pi^2 r^2)$. The remaining constant is written as $-2sA_{\alpha}/r^2$. Multiplication by $-\lambda/2$ gives . Finally, follows from $\int_{\mathbb{R}^2} (r^2 + |y|^2)^{-2} \mathrm{d}^2 y = \pi/r^2$.

Remark (What has been renormalized). Definition is a numerical first-order composite insertion at a regular external point. At the apex it uses the explicitly extended test action obtained by subtracting the boundary value of

G_F^2. At the regular diagonal it uses a separate logarithmic extension. It does not presume that $G_F|_{\mathscr{W}\times\mathscr{W}}$ already has an unambiguous Euclidean composite product. General local normalization principles are discussed in [1, 2]; no interacting singular-background theorem is imported from them.

Closing the finite-constant obligation by a coordinate map

Proposition (Absorption of the finite constant). *Define*

$$c_{\mathrm{eff}} = c - 32\pi^3\alpha^2 s A_\alpha.$$

Then can be written with $A_\alpha = 0$ *and* c *replaced by* c_{eff}.

More generally, the finite scheme change $A_\alpha \mapsto A_\alpha + \Delta A_\alpha$ *is compensated by*

$$c \mapsto c + 32\pi^3\alpha^2 s\,\Delta A_\alpha.$$

This is an equivalence of first-order parametrizations, not a selection of a physical value of c.

Proof. Combine the first and last terms of . They are

$$-\frac{\lambda}{32\pi^3\alpha^2 r^2}\big(c - 32\pi^3\alpha^2 s A_\alpha\big).$$

This proves both formulas. If c has already been calibrated in another fixed scheme, changing the scheme requires the displayed recalibration; one cannot simply erase a measured parameter.

Thus the finite-constant obligation is closed as a finite renormalization with an explicit absorption map. No numerical value for the finite integral in the specific sharp prescription is asserted.

Proposition (Physicalization and its normalization). *With the composite prescription frozen,*

$$\frac{\partial}{\partial c}\delta_\lambda F(r) = -\frac{\lambda}{32\pi^3\alpha^2 r^2}.$$

A free c-number contact therefore changes the first-order one-point function at regular points.

Writing a quadratic defect action as

$$S_{\mathrm{def}} = \frac{g}{2}\int_{\mathscr{W}} \varphi^2,$$

the tadpole contact contribution has

$$g = \frac{\lambda c}{2}, \qquad S_{\mathrm{def}} = \frac{\lambda c}{4}\int_{\mathscr{W}} \varphi^2$$

at this order and in this normalization.

Changing L without changing the free distribution requires

$$c' = c + \sigma_\alpha s \log(L'/L).$$

Consequently $c + \sigma_\alpha s \log(r/L)$ *is independent of this presentation change. It is not invariant under a transverse dilation with all parameters held fixed.*

Proof. The first assertion is differentiation of . A quadratic perturbation $g\varphi^2/2$ changes the covariance by $-g\int_{\mathscr{W}} G_F G_F$. Comparison with $-\lambda c I(r)/2$ gives the factor g. The scale-coordinate relation follows from .

The defect-action factor distinguishes the action coefficient from the self-energy coefficient. Identifying both with $\lambda c/2$ would lose a factor of two.

For a charged Friedrichs sector, the endpoint kernel vanishes at the worldsheet. Therefore a tadpole with the corresponding apex scalar insertion has no contribution from its $c\delta_{\mathscr{W}}$ term, and its apex L-dependence drops out at first order. This endpoint statement does not identify the combinatorial factors of a complex quartic interaction with those of the real interaction used above.

The next-order obstruction remains an obstruction

Proposition (A new logarithmic worldsheet extension is required). *A second insertion of the quadratic defect term encounters, at coincident worldsheet points,*

$$G_{F,E}((y,0),(y',0)) = \frac{1}{4\pi^2 \alpha |y - y'|^2}.$$

Its local integral has the logarithm

$$\int_{\delta < |u| < b} \frac{\mathrm{d}^2 u}{4\pi^2 \alpha |u|^2} = \frac{1}{2\pi\alpha} \log (b/\delta).$$

It is not defined by the free one-point extension U_L or by the first-order diagonal subtraction.

If this is represented by a finite logarithmic zero-channel point-interaction boundary condition $u(r) \propto \log (r/L_b)$, its transverse operator has the negative eigenvalue

$$-4e^{-2\gamma_E}/L_b^2.$$

Proof. The first formula is with both endpoints on $\mathscr{W}$ and separated. Polar integration gives . It is a two-worldsheet-point extension problem, different from either singular locus in Definition .

For the last assertion, $K_0(\kappa r) \sim -\log r + \log (2/\kappa) - \gamma_E$. Its constant-to-log ratio matches $-\log (r/L_b)$ precisely when $\kappa = 2e^{-\gamma_E}/L_b$. This is the familiar logarithmic Bessel extension [5, 4].

Stopping condition (First order is not an interaction completion). No second-order renormalized defect theory is constructed here. Equation names its missing local extension. The negative eigenvalue diagnoses the massless spectral-ground-state difficulty of one finite logarithmic point-interaction realization; it is not a universal nonexistence theorem for every enlarged interacting model.

The tadpole does not select c, remove Proposition 's dilation no-go, construct an interacting Ward identity, or admit an inherited characteristic quantum coefficient. The free ambiguity in VII-H and the non-sourcing statements of XI-A-ST and X-A-ST remain separate [26, 28, 29].

Stage D: ultraviolet matrices and a defined resonance set

Assume two deficient modes:

$$\alpha > \frac{1}{2}, \qquad 1 - \alpha < \beta < \alpha,$$

$$\nu_0 = \beta/\alpha, \qquad \nu_1 = (1 - \beta)/\alpha, \qquad \sigma = \nu_0 + \nu_1 = \alpha^{-1} \in (1, 2).$$

We use 1 as the second matrix index in this section, corresponding to angular mode -1.

The off-diagonal radial coefficient is

$$R_{01}(r) = \frac{1}{4\pi^2\alpha}\int_0^\infty p\, C_{01}(p) K_{\nu_0}(pr) K_{\nu_1}(pr)\, \mathrm{d}p.$$

Its scalar angular contribution is $2\mathrm{Re}\,(e^{i\varphi} R_{01}(r))$.

What the matrix asymptotic proves

Theorem (Ultraviolet matrix and leading finite-chart term). *For finite $H \geq 0$,*

$$C(p) \to S^{-1}, \qquad \lVert C(p) - S^{-1} \rVert = O(p^{-2\nu_{\min}}),$$

and

$$C_{01}(p) = -2S_0^{-1} S_1^{-1} A_0 A_1 H_{01} p^{-\sigma} + O(p^{-\sigma-2\nu_{\min}}).$$

The moment

$$\kappa_{01} = \int_0^\infty p^{1-\sigma} C_{01}(p)\, \mathrm{d}p$$

is absolutely convergent, and

$$R_{01}(r) = \frac{A_0 A_1}{4\pi^2\alpha} \kappa_{01} r^{-\sigma} + o(r^{-\sigma}).$$

For a boundary chart $H = tww^\dagger$, $t \to \infty$, with both components of w nonzero and $\nu_0 < \nu_1$, the ultraviolet limit is Friedrichs in the smaller-order channel and scale-invariant singular in the larger-order channel. Its off-diagonal entry is $O(p^{-(\nu_1-\nu_0)})$.

Proof. Expand the inverse in by a Neumann series for large p. Since $\lVert E \rVert = O(p^{-\nu_{\min}})$, the displayed matrix estimates follow.

For the moment, the large-p integrand is $O(p^{1-2\sigma})$, integrable because $\sigma > 1$. At small p, $0 \leq C(p) \leq S^{-1}$, so the integrand is bounded by $Cp^{1-\sigma}$, integrable because $\sigma < 2$. The function $u^\nu K_\nu(u)$ is bounded for $u > 0$. Therefore the integrand after multiplication by r^σ is dominated by $Cp^{1-\sigma}|C_{01}(p)|$. Dominated convergence gives .

For the boundary chart, the rank-one inverse formula gives

$$C = S^{-1} - \frac{S^{-1}uu^{\dagger}S^{-1}}{u^{\dagger}S^{-1}u}, \qquad u = Ew.$$

The smaller-order component of u dominates as $p \to \infty$. The limiting subtraction removes that coordinate from S^{-1}; the off-diagonal quotient has the stated power.

This closes the finite-dimensional ultraviolet estimate without importing a companion proposition. It does not by itself prove a full stress-tensor finite-part theorem.

Resonance is a property of boundary data

Let

$$\mathfrak{B} = \{(\alpha, \beta, V, H_V) : \tfrac{1}{2} < \alpha < 1,\ 1 - \alpha < \beta < \alpha, ?H_V = H_V^{\dagger} \geq 0\}$$

with a reference length fixed when components of boundary coefficients have different dimensions.

The Bessel Mellin kernel is

$$M(s) = \int_0^{\infty} u^{s-1} K_{\nu_0}(u) K_{\nu_1}(u)\, \mathrm{d}u = \frac{2^{s-3}}{\Gamma(s)} \prod_{\varepsilon_0, \varepsilon_1 = \pm 1} \Gamma\left(\frac{s + \varepsilon_0 \nu_0 + \varepsilon_1 \nu_1}{2}\right).$$

The integral initially converges for $\operatorname{Re} s > \sigma$; the right side defines its meromorphic continuation.

For the actual nonzero entry C_{01}, let

$$\mathscr{C}_{01}(z) = \int_0^{\infty} p^{z-1} C_{01}(p)\, \mathrm{d}p$$

in its convergence strip, with meromorphic continuation where supplied by the chart's endpoint expansions. Then

$$R_{01}(r) = \frac{1}{4\pi^2 \alpha} \frac{1}{2\pi \mathrm{i}} \int_{\operatorname{Re} s = c} r^{-s} M(s) \mathscr{C}_{01}(2 - s)\, \mathrm{d}s$$

on a common fundamental strip.

Definition (Explicit resonance exclusion). For a chart with controlled endpoint expansions, define:

1. $\Re_M$ consists of boundary data for which the actual meromorphic integrand $M(s)\mathscr{C}_{01}(2-s)$ has a pole of order at least two encountered in the small-r contour displacement. This includes collisions between a Bessel pole and a nonzero ultraviolet coefficient pole. Canceled candidate poles are not counted.

2. $\Re_W$ consists of boundary data for which an actual term in the differentiated one-point expansion, paired with a transverse Taylor monomial, has a nonzero angular moment and zero radial cutoff exponent in a Ward boundary term. Explicitly, if that term gives $a\,\varepsilon^{\gamma}(\log\varepsilon)^{k}$, the criterion is $a \neq 0$ and $\gamma = 0$. Set

$$\Re = \Re_M \cup \Re_W.$$

Equal-order data are evaluated by the same actual-pole rule; they are not excluded merely because $v_0 = v_1$. A chart without the required controlled continuation or differentiated remainder is marked unverified rather than declared nonresonant.

This is a criterion on (α, β, V, H_V), including coefficient cancellations. It is not the assertion that a closed countable set of points in the (α,β) plane has been classified. Countably many exponent equations can define positive-dimensional or accumulating strata. For $H \geq 0$ there is no positive-real-p denominator zero in ; such a zero is not the resonance mechanism considered here.

A worked resonant boundary chart

Theorem (A genuine Mellin collision). *Choose*

$$\alpha = \frac{4}{5}, \qquad \beta = \frac{2}{5}, \qquad v_0 = \frac{1}{2}, \qquad v_1 = \frac{3}{4},$$

and the boundary chart $H = tww^{\dagger}$, $t \to \infty$, *with* $w = (1,1)/\sqrt{2}$ *in fixed reference-length units. Write*

$$\delta = v_1 - v_0 = \frac{1}{4}, \qquad \sigma = \frac{5}{4}.$$

Then there are positive constants K, d *such that*

$$C_{01}(p) = -\frac{Kp^{-\delta}}{1 + dp^{-2\delta}},$$

and, as $r$downarrow 0,

$$R_{01}(r) = -\frac{K}{4\pi^2\alpha} M(2-\delta) r^{-2+\delta} + \frac{K d A_0 A_1}{4\pi^2\alpha} r^{-\sigma} \log(1/r) + O(r^{-\sigma}).$$

The logarithmic coefficient is nonzero. The scalar off-diagonal term is nevertheless locally integrable and has a unique extension of scaling degree less than two.

Proof. The rank-one formula in Theorem gives

$$K = S_1^{-1} A_1 / A_0, \qquad d = \frac{S_1^{-1} A_1^2}{S_0^{-1} A_0^2},$$

which are positive. At infinity,

$$C_{01}(p) = -K p^{-\delta} + K d p^{-3\delta} + O(p^{-5\delta}).$$

The first term is integrable against the rescaled Bessel product at zero because $2 - \delta - \sigma = 1/2 > 0$; it gives the first term of .

For the second term, the small-argument Bessel product is

$$K_{\nu_0}(pr) K_{\nu_1}(pr) = A_0 A_1 (pr)^{-\sigma} + \text{less singular terms}.$$

Here

$$1 - 3\delta - \sigma = -1.$$

Thus the intermediate range $1 \ll p \ll r^{-1}$ contributes

$$K d A_0 A_1 r^{-\sigma} \int^{1/r} \frac{\mathrm{d}p}{p}.$$

The ranges below a fixed large momentum and above r^{-1}, and the subtracted Bessel remainders, are $O(r^{-\sigma})$. This can also be read from the double pole at $s = \sigma = 2 - 3\delta$ in . It proves the coefficient and the stated remainder.

The leading scalar degree is $2 - \delta = 7/4 < 2$; the logarithmic term is less singular. Both are locally integrable in two transverse dimensions. A supported distribution has scaling degree at least two, so no difference is possible at the same bound.

Remark (A logarithm is not automatically an apex residue). The logarithm in comes from a Mellin collision and is present off the apex. It is not, by itself, a delta-supported dilation residue. These two uses of the word ``residue" must not be identified.

Stage D concluded: why the broad KH uniqueness claim stops

For stationary, longitudinally translation-invariant mixed tensors define the smooth Killing–homothety operations

$$\mathrm{W1}_A(T) = \partial_\mu T^\mu{}_A,$$

$$\mathrm{W2}(T) = \partial_i(T^i{}_j R^j), \qquad R^j = \varepsilon^j{}_k x^k,$$

$$\mathrm{W3}(T) = \partial_i(T^i{}_j x^j) - T^i{}_i.$$

These are the prescribed operations of I-B-VP-1. They do not impose reference symmetry unless that condition is separately required.

Theorem (Supported KH-Ward counterexample). *Let $v \in \mathbb{R}^2$ be a nonzero constant transverse vector and $D_v = v^k \partial_k \delta_{\mathscr{W}}$. Define the supported mixed tensor Q_v by*

$$(Q_v)^A{}_B = -\frac{2}{3}\delta^A_B D_v,$$

$$(Q_v)^i{}_j = \delta^i_j D_v - \frac{5}{3} v^i \partial_j \delta_{\mathscr{W}} + v_j \partial^i \delta_{\mathscr{W}},$$

$$(Q_v)^A{}_i = (Q_v)^i{}_A = 0.$$

Then

$$\mathrm{W1}(Q_v) = \mathrm{W2}(Q_v) = \mathrm{W3}(Q_v) = 0, \qquad (Q_v)^\mu{}_\mu = 0.$$

It is real, longitudinally translation- and boost-invariant, odd under $x \mapsto -x$, and has transverse scaling degree three. It is jointly rotation covariant when v rotates with the off-diagonal boundary-data phase.

Therefore, whenever an off-diagonal extension exists in a KH-Ward class that permits these supported differences and has stress scaling bound greater than or equal to three, that class does not have uniqueness. This applies in particular to the resonant example of Theorem , conditional only on existence of its full stress extension, and also to the broad nonresonant class stated in I-B-VP-1-v4.

Proof. The identities

$$x^j \partial_k \delta_{\mathscr{W}} = -\delta^j_k \delta_{\mathscr{W}}$$

give

$$(Q_v)^i{}_j x^j = \frac{4}{3} v^i \delta_{\mathscr{W}}, \qquad (Q_v)^i{}_i = \frac{4}{3} D_v.$$

Hence $\mathrm{W3}(Q_v) = 0$. Likewise,

$$(Q_v)^i{}_j R^j = 0,$$

so $\mathrm{W2}(Q_v) = 0$. The longitudinal columns have only longitudinal diagonal entries independent of y, proving $\mathrm{W1} = 0$. The longitudinal trace is $-4D_v/3$ and cancels the transverse trace.

All remaining assertions follow directly from the first-jet formula. A nonzero v gives a nonzero tensor. Adding it changes no regular-part value, no scalar contact, and no trace identity.

Remark (The precise missing symmetry). The transverse block in is not $\bar{g}$-symmetric. For non-diagonal cone data the physical regular stress is g-symmetric but is generally not $\bar{g}$-symmetric. One cannot silently impose $\bar{g}$ -symmetry in the supported algebra and then use it as if it followed from the regular physical symmetry. Multiplication of the supported tensor by the singular conical metric is not available as an alternative.

One may declare an additional rule that the difference of two extensions must be reference symmetric, or impose specific first-jet normalization conditions. Such a rule must be stated and checked; it is not a consequence of

noninteger powers or of .

Stopping condition (Exact *U*(2) endpoint). The present paper proves the ultraviolet matrix estimate, the finite-chart leading scalar moment, a usable boundary-data resonance criterion, and one nonzero resonant logarithm. It also proves that the broad KH uniqueness claim is not justified in the stated contact class.

It does not release:

• a complete classification of $\Re$;

• uniqueness on $\Re$;

• an unconditional complete stress/current extension for every boundary chart and every resonance;

• the claim that every $U(2)$ state's apex freedom lies only in its rotation average. The resonant set remains open as a classification problem. The spatial $U(2)$ relation is an operator domain; an extension of its one-point data is a different object. No conclusion is imported into a non-product extension or the non-product ledger.

Stage E: the optional Dirac comparison is not silently resolved

The Dirac formulas of I-B-VP-1-v4 are not needed in any proof above. We retain them only as a comparison target.

A direct source distinction is important: Spinelly–Bezerra de Mello explicitly analyze a massless left-handed field with finite-radius magnetic-flux models [13]. I-B-VP-1-v4 describes a four-component massless Dirac field. Chirality multiplicity, mixed versus covariant stress components, and flux branches must therefore be normalized before comparing coefficients. Linet and Bezerra de Mello–Saharian are separate comparison sources [11, 12].

After matching the zero-flux normalization as in the supplied comparison, the two disputed polynomial brackets are

$$P_{+}(\alpha,\beta) = (\alpha^2 - 1)(17\alpha^2 + 7) + 120\beta^2(\alpha^2 - 2\beta^2 + 1),$$

$$P_{-}(\alpha,\beta) = (\alpha^2 - 1)(17\alpha^2 + 7) + 120\beta^2(\alpha^2 - 2\beta^2 - 1).$$

Their difference is exactly

$$P_{+} - P_{-} = 240\beta^{2}.$$

A chirality multiplicity or an overall index-convention sign, once fixed at zero flux, does not remove this flux-dependent difference. Flux reversal alone does not remove it either, because both brackets are even in β.

Stopping condition (Dirac audit ending). We do not claim that the polynomial discrepancy has been resolved by the observable distinction just identified. A same-branch, same-chirality, same-flux and same-subtraction derivation is still required. Neither paper is silently corrected here. No new Dirac extension theorem is asserted. The Maxwell printed discrepancy in I-B-VP-4 is a different audit and is not addressed by this comparison [21].

Stage F: reliability layer and exact mathematical release

What was and was not checked

This paper contains analytical calculations in the text. The source and numerical reports of I-B-VP-1-v4 were supplied, but its referenced scripts and raw logs were not executed here.

\texttt{ANALYTIC} A conclusion proved by the displayed argument on its stated domain.

\texttt{REPORTED} A result or computational number attributed to the supplied manuscript, not independently rerun.

\texttt{UNCERTIFIED} A numerical verification or enclosure claim lacking the complete artifacts and total error bounds needed for that interpretation.

No numerical profile in the parent source is used to prove Theorem . No symbolic residual is used to prove Theorem . The first-order coefficients follow from the two explicit local residues in Theorem .

Corrections and qualifications

toprule Parent statement or possible inference Correction in this companion

midrule endfirsthead toprule Parent statement or possible inference Correction in this companion

midrule endhead bottomrule endfoot

A radial Sobolev power rule is an unconditional ``iff'' Smooth polynomial exceptions must be retained. Lemma states the sufficient range actually used.

Sobolev regularity names the field restriction Theorem proves convergence of creation vectors and strong convergence on a declared Fock domain.

A boundary covariance positive for $p^2 > 0$ has positive Lorentzian spectrum Theorem proves a matrix Stieltjes representation and supplies the spectral density.

The normal-ordered square is a generalized free field It is a Wick polynomial of a generalized free field. Its nonzero three-point function excludes Gaussianity.

Scaling degree alone excludes every c-number restriction The approximate-identity calculation proves the obstruction; the identically zero exception is retained.

The code-defined $Q(R)$ has a universal coefficient The observable is explicit. Its coefficient is proved; transfer to a complete contact requires .

A Euclidean weak-binding theorem automatically applies to a conical ring Theorem proves instability directly by a logarithmic variational test on the actual geometry.

Negative coupling excludes every Hadamard state It excludes the nonnegative spectral ground-state construction, not every nonstationary Hadamard state.

The scalar curvature shift determines the complete stress shift It determines δc only. $\delta\tau$ needs a stress-limit calculation.

A_{α} is an unexplained remainder Equations – give its finite-renormalization absorption map.

The defect action and its self-energy have the same coefficient For the convention $S_{\mathrm{def}} = g\int \varphi^2/2$, $g = \lambda c/2$ and the

action coefficient is $\lambda c/4$.

$c + \sigma_{\alpha} s\log(r/L)$ is dilation invariant without qualification It is invariant under a compensating change of presentation (L, c), not under fixed-parameter transverse dilation.

A noninteger leading power proves KH uniqueness The tensor is an admissible supported counterexample unless an additional symmetry or normalization rule excludes it.

The resonance exclusion names a closed countable point set Definition gives actual pole and flux criteria on boundary data; no such topological assertion is made.

Release theorem

Theorem (I-B-VP-1-Ex1 release). *On the prescribed host, completion, density, state kernels, boundary relations, and subtraction conventions of this paper, the following are established:*

1. *the Friedrichs worldsheet restriction exists on $\mathcal{D}_{\mathrm{fin}}$, has covariance , and has the exact one-particle range in Theorem ; it vanishes in nontrivial Friedrichs holonomy sectors;*

2. *deficient weighted boundary fields have covariance and positive matrix spectral measure ;*

3. *their worldsheet Wick quadratics exist on the stated dense spectral domains, while the nonzero covariant c-number finite part has no ordinary restriction;*

4. *the exterior core diagnostic has the analytic double-logarithmic coefficient $-1/(4\pi)$ under the stated profile hypotheses; its finite remainder is model dependent;*

5. *negative curvature coupling gives a negative transverse mode for the declared compact-core class and obstructs the massless spectral ground state;*

6. *the first-order tadpole is defined by two disjoint local subtractions, has the coefficients , and has the finite-constant equivalence ;*

7. *the finite-chart $U(2)$ ultraviolet estimate and leading scalar moment hold; the specified boundary chart*

develops a nonzero Mellin logarithm;

8. *the broad KH-Ward off-diagonal uniqueness inference fails in a class admitting . The complete resonant classification, full core-contact limit, physical stress scale shift, all-orders interaction, and Dirac discrepancy remain unclosed.*

Proof. Items (i)–(iii) are Theorems , , and Propositions , . Item (iv) is Theorem with Proposition . Item (v) is Theorem . Item (vi) is Theorem and Proposition . Items (vii)–(viii) are Theorems , , and . The final sentence records the hypotheses and constructions not supplied by these proofs.

Unchanged programme boundaries

Operator, observable, and algebra are different objects

The following separations are part of the scope:

• a spatial Friedrichs limit is not a renormalized one-point limit;

• a one-point c-number extension is not a covariance extension or an interacting apex algebra;

• a worldsheet restriction of an existing bulk field is not an independent brane factor;

• a first-order Euclidean insertion is not an all-orders time-ordered theory;

• a Mellin logarithm is not automatically a supported dilation anomaly;

• a formal Ward repair is not a singular-metric Levi-Civita derivative of arbitrary tensor distributions.

The scalar wavefront problem of I-B-VP-2, the threshold problem of I-B-VP-3, the Maxwell problem of I-B-VP-4, and the operator-selection problem of I-B-VP-5 are not identified with the present constructions [19, 20, 21, 22]. VII-I's finite-width free algebra and VII-H's free response benchmark remain different domains [25, 26].

Standing stops

This companion does not reopen the product-class ledger. The following supplied programme statuses are retained as boundaries, not proved anew as universal nonexistence results:

• Problems 1, 2, and 8 remain unfinished;

• Problems 3–7 and 9–11 are not reached by these results; in particular Problem 6 is not the implication proved here;

• Problem 12 remains unresolved;

• O.1 remains NOT_CLOSED;

• XVI-N is not discharged and XV-B is not opened;

• H2, I3, and J2 remain unresolved.

No spacetime emergence, Einstein dynamics, backreaction, Page curve, de Sitter state, cosmological bounce, or ultraviolet completion is constructed. No inherited characteristic quantum coefficient is admitted.

The philosophy essay is not a registered theorem premise. The external OpenAI catalogue is not an analytical input. No result from an unavailable manuscript is invented.

Novelty and potential usefulness

The contribution is not priority for generalized free fields, Bessel extensions, codimension-two point interactions, or the long-range effect of cosmic-string cores. Those have substantial existing literature [3, 5, 4, 6].

The narrower contributions are:

1. an explicit same-representation domain and range statement for the worldsheet limit used in the supplied one-point programme;

2. an exactly normalized integrated core diagnostic with a proved double-logarithmic coefficient and an explicit transfer obligation to a complete contact;

3. a first-order subtraction prescription in which apex and regular-diagonal residues are separately calculated;

4. a concrete resonance example and a supported KH-Ward counterexample that prevent an overstatement of uniqueness. Their potential impact is methodological as well as local: they identify which additional estimates and

normalization conditions are needed before an apex formula can be read as an operator, a physical core limit, or an interaction.

appendix

An abstract resolvent audit: no gap at infinite form energy

This appendix addresses the two form-convergence sentences quoted in the supplied I-B-VP-5 material. It is an abstract check, not a new proof of that manuscript's geometric hypotheses.

Lemma (Recovery outside the form domain). *Let Q be a closed nonnegative form on a Hilbert space, extended by $+\infty$ outside* $\operatorname{Dom} Q$*. The Mosco recovery inequality*

$$\limsup_n Q_n[u_n] \le Q[u], \qquad u_n \to u,$$

places no energy restriction when $Q[u] = +\infty$.

Proof. Every extended real number is at most $+\infty$. One may take any strongly convergent sequence, for instance the constant sequence. The substantive recovery requirement concerns $\operatorname{Dom} Q$.

Proposition (Strong resolvent conclusion from the two form bounds). *Suppose Q_n, Q are closed nonnegative forms on one Hilbert space, with:*

1. *$u_n \operatorname{rightharpoonup} u$ implies $Q[u] \le \operatorname{liminf} Q_n[u_n]$;*

2. *every $u \in \operatorname{Dom} Q$ has a strong recovery sequence $w_n \to u$ with $Q_n[w_n] \to Q[u]$. Let B_n, B be their associated operators. For every $s > 0$,*

$$(B_n + s)^{-1} f \to (B + s)^{-1} f$$

strongly for every f.

Proof. Let v_n minimize

$$J_n[w] = Q_n[w] + s\|w\|^2 - 2\operatorname{Re}\operatorname{pair} fw.$$

Comparison with zero bounds lVertv_$n\| \le \|f\|/s$. A weakly convergent subsequence has a limit v. The lower bound and a recovery sequence for every competitor show that v minimizes $J[w] = Q[w] + s\|w\|^2 - 2\mathrm{Re}\,\mathrm{pair}\,fw$. That minimizer is unique and equals $(B + s)^{-1}f$. Thus all weak cluster points coincide, and the minimum values converge.

Choose a recovery sequence w_n for v. The minimizer identity gives

$$J_n[w_n] - J_n[v_n] = Q_n[w_n - v_n] + s\text{]Vertw_}n - v_n\|^2.$$

The left side tends to zero. Therefore $w_n - v_n \to 0$, and $v_n \to v$ strongly.

The proof is the standard variational mechanism behind Mosco convergence [10]. Its validity does not supply the additional one-point estimates absent from a strong resolvent theorem.

Submission-source and verification declarations

Source format. This is one source file with an internal bibliography, standard pdfLaTeX packages, and no external figure, data, code, or bibliography-file dependency. Two pdfLaTeX runs are intended for cross-references and the contents.

Execution boundary. No successful compiler run is reported. No numerical quadrature, symbolic algebra program, proof assistant, repository build, or circuit compiler was run for this draft. The source has been written to avoid the undefined macros and unnumbered-reference problems identified in the supplied version history, but that is not an observed build result.

Attribution. The author name follows the supplied manuscript. The Cheung records are cited with their supplied identifiers and versions. Complete supplied source, supplied excerpts, and online introductory records are not described as one uniformly audited corpus.

Submission boundary. The document supplies a self-contained mathematical statement and proof for its released results, with explicit assumptions and stops. It does not assert referee approval, independent verification,

publication acceptance, or resolution of the standing problems.

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Cheung source and companion records. The entries below distinguish the supplied source and supplied excerpts from the external literature. Their citation does not assert a complete corpus audit, an independent compiler run, or a formal certificate.

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Identification, Cheung Ngo Medical Limited (2026). Framing essay; not a theorem premise. Programme note: https://enter.grok.me/enter.

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[16] N. Cheung, *I-B-G: Intrinsic and holonomy-twisted Green operators on a prescribed conical spacetime*, version 2, supplied source and excerpts, Enter Calculus Companion. thm:positivity is a spatial form-domain statement, not the worldsheet trace constructed here. https://enter.grok.me/browse/I-B-G. Companion record; not a Scholar-indexed journal article.

[17] N. Cheung, *I-B-VP: Vacuum polarization of the Friedrichs ground state on a prescribed conical spacetime: Local Hadamard subtraction, holonomy, and the renormalized stress tensor,* version 3 excerpts and version-qualified source material supplied with the problem, Enter Calculus Companion. Relevant comparison items: thm:friedrichs, lem:infrared, thm:hadamard, and the product boundary-relation convention. The spectral and boundary calculations used in the present proofs are written out here. https://enter.grok.me/browse/I-B-VP. Companion record; not a Scholar-indexed journal article.

[18] N. Cheung, *I-B-VP-1: Prescribed apex finite parts on a conical Enter spacetime: Wick square, stress tensor, and charged current: strong extension, Ward identities, and classified contact freedom*, version 3, October 2026. Supplied complete source I-B-VP-1-v4.tex, Enter Calculus Companion. Principal source being extended and corrected. The historical scripts and numerical logs were not rerun here. https://enter.grok.me/browse/I-B-VP-1. Companion record; not a Scholar-indexed journal article.

[19] N. Cheung, *I-B-VP-2: The exact global wavefront relation of the conical Friedrichs ground state*, version 1, supplied version-qualified documentary material, Enter Calculus Companion. Separation only; its global wavefront theorem is not identified with the present boundary-domain argument. https://enter.grok.me/browse/I-B-VP-2. Companion record; not a Scholar-indexed journal article.

[20] N. Cheung, *I-B-VP-3*, version 1, supplied source excerpts, Enter Calculus Companion. Massive and threshold comparison only. No threshold conclusion is transferred from the present worldsheet restriction. https://enter.grok.me/browse/I-B-VP-3. Companion record; not a Scholar-indexed journal article.

[21] N. Cheung, *I-B-VP-4*, supplied documentary comparison, Enter Calculus Companion. Maxwell contacts and its printed discrepancy are separate from the scalar and Dirac audits here. No unavailable full proof is asserted to have been inspected. https://enter.grok.me/browse/I-B-VP-4. Companion record; not a Scholar-indexed journal article.

[22] N. Cheung, *I-B-VP-5*, version 1, supplied source excerpts, Enter Calculus Companion. Relevant supplied items: thm:selection, cor:measure, thm:nonclosable, prop:contactseparation, and thm:release. Strong resolvent operator selection is not a one-point contact limit. https://enter.grok.me/browse/I-B-VP-5. Companion record; not a Scholar-indexed journal article.

[23] N. Cheung, *I-B-VP-AB*, version 2, companion comparison record, Enter Calculus Companion. Not an analytical premise of this extension. Companion record; not a Scholar-indexed journal article.

[24] N. Cheung, *I-B-VP-SAE*, version 3, companion comparison record, Enter Calculus Companion. Not an analytical premise of this extension. The needed boundary-matrix normalization is derived in Lemma . Companion record; not a Scholar-indexed journal article.

[25] N. Cheung, *VII-I*, version 1, supplied excerpts, Enter Calculus Companion. thm:freealgebra concerns a finite-width free algebra, not this worldsheet field or an interacting apex algebra. https://enter.grok.me/browse/VII-I. Companion record; not a Scholar-indexed journal article.

[26] N. Cheung, *VII-H*, version 1, supplied excerpts, Enter Calculus Companion. thm:ambiguity and thm:release concern a separate free-response normalization problem. https://enter.grok.me/browse/VII-H. Companion record; not a Scholar-indexed journal article.

[27] N. Cheung, *X-L-ST*, version 1, supplied excerpts, Enter Calculus Companion. Width, supported-release, and

complete residual-report comparisons only. Preparation width is not core thickness. https://enter.grok.me/browse/X-L-ST. Companion record; not a Scholar-indexed journal article.

[28] N. Cheung, *XI-A-ST*, version 1, supplied source excerpts, Enter Calculus Companion. Classical action, equation-space, finite-report, and non-sourcing boundaries only. No unprovided XI-A theorem is reconstructed as a premise. https://enter.grok.me/browse/XI-A-ST. https://enter.grok.me/browse/XI-A. Companion record; not a Scholar-indexed journal article.

[29] N. Cheung, *X-A-ST*, version 1, supplied excerpts, Enter Calculus Companion. prop:noeinstein and the separation between finite reports and supplied continuum evaluations. https://enter.grok.me/browse/X-A-ST. Companion record; not a Scholar-indexed journal article.

[30] N. Cheung, *Black-Hole Information as an Access Problem: Enter, Quantum Recovery, and Complete Residuals*, figshare (2026). DOI: 10.6084/m9.figshare.34335024.v1. https://doi.org/10.6084/m9.figshare.34335024.v1.

[31] N. Cheung, *Operational Wormhole Certificates—Existence, Dimension-Dependent Scaling, Operational Functions, and Composition*, figshare (2026). DOI: 10.6084/m9.figshare.34549575.v1. https://doi.org/10.6084/m9.figshare.34549575.v1.

[32] N. Cheung, *Enter, Quantum Magic, and Quantum Gravity: Readout, Recovery, and the Limits of Geometric Response*, figshare (2026). DOI: 10.6084/m9.figshare.34335582.v1. https://doi.org/10.6084/m9.figshare.34335582.v1.

[33] N. Cheung, *Operational Consistency Does Not Select Lorentzian Geometry*, figshare (2026). DOI: 10.6084/m9.figshare.34441758.v1. https://doi.org/10.6084/m9.figshare.34441758.v1.

[34] N. Cheung, *Realization of Relational Time by Destructive Enter on a Clock Register*, figshare (2026). DOI: 10.6084/m9.figshare.34442907.v1. https://doi.org/10.6084/m9.figshare.34442907.v1.