I-B-VP-1-Ex2 Spectral tails, exterior contact limits, and admissibility on a prescribed conical spacetime

A domain-explicit continuation of the worldsheet, core, first-order tadpole, and resonance analysis

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Abstract

We continue the corrective apex analysis of I-B-VP-1-Ex1 on the prescribed conical product, without enlarging its interacting or gravitational claims. The main advance is quantitative control of the operations that precede an apex interpretation. An elementary angular Bessel estimate gives a norm rate for the Friedrichs worldsheet restriction, including its vanishing in nontrivial holonomy sectors. For every nonnegative compressed positive-order boundary chart, we prove a matrix Källén–Lehmann formula, uniform threshold and ultraviolet bounds, and an untruncated weighted boundary limit in the original spectral Fock representation. These estimates also establish radial convergence of Wick quadratics on a common dense, energy-cutoff finite-particle domain; they do not assert convergence on arbitrary finite-particle vectors.

A second result upgrades the exterior core diagnostic from a scalar asymptotic to strong distributional concentration. After subtraction of its analytic $-1/(4\pi)\log\log(b/R)$ term, the exterior zero-mode correction converges to a worldsheet contact. A separate moment criterion states exactly when additional core contributions preserve that coefficient. An explicit shrinking-core perturbation shows why exterior data alone cannot determine the complete contact.

We also prove fixed-external-support estimates for the two independent tadpole subtractions, retain the finite contact reparametrization, exhibit an infinite family of actual boundary-chart Mellin logarithms, and classify the

Killing–homothety first-jet counterexample in a specified rotation-equivariant submodule. The potential usefulness is a controlled separation between bulk-derived boundary operators, exterior concentration, and a physically normalized defect observable. No priority is claimed for generalized free fields, Bessel extensions, or codimension-two renormalization. The full core-contact problem, unrestricted resonance and contact classifications, Dirac discrepancy, and interacting completion remain open.

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Book expansion: separated estimate tracks

Three tracks are kept distinct: (i) quantitative boundary-field estimates; (ii) exterior-core concentration; (iii) tadpole. Test spaces and the energy-cutoff domain are declared *before* theorems. Each estimate is labelled proved here or supplied input; removing a spectral cutoff is never silently inherited.

Opening boundaries, background, and exact scope

Enter does not extract the continuum objects constructed here

The philosophy of Enter distinguishes destructive readout, quantum return, a supplied description, and physical identification [14]. The spacetime model of I-B prescribes a classical conical background and a separate finite operational interface [15]. Neither the philosophy nor that finite interface is an analytical premise for a boundary-field or contact-limit theorem.

For clarity, the finite instrument is recalled explicitly. Let C have calibrated orthonormal basis $\{|\tau_j\rangle\}$, let A contain every surviving finite register, and install classical words w_j, including unused and failure labels. Set

$$D_j = \langle \tau_j|_C \otimes I_A, \qquad \mathscr{M}(X) = \sum_j \mathrm{lvert j, } w_j \rangle \mathrm{langle j, } w_j|_R \otimes D_j X D_j^\dagger .$$

Its recorded Kraus maps satisfy

$$\sum_j D_j^\dagger D_j = I_{CA}, \qquad \mathrm{Tr}_R \mathscr{M}(X) = \mathrm{Tr}_C X.$$

Both identities remain true after adjoining any untouched finite reference: expand X in clock matrix units and sum its diagonal blocks. A positive-probability branch retains its actual conditional A-state; no normalized state is assigned to a zero branch. The returned quantum interface does not contain C.

This is the complete supplied-report instrument of I-B and En09-A, not a continuum acquisition theorem [15, thm:instrument] [16]. It does not acquire an unknown worldsheet field, choose a self-adjoint domain, prepare the spectral state, or normalize an apex contact. The geometric time coordinate is not consumed.

Conservation is not a source law

All geometry below is prescribed. A conserved contact or a calculated first-order response does not impose

$$G_{\mu\nu} + \Lambda g_{\mu\nu} = \kappa T_{\mu\nu}.$$

The non-sourcing distinction of I-B prop:not-source and X-A-ST prop:noeinstein is retained [15, 24]. No singular-metric product is defined merely by writing that comparison equation.

Relationship to the supplied manuscripts

The complete sources I-B-VP-1-v4.tex and I-B-VP-1-Ex1-v2.tex are the principal companion records [19, 20]. The present manuscript strengthens their domain estimates and makes several qualifications explicit.

The additional companion citations have documentary or methodological roles:

• I-B-G distinguishes the spatial form domain, extension choice, and supported Green-operator transfer [17].

• I-B-VP supplies the comparison with compact-test infrared estimates and regular-host state constructions [18].

• I-B-VP-5 distinguishes spatial operator selection from one-point contact normalization [21].

• I-B-VP-AB and I-B-VP-SAE are normalization and extension-ledger comparisons, not imported proofs [22, 23].

• VII-H, VII-I, and the IX sheets concern different response, free-product, and causal-product domains [27, 28].

All estimates required for the results released here are given below. A documentary relationship is not a transfer of hypotheses.

What is new and what is not

The principal new statements are:

1. an all-angular Friedrichs restriction estimate with an explicit norm rate;

2. compressed-chart spectral tail bounds and a direct same-representation weighted boundary-limit proof;

3. a strong distributional exterior-contact limit, together with a separate sufficient full-core transfer criterion;

4. fixed-external-support tadpole remainder estimates;

5. an infinite family of evaluated Mellin collisions and a complete calculation in one specified KH first-jet module.

The established background includes generalized free fields, self-adjoint Bessel extensions, local distributional renormalization, and core-sensitive cosmic-string effects [1, 2, 3, 7, 6]. No exhaustive historical priority claim is made.

Host, completion, spectral convention, and operator domains

Fix

$$0 < \alpha < 1, \qquad 0 \le \beta < 1.$$

The regular host is

$$M = \mathbb{R}_t \times (0,\infty)_r \times S^1_\varphi \times \mathbb{R}_z, \qquad g = -\mathrm{d}t^2 + \mathrm{d}r^2 + \alpha^2 r^2 \mathrm{d}\varphi^2 + \mathrm{d}z^2.$$

Its declared completion adds the worldsheet

$$\mathscr{W} = \mathbb{R}_t \times \{0\} \times \mathbb{R}_z.$$

Use the auxiliary smooth structure

$$\overline{M} = \mathbb{R}^{1+1}_y \times \mathbb{R}^2_x, \qquad y = (t,z), \qquad x = (r\cos\varphi, r\sin\varphi),$$

the reference metric

$$\bar{g} = -\mathrm{d}t^2 + \mathrm{d}z^2 + (\mathrm{d}x^1)^2 + (\mathrm{d}x^2)^2,$$

and density

$$\mathrm{d}\Omega = \alpha\, \mathrm{d}^2y\, \mathrm{d}^2x, \qquad \langle \delta_{\mathscr{W}}, f \rangle = \int_{\mathbb{R}^2} f(y,0)\, \mathrm{d}^2y.$$

Thus $\delta_{\mathscr{W}} = \alpha^{-1}\delta^{(2)}(x)$ relative to $\mathrm{d}\Omega$. The coordinates x are not developing coordinates, and g is not assigned a smooth value at $x = 0$.

For the charged field,

$$\Phi(\varphi + 2\pi) = e^{2\pi i\beta}\Phi(\varphi).$$

Use the periodic gauge $e^{-i\beta\varphi}\Phi$ when comparing transverse point limits. Set

$$q_n = n + \beta, \qquad \nu_n = \frac{|n + \beta|}{\alpha}.$$

This is a gauge convention on the punctured plane, not an assertion that the flat holonomy extends smoothly through the apex.

Fourier and mass-shell conventions

For $h \in \mathcal{S}(\mathbb{R}^{1+1})$, let

$$\widehat{h}(\omega, k) = \int_{\mathbb{R}^2} e^{i\omega t - ikz} h(t, z)\, dt\, dz, \qquad V_+ = \{(\omega, k) : \omega > |k|\}.$$

For transverse spectral parameter $\lambda > 0$, write

$$\omega_{\lambda,k} = \sqrt{\lambda^2 + k^2}.$$

The Friedrichs covariance is normalized by

$$W_\beta(X, X') = \frac{1}{8\pi^2\alpha} \sum_{n \in \mathbb{Z}} e^{iq_n(\varphi - \varphi')} \int_{\mathbb{R}} dk \int_0^\infty \frac{\lambda\, d\lambda}{\omega_{\lambda,k}} \cdot e^{-i\omega_{\lambda,k}(t - t' - i0) + ik(z - z')} J_{\nu_n}(\lambda r) J_{\nu_n}(\lambda r').$$

This formula defines the spectral prescription used here. At $\alpha = 1, \beta = 0$ it has the ordinary flat scalar normalization.

For a positive matrix spectral density $\rho(t)$, where t is mass squared, use

$$\Delta_+^{(1+1)}(y; t) = \int_{\mathbb{R}} \frac{dk}{2\pi\, 2\sqrt{k^2 + t}} e^{-i\sqrt{k^2 + t}(t_y - i0) + ikz_y}.$$

The time coordinate in this display is t_y, not the spectral variable t. Equivalently, the Fourier density of

$\int_0^\infty \rho(t)\Delta_+^{(1+1)}(y;t)\mathrm{d}t$ is

$$2\pi 1_{V_+}(p)\rho(p_0^2 - p_1^2).$$

Two different finite-particle domains

Let $\mathscr{H}_1$ be the one-particle space of the relevant spectral covariance, and let $\mathscr{F}_s(\mathscr{H}_1)$ be its bosonic Fock space.

For charged fields use the particle and conjugate antiparticle spaces.

The linear-field domain is

$$\mathcal{D}_{\mathrm{fin}} = \bigcup_{N \geq 0} \mathrm{bigoplus}_{m=0}^{N} \mathscr{H}_1^{\otimes_s m}.$$

For $v \in \mathscr{H}_1$,

$$\|(a(v) + a^*(v))\Psi\| \leq 2\|v\| \, \|(N+1)^{1/2}\Psi\|.$$

For quadratic fields we use the smaller dense domain

$$\mathcal{D}_{\mathrm{c}} = \bigcup_{\mathrm{substack}\, N < \infty}$$

No smoothness of the spectral wave functions is required. This is a common dense domain, not an assertion that it is invariant under every displayed operator. In particular, a Wick quadratic can create particles of unbounded energy.

Remark (A domain correction). A finite particle number does not bound the energy. The convergence of a quadratic field on $\mathcal{D}_{\mathrm{c}}$ must not be promoted to convergence on all of $\mathcal{D}_{\mathrm{fin}}$. No essential self-adjointness theorem for the quadratic operators is claimed.

Stage A: an all-angular Friedrichs restriction estimate

An elementary angular estimate

Define

$$\gamma_\beta = \begin{cases} \min\{2, \alpha^{-1}\}, & \beta = 0, \\ \min(\beta, 1-\beta)/\alpha, & 0 < \beta < 1. \end{cases}$$

Lemma (Uniform angular Bessel control). *For fixed α, β, there is $C < \infty$ such that*

$$\sum_{n \in \mathbb{Z}} |J_{\nu_n}(u)|^2 \le C(1+u), \qquad u \ge 0.$$

For $0 \le u \le 1$,

$$|J_0(u) - 1|^2 + \sum_{n \ne 0} |J_{|n|/\alpha}(u)|^2 \le C u^{2\gamma_0},$$

$$\sum_n |J_{\nu_n}(u)|^2 \le C u^{2\gamma_\beta}, \qquad 0 < \beta < 1.$$

Consequently the left sides of and are bounded for all $u \ge 0$ by

$$C u^{2\gamma_\beta}(1+u).$$

Proof. For real $u > 0$ and $\nu > 0$, Schläfli's integral gives

$$J_\nu(u) = \frac{1}{\pi}\int_0^\pi \cos(u\sin\theta - \nu\theta)\,\mathrm{d}\theta - \frac{\sin(\pi\nu)}{\pi}\int_0^\infty e^{-u\sinh\nu - \nu\nu}\,\mathrm{d}\nu.$$

Hence $|J_\nu(u)| \le 1 + 1/(\pi\nu)$. The positive orders in the fixed angular family have a positive minimum, while $|J_0(u)| \le 1$.

The power series also gives

$$|J_\nu(u)| \le \frac{(u/2)^\nu}{\Gamma(1+\nu)} \exp\left(\frac{u^2}{4(\nu+1)}\right).$$

For $\nu \ge \max\{2u, 2\}$, Stirling's lower bound for $\Gamma(1+\nu)$ makes the right side at most $C\theta^\nu$ for some fixed $\theta < 1$.

There are only $O(1+u)$ remaining orders. Their uniform bound and the summable tail prove .

For $u \le 1$, the same power series and gamma denominator show that the smallest positive order controls the sum.

Also $J_0(u) - 1 = O(u^2)$. These observations prove –. For $u \ge 1$, use . The identities and series used here are classical [4, Sections 10.2 and 10.9].

Norm convergence and spectral tails

Let $v_{h,r,\varphi}$ be the one-particle creation vector obtained by smearing the field in y with h and evaluating transversely at (r,φ), in the periodic gauge. For $\beta = 0$, define $v_{h,0}$ by the zero angular channel with radial factor 1. For $0 < \beta < 1$, set $v_{h,0} = 0$.

Theorem (Quantitative Friedrichs restriction). *For* $0 < r \le 1$, *uniformly in* φ,

$$\mathrm{lVertv_}h, r, \varphi - v_{h,0}\| \le C r^{\gamma_\beta} P_M(h),$$

where P_M *is a Schwartz seminorm.*

The associated linear fields therefore converge strongly on $\mathcal{D}_{\mathrm{fin}}$. *At* $\beta = 0$, *the limit has covariance*

$$W_{\mathscr{H}}(t,z) = \frac{1}{4\pi^2 \alpha} \frac{1}{z^2 - (t - \mathrm{i}0)^2}$$

and creation-vector norm

$$\mathrm{lVertv_}h, 0\|^2 = \frac{1}{8\pi^2 \alpha} \int_{\mathbb{R}} \mathrm{d}k \int_{|k|}^{\infty} |\hat{h}(\omega, k)|^2 \, \mathrm{d}\omega.$$

Its completed one-particle range is exactly the zero-angular bulk channel.

For $0 < \beta < 1$, *the limit is zero. The same conclusions and rate hold after averaging against nonnegative normalized transverse approximate identities supported in* $r \le \varepsilon$.

Uniformly for $0 \le r \le 1$, *the squared creation-vector norm contributed by* $\omega < \eta$, $0 < \eta < 1$, *is* $O(\eta^2)$. *For* $\omega > \Lambda \ge 1$, *it is bounded by*

$$C_M P_M(h)^2 \Lambda^{3-2M}, \qquad M \ge 2.$$

Proof. In the periodic gauge the angular feature vector is

$$\ell_{r,\varphi}(\lambda) = (e^{\mathrm{i}n\varphi} J_{\nu_n}(\lambda r))_{n \in \mathbb{Z}}.$$

Its limiting vector is the zero-channel unit vector for $\beta = 0$, and zero otherwise. By Lemma ,

$$\|\ell_{r,\varphi}(\lambda) - \ell_0\|^2 \le C r^{2\gamma_\beta} \lambda^{2\gamma_\beta}(1+\lambda), \qquad r \le 1.$$

Insert this bound in and use

$$\frac{\lambda\, d\lambda}{\omega_{\lambda,k}} = d\omega, \qquad |k| < \omega.$$

The remaining integral is controlled by a sufficiently high Schwartz seminorm of h, proving . Equation proves the operator limit.

At $\beta = 0$, the same change of variables gives ; its Fourier boundary value is . Restrictions of Schwartz functions to V_+ are dense in $L^2(V_+)$, so the completed range is the full zero angular channel.

For the tails use

$$\|\ell_{r,\varphi}(\lambda)\|^2 \le C(1+\omega).$$

The region $\omega < \eta$, $|k| < \omega$, has area $O(\eta^2)$. On $\omega > \Lambda$, integration of $C\omega(1+\omega)^{1-2M}$ gives . The target vector obeys the same bounds. A normalized positive transverse average preserves by Minkowski's inequality.

Remark (What has been removed from the proof obligation). Theorem needs no unproved uniform-large-order Bessel assertion. The angular tail is estimated explicitly. It also does not infer a transverse point trace from membership in a spatial H^1 form domain. The infrared comparison with I-B-VP lem:infrared is methodological, not a transfer of its compact spatial test estimate to a worldsheet delta test [18, 17].

Non-extraction. The limiting vectors belong to the original spectral Fock representation. Their existence is an analytical statement, not continuum field acquisition by . No contact parameter is selected.

Stage A continued: compressed boundary charts and exact tails

The declared positive-order extension class

Consider finitely many deficient modes with

$$0 < \nu_j < 1, \qquad j = 1, \dots, d.$$

The logarithmic order $\nu = 0$ is not included in this section. A radial component has expansion

$$f_j(r) = a_j r^{-\nu_j} + b_j r^{\nu_j} + o(r^{\nu_j}).$$

Choose a subspace $V \subseteq \mathbb{C}^d$, its orthogonal projection P_V, and a Hermitian $H_V \geq 0$ on V. The boundary relation is

$$a \in V, \qquad P_V \mathrm{diag}\,(\nu_j) b = H_V a.$$

The orthogonal complementary directions are Friedrichs. This is a product boundary law, independent of longitudinal momentum.

Set

$$A_j = 2^{\nu_j - 1}\Gamma(\nu_j), \qquad S_j = \frac{\pi}{2\sin(\pi\nu_j)}, \qquad d_j = \frac{S_j}{A_j^2}, \qquad a_* = \min_j \nu_j, \quad b_* = \max_j \nu_j.$$

The symbol a_* is an exponent, not a boundary coefficient. Let $\iota : V \hookrightarrow \mathbb{C}^d$ be inclusion, and define

$$B(z) = 2H_V + P_V \mathrm{diag}\,(d_j z^{\nu_j})|_V, \qquad F(z) = \iota B(z)^{-1} \iota^{\dagger},$$

with principal powers on $\mathbb{C} \setminus (-\infty, 0]$. When $V = \{0\}$, set $F = 0$.

Lemma (Resolvent normalization). *For $z > 0$, put $E(z) = \mathrm{diag}\,(A_j z^{-\nu_j/2})$. The radial resolvent is*

$$\mathscr{R}_z(r, r') = \mathrm{diag}\left(I_{\nu_j}(\sqrt{z}\, r_<) K_{\nu_j}(\sqrt{z}\, r_>)\right)$$
$$+ \mathrm{diag}\,(K_{\nu_j}(\sqrt{z}\, r))\, E(z)^{-1} F(z) E(z)^{-1}\, \mathrm{diag}\,(K_{\nu_j}(\sqrt{z}\, r')).$$

In a full chart,

$$E^{-1} F E^{-1} = (S + 2EHE)^{-1}.$$

Proof. The radial Wronskian of I_ν, K_ν gives the Friedrichs Green normalization. At zero,

$$I_\nu(\sqrt{z}\, r) \sim \frac{2^{-\nu} z^{\nu/2}}{\Gamma(1+\nu)} r^\nu,$$

$$K_\nu(\sqrt{z}\, r) \sim A_\nu z^{-\nu/2} r^{-\nu} + 2^{-\nu-1}\Gamma(-\nu) z^{\nu/2} r^\nu.$$

The identities

$$2\nu\frac{2^{-\nu}}{\Gamma(1+\nu)}A_{\nu}=1,\qquad \frac{2^{-\nu}}{\Gamma(1+\nu)}S_{\nu}+2^{-\nu-1}\Gamma(-\nu)=0$$

give upon imposing . Compression is imposed on the boundary coefficients before inversion. The standard Bessel-extension setting is discussed in [2, 3]; the normalization used here follows from the displayed calculation.

A uniform matrix estimate

For a matrix T, write $\operatorname{Im} T=(T-T^{\dagger})/(2\mathrm{i})$.

Lemma (Coercivity and cut bounds). *The matrix $B(z)$ is invertible off the negative axis. There is C, depending on the fixed orders and d_j, but not on V or $H_V\ge 0$, such that*

$$\mathrm{lVertF}(z)\|\le\begin{cases}C|z|^{-b_*}, & 0<|z|\le 1,\\ C|z|^{-a_*}, & |z|\ge 1.\end{cases}$$

The same estimates hold on both shores of the cut.

For $t>0$, set

$$Y(t)=P_V\operatorname{diag}\left(d_j\sin(\pi\nu_j)t^{\nu_j}\right)|_V.$$

Then

$$0\le -\operatorname{Im}B(-t+\mathrm{i}0)^{-1}=B(-t+\mathrm{i}0)^{-1}Y(t)B(-t+\mathrm{i}0)^{-*}\le Y(t)^{-1}.$$

Proof. For $z=|z|e^{\mathrm{i}\theta}$, $|\theta|\le\pi$, the arguments of the terms in the quadratic form of $B(z)$ lie between 0 and $b_*\theta$, in the appropriate orientation. Rotate the form by $e^{-\mathrm{i}b_*\theta/2}$. Its real part is at least

$$\cos(\pi b_*/2)\Big(2\mathrm{langleu},H_V u\rangle+\sum_j d_j|z|^{\nu_j}|(\iota u)_j|^2\Big).$$

Since $b_*<1$, the cosine is strictly positive. This proves invertibility and .

For the last estimate, write $B=X+\mathrm{i}Y$ on the upper shore and conjugate by $Y^{-1/2}$. The matrix $Y^{-1/2}XY^{-1/2}$ is Hermitian, and

$$\|(Y^{-1/2}XY^{-1/2} + \mathrm{i}I)^{-1}\| \le 1.$$

This proves the upper bound in . The identity in the middle follows by subtracting the inverse and its adjoint.

Theorem (Compressed matrix Källén–Lehmann measure). *The Euclidean boundary covariance*

$$\widetilde{W}_a(z) = \frac{1}{2\pi\alpha}F(z)$$

has the Stieltjes representation

$$\widetilde{W}_a(z) = \int_0^\infty \frac{\rho_a(t)}{z+t}\,\mathrm{d}t, \qquad \rho_a(t) = \frac{1}{2\pi^2\alpha}\iota B_+(t)^{-1}Y(t)B_+(t)^{-*}\iota^\dagger \ge 0,$$

where $B_+(t) = B(-t + \mathrm{i}0)$. There are no additional polynomial terms or zero-mass atoms in this representation.

Uniformly over $V, H_V \ge 0$,

$$\|\rho_a(t)\| \le \begin{cases} Ct^{-b_*}, & 0 < t \le 1, \\ Ct^{-a_*}, & t \ge 1. \end{cases}$$

It defines a continuous positive Wightman covariance on $\mathcal{S}(\mathbb{R}^{1+1};\mathbb{C}^d)$:

$$W_a(y) = \int_0^\infty \rho_a(t)\Delta_+^{(1+1)}(y;t)\,\mathrm{d}t.$$

Proof. The cut identity in Lemma gives the stated positive density and its estimates. Apply a keyhole contour entrywise to $F(z)/(z - z_0)$. The large circle vanishes because $a_* > 0$. The small circle vanishes because $b_* < 1$. This gives the Stieltjes formula with no polynomial or zero-mass term.

To verify the Wightman domain explicitly, set

$$q_h(t) = \int_{\mathbb{R}} \frac{|\widehat{h}(\sqrt{k^2+t},k)|^2}{2\pi\,2\sqrt{k^2+t}}\,\mathrm{d}k.$$

For every sufficiently large M,

$$q_h(t) \le \begin{cases} CP_M(h)^2(1 + |\log t|), & 0 < t \le 1, \\ C_M P_M(h)^2 t^{-M}, & t \ge 1. \end{cases}$$

For the first bound split $|k| \le 1$ from its complement. The only threshold integral is $\int_0^1 (k^2+t)^{-1/2}\mathrm{d}k = O(1 + |\log t|)$.

For the second put $k = \sqrt{t}\, v$ and use Schwartz decay. Equations and prove continuity and finiteness. Positivity follows from the positive spectral measure.

Corollary (Quantitative boundary spectral tails). *The squared boundary creation-vector norm contributed by* $0 < t < \varepsilon < 1$ *is bounded by*

$$CP_M(h)^2 \varepsilon^{1-b_*}(1 + |\log \varepsilon|).$$

For $t > T \geq 1$*, it is bounded by*

$$C_M P_M(h)^2 T^{1-a_*-M}, \qquad M + a_* > 1.$$

Proof. Integrate the products of and .

The untruncated limit in the bulk representation

Define the weighted modal fields

$$A_{r,j}(h) = r^{\nu_j} \frac{1}{2\pi} \int_0^{2\pi} e^{-\mathrm{i} q_j \varphi} \Phi(h, r, \varphi)\, \mathrm{d}\varphi.$$

Theorem (Same-representation compressed boundary limit). *For every relation and* $h \in \mathcal{S}(\mathbb{R}^{1+1}; \mathbb{C}^d)$*, the fields* $A_r(h)$ *have a strong limit* $a(h)$ *on* $\mathcal{D}_{\mathrm{fin}}$ *in the original spectral Fock representation. Its covariance is –, and its boundary coefficient vector lies in V.*

For fixed chart and orders there is a Schwartz seminorm P_M *such that the creation vectors satisfy*

$$\mathrm{lVertv_}r(h) - v_a(h)\| \leq C r^{2a_*} P_M(h), \qquad 0 < r \leq r_0 \leq 1.$$

The constants can be chosen uniformly over $V, H_V \geq 0$ *with the orders fixed.*

No spectral cutoff remains in the conclusion.

Proof. Introduce the diagonal matrix

$$D_r(z) = \mathrm{diag}\left(\frac{(\sqrt{z}\, r)^{\nu_j} K_{\nu_j}(\sqrt{z}\, r)}{A_j} \right).$$

After multiplication of the two endpoints by their respective powers of r, the correction term in is

$$D_r(z)F(z)D_{r'}(z).$$

Its boundary-to-boundary limit is $F(z)$.

On a compact spectral interval $0 < \varepsilon \le t \le T < \infty$, the small-argument Bessel expansions are uniform and give the mixed endpoint limits. Stone's formula therefore gives convergence of the corresponding finite spectral covariance blocks, including the mixed bulk–boundary block.

Here is the estimate that removes both spectral cutoffs. Let $\rho_{\mathrm{diff},r}(t)$ denote the positive spectral matrix of the difference between the weighted radial evaluation and its boundary limit. By the resolvent identity just displayed, its correction part is the cut jump of

$$(D_r(z) - I)F(z)(D_r(z) - I).$$

Its Friedrichs part is diagonal and proportional to

$$r^{2\nu_j}J_{\nu_j}(\sqrt{t}\,r)^2.$$

Positivity follows either from the finite spectral Gram blocks or their endpoint limit.

For $t \le 1$, the Bessel expansions on the cut give

$$\mathrm{lVertD_}r(-t + \mathrm{i}0) - I\| \le Cr^{2a_*}t^{a_*}.$$

The free term is bounded by $Cr^{4a_*}t^{a_*}$. Lemma thus yields

$$\|\rho_{\mathrm{diff},r}(t)\| \le Cr^{4a_*}(t^{a_*} + t^{2a_* - b_*}), \qquad 0 < t \le 1.$$

Since $2a_* - b_* > -1$, this is integrable against $1 + |\log t|$.

For $t \ge 1$, the fixed-order Hankel asymptotics of $K_\nu(\mathrm{i}u)$, together with the small-argument series when $u \le 1$, imply

$$\|\rho_{\mathrm{diff},r}(t)\| \le Cr^{4a_*}(1 + t)^m$$

for some fixed integer m. To see that the factor r^{4a_*} is legitimate also when $\sqrt{t}\, r \geq 1$, use $r^{-2a_*} \leq t^{a_*}$ and absorb that power into $(1 + t)^m$. All remaining Bessel factors have fixed orders and polynomial growth on the cut. The same argument bounds the individual radial spectral matrices by Ct^{-b_*} at zero and a fixed polynomial at infinity.

Multiplying – by proves . One may first establish these estimates for finite spectral blocks. They make the radial creation vectors Cauchy in the original one-particle space, so their limit belongs to that space. The mixed endpoint calculation identifies its covariance with F, not merely with an abstract reconstructed copy. Finally apply .

The range is contained in V because $F = \imath B^{-1} \imath^{\dagger}$. All inverse estimates used in the proof are uniform in the nonnegative compressed chart.

Remark (Compression must precede inversion). In general

$$\left(P_V D|_V\right)^{-1} \neq P_V D^{-1}|_V.$$

Replacing the left side by the right side changes the boundary field. Theorem uses the actual compressed inverse throughout. Neither a spatial form-domain statement nor a positive Euclidean matrix at one momentum would prove this result.

An explicit compressed-chart crossover

Proposition (Which order controls a mixed rank-one chart). *Let $V = \mathbb{C}v$, $\|v\| = 1$, and $H_V = hI_V$, $h \geq 0$. Suppose v has nonzero components in two different orders $0 < \nu_- < \nu_+ < 1$. Put*

$$a_{\pm} = d_{\pm}|v_{\pm}|^2.$$

The nonzero covariance is $vv^{\dagger} f(z)$, where

$$f(z) = \frac{1}{2\pi\alpha} \frac{1}{2h + a_- z^{\nu_-} + a_+ z^{\nu_+}}.$$

Its scalar spectral density is

$$\rho_v(t) = \frac{a_- \sin(\pi\nu_-) t^{\nu_-} + a_+ \sin(\pi\nu_+) t^{\nu_+}}{2\pi^2 \alpha |2h + a_- e^{i\pi\nu_-} t^{\nu_-} + a_+ e^{i\pi\nu_+} t^{\nu_+}|^2}.$$

At separated Euclidean points its short-distance power has dimension $1 - \nu_+$. *If* $h = 0$, *its long-distance power has dimension* $1 - \nu_-$. *If* $h > 0$, *its leading long-distance noncontact power has dimension* $1 + \nu_-$. *All leading separated-point coefficients are positive.*

Proof. Compression gives , and its cut jump gives . At large t, the density is a positive constant times $t^{-\nu_+}(1 + o(1))$. At small t, it is a positive constant times $t^{-\nu_-}$ when $h = 0$, and times t^{ν_-} when $h > 0$.

The Euclidean separated-point covariance is

$$\frac{1}{2\pi}\int_0^\infty \rho_\nu(t) K_0(\sqrt{t}\, D)\, \mathrm{d}t, \qquad D > 0.$$

Substitute $t = u^2/D^2$. The endpoint bounds of Theorem , the exponential decay of K_0, and its logarithmic behavior at zero justify dominated convergence in the relevant limit. The integral

$$\int_0^\infty u^{\lambda-1} K_0(u)\, \mathrm{d}u = 2^{\lambda-2}\Gamma(\lambda/2)^2, \qquad \lambda > 0,$$

is strictly positive. It gives the stated powers and positive coefficients.

This calculation is a compressed-chart result, not a claim that arbitrary off-diagonal boundary fields are independent scalar channels. The spectral generalized-free-field terminology is used in its Gaussian sense [1].

Non-extraction and scope. The boundary fields are limits in a supplied free spectral representation. They are not independently acquired defect degrees of freedom, and the instrument does not choose V, H_V. The logarithmic channel, non-product boundary laws, and interacting boundary dynamics remain outside this theorem.

Stage A concluded: Wick quadratics with a common domain

A spectral criterion

Lemma (Positive-energy convolution estimate). *Let a finite-component Gaussian field have positive Fourier density* $w(p)$ *supported in* V_+, *and suppose*

$$\operatorname{Tr} w(p) \le C(1 + p_0)^m (p_+ p_-)^{-b}, \qquad p_\pm = p_0 \pm p_1, \qquad 0 \le b < 1.$$

Then:

1. *its quadratic pair-creation kernels, smeared by* $h \in \mathcal{S}(\mathbb{R}^{1+1})$*, are Hilbert–Schmidt;*

2. *its number-preserving quadratic kernels map every one-particle energy band* $p_0 \le E$ *boundedly into the one-particle space;*

3. *the normal-ordered quadratic fields are defined on* $\mathcal{D}_c$*. Hermitian quadratic combinations with real tests are symmetric and closable.*

Proof. For total momentum $P \mathrm{in} V_+$, both p and $P - p$ lie in V_+ precisely when

$$0 < p_+ < P_+, \qquad 0 < p_- < P_-.$$

The Jacobian is $\mathrm{d}p_0 \mathrm{d}p_1 = \frac{1}{2}\mathrm{d}p_+ \mathrm{d}p_-$. Consequently the convolution of the two singular factors is bounded by

$$C(1 + P_0)^{2m} P_+^{1-2b} P_-^{1-2b},$$

using twice the beta integral

$$\int_0^A u^{-b}(A-u)^{-b}\,\mathrm{d}u = A^{1-2b}\mathrm{B}(1-b, 1-b).$$

The result is locally integrable because $b < 1$, and its growth is polynomial. Multiplication by $|\widehat{h}(P)|^2$ is therefore integrable. This is the Hilbert–Schmidt pair-creation estimate.

For the number-preserving kernel, the relevant factor is $\widehat{h}(p - q)$. On $q_0 \le E$, the q-measure has finite mass by . The p-integral is bounded uniformly in that compact energy region by Schwartz decay. The kernel with the input restricted to that band is Hilbert–Schmidt, hence bounded. The usual creation and annihilation estimates on each finite particle sector now define the quadratic operator on $\mathcal{D}_c$. For Hermitian combinations its adjoint contains the same dense domain, proving closability.

Theorem (Quadratic boundary limits). *The following Wick quadratics exist on* $\mathcal{D}_c$*:*

1. *the neutral Friedrichs worldsheet square* $:\Phi^2_{\mathscr{M}}:(h)$*;*

2. *the finite-component quadratics* $:a_j a_k^\dagger:(h)$ *of Theorem , with conjugate sectors included for charged fields. They are strong limits on that domain of the corresponding worldsheet-smeared radial Wick quadratics. For every fixed particle-number and input-energy bound,*

$$\|(Q_{r,\varphi}(h)-Q_{\mathscr{W}}(h))\Psi\| \le C_{N,E} r^{\gamma_\beta} P_M(h)\|\Psi\|,$$

$$\left\|(Q_r^a(h)-Q^a(h))\Psi\right\| \le C_{N,E} r^{2a_*} P_M(h)\|\Psi\|.$$

In a nontrivial Friedrichs holonomy sector the limiting normal-ordered boundary quadratic is zero.

For the neutral Friedrichs square,

$$\left\langle \Omega, :\Phi^2_{\mathscr{W}}:(y):\Phi^2_{\mathscr{W}}:(y')\Omega\right\rangle = 2W_{\mathscr{W}}(y-y')^2,$$

and its exact Fourier density is

$$\widetilde{W}_Q(P)=\frac{P_0^2-P_1^2}{16\pi^2\alpha^2}1_{V_+}(P).$$

Proof. The Friedrichs radial evaluation density is bounded by $C(1+p_0)$ by Lemma . The boundary limit has constant density $(2\alpha)^{-1}1_{V_+}$. Thus Lemma applies with $b=0$.

For the weighted fields, Theorem gives with $b=b_*<1$, after multiplication by a fixed polynomial in p_0. The individual weighted radial densities obey the same type of bound by the proof of Theorem .

To prove the radial convergence, realize the radial and boundary creation features in their common one-particle spectral space. For Friedrichs their difference obeys the feature estimate used in . For the weighted fields its squared spectral norm obeys –. The elementary identity

$$u_r\otimes v_r-u_0\otimes v_0=(u_r-u_0)\otimes v_r+u_0\otimes(v_r-v_0)$$

and Cauchy–Schwarz reduce the quadratic kernel differences to the same convolution estimates with one difference feature. They give the factors r^{γ_β} and r^{2a_*}, respectively. For the number-preserving terms restrict the input to the energy band before applying the Hilbert–Schmidt estimate. The finite-sector creation and annihilation

bounds give –. All spectral tails are controlled by Schwartz decay; no fixed ultraviolet cutoff remains.

Wick's rule proves . For , convolve the two densities $(2\alpha)^{-1}1_{V_+}$, with the Wick factor 2 and Fourier factor $(2\pi)^{-2}$. The diamond $\{p\text{in}V_+ : P - p\text{in}V_+\}$ has area $(P_0^2 - P_1^2)/2$. This gives the displayed coefficient.

Remark (Quadratic does not mean Gaussian). The nonzero Wick square is not itself a Gaussian generalized free field. At separated points its three-point function contains

$$8W(y_1,y_2)W(y_2,y_3)W(y_3,y_1),$$

which is not identically zero. This preserves the correction made in Ex1 [20].

The c-number obstruction is unchanged

Put

$$E_x = x^i\partial_i, \qquad \sigma_\alpha = 2\pi\alpha,$$

and define the radial finite part

$$\text{langleU_}L, f\rangle = \lim_{\varepsilon\text{downarrow}\,0}\Big[\int_{r>\varepsilon} r^{-2}f\,\mathrm{d}\Omega + \sigma_\alpha\log(\varepsilon/L)\int f(y,0)\,\mathrm{d}^2y\Big].$$

Angular Taylor averaging proves existence, with $O(\varepsilon^2 p_2(f))$ error, and

$$(E_x + 2)U_L = \sigma_\alpha\delta_{\mathscr{W}}, \qquad U_{L'} - U_L = -\sigma_\alpha\log(L'/L)\delta_{\mathscr{W}}.$$

These are the one-point finite-part conventions of the parent series [19, 20].

Proposition (No ordinary restriction of the nonzero c-number). *If $s \neq 0$, no distribution $sU_L + c\delta_{\mathscr{W}}$ has an approximate-identity-independent ordinary restriction to $\mathscr{W}$. If $s = 0, c \neq 0$, the same is true. The identically zero case is an exception.*

Proof. For $\eta_\varepsilon(x) = \varepsilon^{-2}\eta(x/\varepsilon)$, with $\eta(0) \neq 0$, gives

$$\text{langleU_}L, \eta_\varepsilon\rangle = \varepsilon^{-2}\big(\text{langleU_}L, \eta\rangle + \sigma_\alpha\log\varepsilon\,\eta(0)\big).$$

The contact adds $c\varepsilon^{-2}\eta(0)$. No fixed c removes the logarithm when $s \neq 0$. For $s = 0, c \neq 0$, the inverse-square divergence remains. Longitudinal smearing does not change the argument.

Stopping condition (Exact free-domain ending). Theorems , , and establish free boundary operators and their radial limits on the specified domains. They do not establish:

• an interacting defect algebra;

• every bulk Wick product smeared by an arbitrary completion test touching $\mathscr{W}$;

• essential self-adjointness of every quadratic field;

• convergence of quadratics on all finite-particle vectors;

• a restriction of the nonzero covariant c-number part. A finite Enter report does not supply any missing domain or product. The free algebras and response maps of VII-H and VII-I remain separate [27].

Stage B: the exterior diagnostic as a distributional contact limit

Explicitly normalized reflection profiles

For the flower-pot model use a flat interior disk of geodesic radius αR, matched to an exterior cone at exterior radius R. The integrated scalar curvature is $4\pi(1-\alpha)$. For the neutral zero angular mode put

$$\ell_\xi(u) = u\frac{I_1(\alpha u)}{I_0(\alpha u)} + \frac{2\xi(1-\alpha)}{\alpha},$$

$$B_\xi(u) = \frac{uI_1(u) - \ell_\xi(u)I_0(u)}{uK_1(u) + \ell_\xi(u)K_0(u)}.$$

The derivative jump

$$v'_+ - v'_- = \frac{2\xi(1-\alpha)}{\alpha R}v$$

gives this formula by matching $I_0(pr) + B_\xi(pR)K_0(pr)$ to the interior regular solution.

For $\xi \geq 0$ the denominator is positive. The Bessel endpoint expansions give

$$B_0(u) = \frac{1-\alpha}{2}u^2 + O(u^4|\log u|),$$

$$B_\xi(u) = -\frac{1}{\log(\kappa_\xi/u)} + O(\log^{-2}(1/u)), \qquad \xi > 0,$$

$$B_\xi(u) = O(e^{2u}/u), \qquad u \to \infty.$$

Only these explicitly stated profile properties are needed below.

Define

$$f_R(r) = \frac{1}{4\pi^2\alpha}\int_0^\infty p\,B(pR)K_0(pr)^2\,\mathrm{d}p, \qquad r > R,$$

and

$$A(u) = \int_u^\infty vK_0(v)^2\,\mathrm{d}v, \qquad Q_R(b) = 2\pi\alpha\int_R^b rf_R(r)\,\mathrm{d}r.$$

Here $b > 0$ is fixed. The Bessel integral gives

$$A(0) = \frac{1}{2}, \qquad A(u) - \frac{1}{2} = O(u^2\log^2 u), \qquad A(u) = O(e^{-2u}) \quad (u \to \infty).$$

Assumption (Profile class for the exterior theorem). The profile B is locally bounded on $(0,\infty)$, has at most $C(1+u)^m e^{2u}$ growth, and

$$\int_1^\infty |B(u)|A(u)\frac{\mathrm{d}u}{u} < \infty.$$

At zero it obeys one of:

1. $B(u) = O(u^\eta)$ for some $\eta > 0$;

2. $B(u) = -1/\log(\kappa/u) + O(\log^{-2}(1/u))$, with $\kappa > 0$.

The coefficient and the stronger limit

Theorem (Exterior mass and strong concentration). *Under Assumption :*

1. *in the power-tail case,*

$$Q_R(b) \to Q_0 = \frac{1}{2\pi}\int_0^\infty B(u)A(u)\,\frac{\mathrm{d}u}{u};$$

2. *in the logarithmic case, there is a finite* C_B *such that*

$$Q_R(b) = -\frac{1}{4\pi}\log\log(b/R) + C_B + o(1);$$

3. *let* $\mathscr{E}_R$ *be the distribution equal to* $f_R(r)$ *on* $R < r < b$ *and zero elsewhere transversely, with longitudinally constant coefficient. On tests compactly supported in* $r < b$*,*

$$\mathscr{E}_R - Q_R(b)\delta_{\mathscr{W}} \to 0 \quad \text{strongly in} \mathcal{D}'.$$

Consequently, in the logarithmic case,

$$\text{boxed}\,\mathscr{E}_R + \frac{1}{4\pi}\log\log(b/R)\delta_{\mathscr{W}} \to C_B\delta_{\mathscr{W}}$$

strongly on every fixed compact test-support stage in $r < b$*.*

Proof. Changing variables in gives the exact identity

$$Q_R(b) = \frac{1}{2\pi}\int_0^\infty B(u)[A(u) - A((b/R)u)]\frac{\mathrm{d}u}{u}.$$

Absolute convergence follows from the endpoint assumptions. In the power-tail case dominated convergence proves (i).

For the logarithmic case put $T = b/R$, and choose u_0 small enough that $\log(\kappa/u) > 0$ below u_0. The $O(\log^{-2})$ remainder contributes a finite limit, because

$$\int_0^{u_0} \frac{\mathrm{d}u}{u\log^2(\kappa/u)} < \infty.$$

Replacing $A(u)$ there by 1/2 also changes only a convergent integral. The smooth lower cutoff furnished by $A(Tu)$ may be replaced by the sharp cutoff $u = T^{-1}$ with $o(1)$ error. Indeed, after $v = Tu$, the discrepancy is integrable in $\mathrm{d}v/v$: it is $O(v^2\log^2 v)$ at zero and exponentially small at infinity. On $v \le \sqrt{T}$, its logarithmic denominator is bounded below by a fixed multiple of $\log T$; the complementary tail is exponentially small. The divergent part is

therefore exactly

$$-\frac{1}{4\pi}\int_{T^{-1}}^{u_0}\frac{\mathrm{d}u}{u\log(\kappa/u)} = -\frac{1}{4\pi}\log\log T + \text{constant} + o(1).$$

This proves (ii), including existence of its finite remainder.

We next prove the stronger distributional assertion. For $r \geq 2R$, the change $v = pr$ and the growth assumption give

$$|f_R(r)| \leq Cr^{-2}, \qquad f_R(r) \to 0 \quad \text{for each fixed} r > 0.$$

The dominating integrand is a polynomial times $vK_0(v)^2e^v$, which is integrable at infinity; near zero use the boundedness of B. On the inner annulus,

$$2\pi\alpha\int_R^{2R} r|f_R(r)|\,\mathrm{d}r \leq \frac{1}{2\pi}\int_0^{\infty}|B(u)|[A(u) - A(2u)]\,\frac{\mathrm{d}u}{u} < \infty,$$

uniformly in R. At zero the difference of the two A's is $O(u^2\log^2 u)$, so this integral converges also for a logarithmic profile.

For a smooth test, let $\bar{f}(y, r)$ denote its angular average. Taylor averaging gives

$$|\bar{f}(y, r) - f(y, 0)| \leq Cr^2 p_2(f).$$

Thus

$$\left|\langle \mathscr{E}_R - Q_R(b)\delta_{\mathscr{W}}, f\rangle\right| \leq C_K p_2(f)\int_R^b r^3|f_R(r)|\,\mathrm{d}r.$$

The part below $2R$ tends to zero by . Above $2R$, supplies the integrable dominator Cr and pointwise convergence to zero. The last integral therefore tends to zero. The estimate is uniform on bounded sets of tests with fixed compact support, proving strong convergence. Combining it with (ii) gives .

Remark (The advance beyond the scalar diagnostic). Equation proves a distributional contact limit for the exterior zero-mode correction itself. It is stronger than the assertion that one radial integral has a double logarithm. It is still not the complete renormalized core Wick square or the core stress tensor. The long-range exterior context

is the work of Allen–Kay–Ottewill [6]; no complete interior normalization is imported from that comparison.

Stage B continued: a full-core transfer criterion and its obstruction

Theorem (Moment-controlled transfer to an assembled correction). *Suppose a separately specified renormalized core correction, in the same density and smooth structure, has decomposition*

$$\mathscr{F}_R = \mathscr{E}_R + \mathscr{I}_R + \mathscr{T}_R.$$

Assume:

1. $\mathscr{I}_R = i_R(x)$ *is radial, supported in* $r \le c_0 R$*;*

2. *its mass*

$$m_R = \int_{\mathbb{R}^2} i_R(x)\alpha\, \mathrm{d}^2x \to m_0 \in \mathbb{R};$$

3. *its second absolute moment satisfies*

$$\int_{\mathbb{R}^2} |x|^2 |i_R(x)|\alpha\, \mathrm{d}^2x \to 0;$$

4. $\mathscr{T}_R \to \mathscr{T}_0$ *strongly in* $\mathcal{D}'$*, with a fixed specified limit. Then, in the logarithmic-profile case,*

$$\mathscr{F}_R + \frac{1}{4\pi}\log\log(b/R)\delta_{\mathscr{W}} \to \mathscr{T}_0 + (C_B + m_0)\delta_{\mathscr{W}}.$$

The statement also holds for uniformly controlled longitudinally smeared versions of these hypotheses.

Proof. Angular Taylor averaging gives

$$\left|\langle \mathscr{I}_R - m_R\delta_{\mathscr{W}}, f\rangle\right| \le C_K p_2(f) \int |x|^2 |i_R(x)|\alpha\, \mathrm{d}^2x.$$

Hence $\mathscr{I}_R \to m_0\delta_{\mathscr{W}}$ strongly. Apply Theorem and add the given strong limit of $\mathscr{T}_R$. The same estimate after longitudinal integration proves the last assertion.

Proposition (Exterior information does not determine the complete coefficient). *Choose radial* $\psi \in C_c^\infty(\{r < 1\})$ *with* $\int \psi\, \alpha \mathrm{d}^2x = 1$*. For any real sequence* a_R *satisfying* $R^2|a_R| \to 0$*, the shrinking-core modification*

$$j_R(x) = a_R R^{-2}\psi(x/R)$$

obeys

$$j_R - a_R\delta_{\mathscr{M}} \to 0 \quad \text{strongly in} \mathcal{D}'.$$

It changes no exterior value for $r > R$.

Taking $a_R = a$ changes the finite contact arbitrarily. Taking $a_R = \eta\log\log(b/R)$ changes the complete double-logarithmic coefficient while leaving the exterior diagnostic unchanged.

Proof. The radial first moment vanishes. Taylor's theorem gives

$$|\text{langlej_}R - a_R\delta_{\mathscr{M}}, f\rangle| \le C_K|a_R|R^2 p_2(f).$$

This proves and the asserted examples.

Remark (Meaning of the counterexample). Proposition is a nonimplication about incomplete normalization data. It does not assert that an arbitrary j_R is the Wick square of a fixed physical core state in a fixed renormalization prescription. Rather, a claim about that fixed physical prescription must exclude this freedom by proving its interior and subtraction estimates. Calling the exterior calculation a complete core theorem would omit those estimates.

The other retained core boundaries

For negative coupling, the direct variational conclusion of Ex1 remains valid on its compact-curvature core class [20]. Its elementary proof is worth recalling. Take a radial function equal to one on the curvature support, then equal to

$$u_b(r) = \frac{\log(b/r)}{\log(b/R)} \quad (R < r < b).$$

Its exterior kinetic energy is $2\pi\alpha/\log(b/R)$, whereas its curvature energy is $4\pi\xi(1-\alpha) < 0$. For sufficiently large b the quadratic form is negative. For the compact smooth or circle-trace perturbations in question, the negative

spectrum is discrete below $[0,\infty)$. Thus the massless nonnegative spectral-ground-state construction is obstructed. This does not exclude every nonstationary Hadamard state.

The scalar Hadamard scale shift can determine a curvature contribution to the scalar contact, but it does not determine the finite traceless stress contact. The smooth-core stress counterterms contain derivatives of curvature as well as curvature products [10, 9]. Their thin-core finite remainder must be calculated, not set to zero because a singular expression has not been defined.

Stopping condition (Physical transfer remains open). Theorem is a proved sufficient implication, with a nonempty mathematical profile class. Its hypotheses have not been verified here for the complete flower-pot one-point prescription, its nonzero angular modes, or the stress-contact coordinate τ. The physical full-contact problem is therefore still open.

Strong resolvent selection in I-B-VP-5 does not establish these one-point hypotheses [21]. Nor does a contact or its conservation supply Einstein dynamics [15, 24].

Stage C: two-locus tadpole subtraction with test-uniform errors

Kernel and prescription

Take the neutral real massless Friedrichs field, with interaction $\lambda\varphi^4/4!$. The Euclidean kernel is

$$G_F(X,X') = \frac{1}{8\pi^2\alpha r r'}\frac{\sinh(\eta/\alpha)}{\sinh\eta\cosh(\eta/\alpha)-\cos(\varphi-\varphi')}, \qquad \cosh\eta = \frac{r^2+(r')^2+|y-y'|^2}{2rr'}.$$

This is the same normalization used in Ex1 [20]. It follows by summing the angular geometric series in the separated Euclidean Green kernel. At an apex endpoint,

$$G_F(X,(y',0)) = \frac{1}{4\pi^2\alpha(r^2+|y-y'|^2)}.$$

Therefore

$$I(r) = \int_{\mathscr{M}} G_F(X,(y',0))^2\,\mathrm{d}^2y' = \frac{1}{16\pi^3\alpha^2r^2}.$$

Let

$$s = \frac{\alpha^{-2} - 1}{48\pi^2} > 0.$$

For external $r > 0$, define

$$\mathscr{T}_{\varepsilon,\delta}(X) = \int_{\text{substack } r' > \varepsilon}$$

$$+\sigma_\alpha s \log(\varepsilon/L) I(r) + \frac{s}{8\pi^2 r^2} \log(\mu\delta) + cI(r).$$

The coincidence distance is measured in a regular flat developing chart near X. The apex and regular-diagonal subtraction neighborhoods are chosen disjoint. The integral at infinity is improper.

Theorem (External-support uniform convergence). *Let* $K \text{Subset } M_E$ *be a compact external Euclidean set with* $r \geq r_0 > 0$. *For every fixed* $k \geq 0$, *there are constants* $C_{K,k}$ *such that*

$$\| \mathscr{T}_{\varepsilon,\delta} - \mathscr{T}_{L,\mu,c} \|_{C^k(K)} \leq C_{K,k}(\varepsilon^\gamma + \delta), \qquad \gamma = \min\{2, \alpha^{-1}\},$$

for sufficiently small independent cutoffs. In particular, the limit exists independently of their relative approach to zero and is smooth for $r > 0$.

For the fixed finite subtraction convention there is a dimensionless constant A_α *such that*

$$\delta_\lambda F(r) := -\frac{\lambda}{2} \mathscr{T}_{L,\mu,c}(r)$$

$$= -\frac{\lambda c}{32\pi^3 \alpha^2 r^2} - \frac{\lambda s}{16\pi^2 \alpha r^2} \log(r/L) - \frac{\lambda s}{16\pi^2 r^2} \log(\mu r) + \frac{\lambda s A_\alpha}{r^2}.$$

Proof. Near the apex, expand in $e^{-\eta/\alpha}$ and in $r'\wedge 2$. Uniformly for $X \in K$, the zero angular term has its endpoint value , and the first correction is bounded by $Cr'\wedge\gamma$. After any fixed external derivatives, the same estimate holds, with an integrable decay factor in $y - y'$. Indeed $e^{-\eta} \text{asymp } rr'/(r^2 + |y - y'|^2)$ there. Consequently the apex-subtracted integrand has omitted integral $O(\varepsilon^\gamma)$, uniformly in the $C^k(K)$ norm. Its logarithmic constant term is $\sigma_\alpha s I(r) \log(1/\varepsilon)$.

Near the regular diagonal, use flat local coordinates $u = X' - X$. The kernel is

$$G_F(X, X') = \frac{1}{4\pi^2 |u|^2} + \text{smooth remainder.}$$

The only nonintegrable term in the tadpole is

$$\frac{s}{16\pi^4 r^2 |u|^4}.$$

The unit three-sphere has area $2\pi^2$, giving residue $s/(8\pi^2 r^2)$. After subtracting this term, the variation of $s/(r')^2$ is $O(|u|)$, so the remaining absolute local integrand is at worst $C|u|^{-3}$. Its omitted integral is $O(\delta)$. External derivatives are estimated after centering the local coordinates at the moving external point; the subtracted coefficients and remainders are smooth uniformly on K. This proves the diagonal part of .

The two neighborhoods are disjoint, so there is no overlap subtraction and no joint-cutoff condition. At infinity, has inverse-square four-dimensional decay. After integration over the two longitudinal variables, the large-r' tadpole tail is $O(r'\wedge -3 dr')$. The apex-subtracted longitudinal tail is likewise integrable. These bounds hold after fixed external derivatives on K, completing the convergence and smoothness proof.

Translation and rotation invariance make the result a function of r. Rescale all integration variables by r. The two local residues then give

$$\sigma_\alpha s I(r) \log(r/L) + \frac{s}{8\pi^2 r^2} \log(\mu r).$$

The remaining finite dimensionless integral is a constant times s/r^2; write it as $-2sA_\alpha/r^2$. The contact term is $cI(r)$. Multiplication by $-\lambda/2$ gives .

Remark (Why this is a first-order numerical insertion). The proof uses an extended test action at the apex and a separate extension at the regular diagonal. It never multiplies two apex-supported distributions. The construction is not a time-ordered interacting product on an enlarged field algebra. The distributional extension and smooth-background normalization frameworks provide context [7, 8, 9]; the two-locus estimate is proved directly here.

Finite changes of coordinates and schemes

Proposition (The finite constant and contact response). *Define*

$$c_{\mathrm{eff}} = c - 32\pi^3\alpha^2 s A_\alpha .$$

Then has the same form with $A_\alpha = 0$ *and c replaced by* c_{eff}*. The finite scheme change* $A_\alpha \mapsto A_\alpha + \Delta A_\alpha$ *is compensated at first order by*

$$c \mapsto c + 32\pi^3\alpha^2 s \Delta A_\alpha .$$

Moreover,

$$\frac{\partial}{\partial c}\delta_\lambda F(r) = -\frac{\lambda}{32\pi^3\alpha^2 r^2}.$$

Thus a free contact changes this prescribed first-order observable at regular external points.

Proof. Combine the first and last terms of , then differentiate with the subtraction convention held fixed.

For the quadratic defect convention

$$S_{\mathrm{def}} = \frac{g}{2}\int_{\mathscr{M}} \varphi^2,$$

its first covariance correction is $-g\int_{\mathscr{M}} G_F G_F$. Comparison with the contact contribution gives

$$g = \frac{\lambda c}{2}, \qquad S_{\mathrm{def}} = \frac{\lambda c}{4}\int_{\mathscr{M}} \varphi^2 .$$

The self-energy coefficient and the coefficient multiplying the action integral are not the same number.

A change of presentation of the same free distribution is

$$L \mapsto L', \qquad c \mapsto c + \sigma_\alpha s \log(L'/L).$$

It leaves $c + \sigma_\alpha s \log(r/L)$ unchanged. This is not fixed-parameter transverse dilation invariance.

A change of the regular-diagonal scale $\mu \mapsto \mu'$ can instead be compensated in the regular first-order response by

$$c \mapsto c - 2\pi\alpha^2 s\log(\mu'/\mu),$$

if that recalibration is permitted. It changes the free apex prescription and is therefore not an invariance of the entire free one-point distribution.

For a charged Friedrichs sector with nontrivial holonomy, the kernel with an apex endpoint vanishes. Its analogous first-order apex scalar insertion consequently has no $c\delta_{\mathscr{M}}$ contribution. This endpoint statement does not identify the real and complex quartic combinatorics.

Non-sourcing. Equation is a response in the frozen free/first-order prescription. It neither selects c nor supplies an Einstein equation [15, 24].

The precise next-order logarithm, without an interacting release

Proposition (Worldsheet primitive and its residue). *At separated Euclidean worldsheet points,*

$$G_{F,E}((y,0),(y',0)) = \frac{1}{4\pi^2\alpha|y-y'|^2}.$$

For $f \in C_c^\infty(\mathbb{R}^2)$,

$$\int_{|u|>\delta} \frac{f(u)}{4\pi^2\alpha|u|^2}\,\mathrm{d}^2u = \frac{f(0)}{2\pi\alpha}\log(1/\delta) + \text{finite part} + O(\delta^2 p_2(f)).$$

Any extension of this primitive preserving its scaling degree has a scalar delta ambiguity. Its Euler residue is $(2\pi\alpha)^{-1}\delta^{(2)}(u)$.

In a second quadratic-defect insertion with smooth local switches, the divergent local coefficient is consequently proportional to

$$\frac{g^2}{2\pi\alpha}\log(1/\delta).$$

The switch product at coincidence must be retained.

Proof. Use with both endpoints on the worldsheet. Angular Taylor averaging of f gives $f(0) + O(|u|^2 p_2(f))$. Polar

integration proves . The supported-difference and Euler statements follow by the same calculation as –, now in the relative worldsheet variables. At a repeated insertion, the smooth factors multiply $f(0)$; their product, not an independently chosen constant, is the residue.

This is a new relative-worldsheet-point extension problem. It is not normalized by the free transverse finite part U_L , nor by the regular-diagonal subtraction in .

For comparison, a finite logarithmic zero-channel boundary condition $u(r) \propto \log(r/L_b)$ has a negative transverse eigenvalue

$$-4e^{-2\gamma_E}/L_b^2.$$

Indeed $K_0(\kappa r) \sim -\log r + \log(2/\kappa) - \gamma_E$, so matching gives $\kappa = 2e^{-\gamma_E}/L_b$ [2, 3].

Stopping condition (No all-orders inference). Proposition evaluates the local primitive and identifies the missing normalization. It does not define the complete second-order interacting coefficient, prove a renormalization-group equation, resum a defect interaction, or classify all stable enlarged models.

The negative logarithmic point-interaction eigenvalue is a difficulty for that massless spectral-ground-state realization, not a universal nonexistence theorem for every interacting defect theory. The distinct VII–IX singular products are not imported [27, 28].

Stage D: actual Mellin poles and an infinite resonant family

Assume two deficient orders

$$0 < \nu_0 < \nu_1 < 1, \qquad \sigma = \nu_0 + \nu_1 = \alpha^{-1} \in (1, 2), \qquad \delta = \nu_1 - \nu_0 > 0.$$

They correspond to

$$\beta = \alpha\nu_0, \qquad 1 - \beta = \alpha\nu_1.$$

Fix a reference length for dimensional boundary coefficients; the following Mellin variables are expressed in those

units.

For the rank-one infinite boundary energy $H = tww^{\dagger}$, $t \to \infty$, with $w = (1, 1)/\sqrt{2}$, the matrix inverse formula gives

$$C_{01}(p) = -\frac{Kp^{-\delta}}{1 + dp^{-2\delta}}, \qquad K, d > 0.$$

For example,

$$K = S_1^{-1}A_1/A_0, \qquad d = \frac{S_1^{-1}A_1^2}{S_0^{-1}A_0^2}.$$

The radial scalar coefficient is

$$R_{01}(r) = \frac{1}{4\pi^2\alpha}\int_0^{\infty} pC_{01}(p)K_{\nu_0}(pr)K_{\nu_1}(pr)\,\mathrm{d}p.$$

A coefficient-sensitive criterion

The Bessel Mellin factor is

$$M(s) = \frac{2^{s-3}}{\Gamma(s)}\prod_{\varepsilon_0,\varepsilon_1 = \pm 1}\Gamma\left(\frac{s + \varepsilon_0\nu_0 + \varepsilon_1\nu_1}{2}\right).$$

Initially it equals $\int_0^{\infty} u^{s-1}K_{\nu_0}(u)K_{\nu_1}(u)\mathrm{d}u$ for $\mathrm{Re}\, s > \sigma$. The formula follows, for example, by inserting the two Schwinger integral representations of K_ν, performing the u-integral, and then a beta integral. It is a classical Bessel Mellin identity [4, 5].

For the actual entry ,

$$\mathscr{C}(z) = \int_0^{\infty} p^{z-1}C_{01}(p)\,\mathrm{d}p = -\frac{K\pi}{2\delta}d^{(z-\delta)/(2\delta)}\csc\left(\frac{\pi(z + \delta)}{2\delta}\right), \qquad -\delta < \mathrm{Re}\, z < \delta.$$

Substitution $p^{2\delta} = du$ proves this formula. On a common fundamental strip,

$$R_{01}(r) = \frac{1}{4\pi^2\alpha}\frac{1}{2\pi\mathrm{i}}\int r^{-s}M(s)\mathscr{C}(2 - s)\,\mathrm{d}s.$$

Definition (Actual-pole and Ward-flux resonance). For boundary data with controlled endpoint expansions:

1. a Mellin resonance is a pole of order at least two of the actual product $M(s)\mathscr{C}(2 - s)$ encountered in its justified

small-r contour displacement;

2. a Ward-flux resonance is a nonzero coefficient of a term $\varepsilon^{0}(\log \varepsilon)^{k}$ in an actual differentiated boundary flux after pairing with a transverse Taylor monomial and taking its angular moment. Candidate poles removed by zero coefficients or meromorphic cancellation are not counted. A chart without the necessary continuation or differentiated remainder is unverified, not declared nonresonant.

The criterion depends on the boundary chart and its coefficients, not only on (α, β). It asserts neither a closed countable point set nor a complete classification.

A family with a proved nonzero logarithm

Theorem (Infinite family of evaluated Mellin collisions). *Choose any*

$$1 < \sigma < 2, \qquad m \in \mathbb{N}, \quad m \geq 1,$$

and set

$$\delta = \frac{2-\sigma}{2m+1}, \qquad \nu_0 = \frac{\sigma-\delta}{2}, \qquad \nu_1 = \frac{\sigma+\delta}{2}.$$

These are admissible two-channel orders. Let

$$\eta_\ell = (2\ell+1)\delta, \qquad c_\ell = -K(-d)^{\ell}.$$

For the boundary chart ,

$$R_{01}(r) = \frac{1}{4\pi^2\alpha}\Big[\sum_{\ell=0}^{m-1} c_\ell M(2-\eta_\ell) r^{-2+\eta_\ell} + c_m A_0 A_1 r^{-\sigma}\log(1/r)\Big] + O(r^{-\sigma}).$$

The logarithmic coefficient is nonzero. The remainder estimate remains valid after any fixed number of applications of $r\partial_r$, with a changed constant.

For $m = 1, \sigma = 5/4$, this recovers

$$\alpha = 4/5, \quad \beta = 2/5, \quad \nu_0 = 1/2, \quad \nu_1 = 3/4$$

and the positive logarithmic coefficient of Ex1.

Proof. The inequalities $0 < \nu_0 < \nu_1 < 1$ follow from $\sigma > 1$, $\delta > 0$, and $\sigma + \delta < 2$. The large-p expansion is

$$C_{01}(p) = \sum_{\ell=0}^{m} c_\ell p^{-\eta_\ell} + O(p^{-\eta_{m+1}}).$$

For $\ell < m$,

$$2 - \eta_\ell > \sigma.$$

Thus the full rescaled Bessel integral of that term converges at zero and equals $M(2 - \eta_\ell) r^{-2+\eta_\ell}$.

At the critical term,

$$\eta_m + \sigma = 2.$$

The product of the small-argument leading Bessel terms is

$$K_{\nu_0}(pr) K_{\nu_1}(pr) = A_0 A_1 (pr)^{-\sigma} + \text{less singular terms}.$$

Therefore the intermediate range $1 \ll p \ll r^{-1}$ contributes

$$c_m A_0 A_1 r^{-\sigma} \int^{1/r} \frac{\mathrm{d}p}{p}.$$

The low-momentum part, the high-momentum part above r^{-1}, and the subtracted Bessel remainders are $O(r^{-\sigma})$. For the last matrix remainder use

$$\eta_{m+1} + \sigma > 2, \qquad \sup_{u>0} |u^\nu K_\nu(u)| < \infty.$$

The same fixed-order estimates hold after applying $u\partial_u$ to the Bessel factors. They prove the differentiated remainder assertion.

Equivalently, $M(s)$ has residue $A_0 A_1$ at $s = \sigma$, while $\mathscr{C}(2 - s)$ has residue c_m there. Their double pole yields exactly the displayed logarithm. All its factors are nonzero.

Remark (Two distinct logarithms). The logarithm in is present off the apex. It is not automatically a delta-supported dilation residue. The leading scalar degree is $2 - \delta < 2$, so this scalar off-diagonal term is locally integrable. A complete stress or current extension still requires its actual differentiated coefficients and Ward-flux analysis.

Stopping condition (General $U(2)$ ending). Theorem evaluates a nonempty infinite family, not all resonances. No full classification of the actual-pole set or the Ward-flux set is released. No uniqueness of every resonant or nonresonant off-diagonal one-point extension follows. The matrix boundary-field theorem of Section and the one-point extension problem remain different constructions.

Stage D concluded: the exact admissibility of the KH counterexample

Set

$$R^i = \varepsilon^i_{\ j} x^j, \qquad \varepsilon = \begin{pmatrix} 0 & -1 \\ 1 & 0 \end{pmatrix}.$$

For stationary, longitudinally translation-invariant mixed tensors define

$$\mathrm{W1}_A(T) = \partial_\mu T^\mu_{\ A},$$

$$\mathrm{W2}(T) = \partial_i (T^i_{\ j} R^j),$$

$$\mathrm{W3}(T) = \partial_i (T^i_{\ j} x^j) - T^i_{\ i}.$$

These are smooth operations on the auxiliary completion. They do not define multiplication of a supported tensor by the singular conical metric.

A completely specified first-jet module

Definition (Polar-vector first-jet submodule). Fix a real vector $v \in \mathbb{R}^2$, $v \neq 0$, and set $D_v = v^k \partial_k \delta_{\mathscr{W}}$. Consider tensors with no mixed longitudinal–transverse blocks and

$$Q^A_{\ B} = u\, \delta^A_B D_v,$$

$$Q^i{}_j = a\,\delta^i_j D_v + b\,v^i \partial_j \delta_{\mathscr{W}} + c\,v_j \partial^i \delta_{\mathscr{W}},$$

where $u, a, b, c \in \mathbb{R}$. This is the parity-even, $O(2)$-equivariant module linear in the polar vector v, with longitudinal boost invariance. No independent pseudo-vector structure is included.

Theorem (Exact KH first-jet calculation). *Within Definition , the simultaneous equations*

$$\mathrm{W1} = \mathrm{W2} = \mathrm{W3} = 0, \qquad Q^\mu{}_\mu = 0$$

have exactly the solutions

$$Q^A{}_B = -\frac{2a}{3}\delta^A_B D_v,$$

$$Q^i{}_j = a\Big(\delta^i_j D_v - \frac{5}{3} v^i \partial_j \delta_{\mathscr{W}} + v_j \partial^i \delta_{\mathscr{W}}\Big).$$

Thus Ex1's Q_v spans this specified scalar-coefficient solution module.

Any one of the following additional requirements removes this nonzero module:

1. *symmetry after lowering with $\bar{g}$;*

2. *the full smooth-reference divergence $\partial_i Q^i{}_j = 0$;*

3. *the massless stress almost-homogeneity equation $(E_x + 4)^2 Q = 0$. These eliminations concern this module only, not every supported tensor.*

Proof. Use $x^\ell \partial_k \delta_{\mathscr{W}} = -\delta^\ell_k \delta_{\mathscr{W}}$. For ,

$$Q^i{}_j x^j = -(a + 2b + c) v^i \delta_{\mathscr{W}}, \qquad Q^i{}_i = (2a + b + c) D_v.$$

Hence W3 is equivalent to

$$3a + 3b + 2c = 0.$$

Similarly,

$$Q^i{}_j R^j = (c - a)(\varepsilon v)^i \delta_{\mathscr{W}},$$

so W2 gives $c = a$. The trace equation is

$$2u + 2a + b + c = 0.$$

Solving gives $c = a$, $b = -5a/3$, $u = -2a/3$. W1 vanishes identically by longitudinal translation invariance.

The transverse antisymmetric part is

$$Q_{ij} - Q_{ji} = -\frac{8a}{3}(v_i\partial_j - v_j\partial_i)\delta_{\mathscr{W}},$$

which is nonzero for $a \neq 0$. This proves (i). The reference divergence is

$$\partial_i Q^i{}_j = a\Big(v_j\Delta - \frac{2}{3}\partial_j v^k\partial_k\Big)\delta_{\mathscr{W}},$$

where v is constant and $\partial_j v^k \partial_k$ means $v^k\partial_j\partial_k$. The polynomial multiplier in is not identically zero for nonzero v, proving (ii).

Finally Q is a first normal delta jet, so

$$(E_x + 3)Q = 0, \qquad (E_x + 4)^2 Q = Q.$$

This proves (iii).

An explicit admissibility ledger

toprule Condition Nonzero Q_v

midrule endfirsthead toprule Condition Nonzero Q_v

midrule endhead bottomrule endfoot

Real, supported on $\mathscr{W}$, longitudinal translations and boosts Satisfied.

KH equations W1–W3 and zero mixed trace Satisfied exactly.

Stress scaling-degree upper bound at least 3 Satisfied: $\mathrm{sd}_\perp Q_v = 3$. A noninteger upper bound between 3 and 4 does not exclude it.

Joint rotations of x and the boundary-data vector v Satisfied.

Rotational invariance with the nonzero v held fixed Not satisfied. This is stronger than joint equivariance.

Reference-lowered symmetry Violated by .

Full reference divergence Violated by .

Stress almost homogeneity of degree −4 Violated: $(E_x + 4)^2 Q_v = Q_v$.

A prescription allowing no first normal jets Violated by definition. Such a prescription is additional normalization data.

Symmetry obtained by lowering with the singular conical metric Not a defined admissibility test on these supported distributions without an additional construction.

Remark (Dimensional and symmetry qualifications). A physical dimensional rule for the coefficient v must be stated with its dependence on boundary data and reference scales. The present contact calculation does not manufacture that rule. Likewise, an auxiliary reference-symmetry condition must not be inferred from regular g -symmetry of an off-diagonal cone stress.

Excluding Q_v is not, by itself, a proof that every other ambiguity vanishes. For example, outside the polar-vector submodule there are reference-symmetric first jets with higher rotation weight. A full classification must specify and solve its entire module.

Corollary (Precise persistence of the nonuniqueness warning). *If a KH-Ward extension class admits the supported differences , then any one extension, when it exists, generates a nonunique family by adding them. Noninteger leading asymptotic powers do not restore uniqueness. If one of Theorem 's added conditions is imposed, this particular counterexample is excluded, but a full uniqueness theorem remains a separate obligation.*

Proof. The additions have zero regular restriction, zero trace, and zero KH residual. The remaining assertions are exactly the module calculation and its stated scope.

Non-sourcing. These are contact-compatibility statements on the declared completion. They do not supply

singular Einstein dynamics [15, 24].

Dirac audit: retained discrepancy and a bibliographic correction

No Dirac formula is needed in the preceding proofs. The unresolved comparison in I-B-VP-1-v4 and Ex1 is retained [19, 20].

In the supplied normalization comparison, the disputed brackets are

$$P_{\pm} = (\alpha^2 - 1)(17\alpha^2 + 7) + 120\beta^2(\alpha^2 - 2\beta^2 \pm 1).$$

Their difference is the elementary identity

$$P_{+} - P_{-} = 240\beta^2.$$

Once a zero-flux overall normalization has been fixed, an overall multiplicity or sign does not remove that flux-dependent difference. This observation does not determine whether a branch, subtraction, transcription, or physical-model difference accounts for it.

The primary arXiv record of Spinelly–Bezerra de Mello identifies a massless left-handed field and finite flux-tube models [12]. The journal metadata for hep-th/0306103 is *International Journal of Modern Physics D* **13** (2004), 607–624. The *A* **19**, 3045–3064 entry in Ex1 should not be retained for this arXiv identifier. This is a bibliographic correction, not a resolution of the physical discrepancy.

Bezerra de Mello–Saharian is a distinct current calculation with its own compactification and flux conventions [13]. No new same-branch spinor calculation is performed here.

Stopping condition (Dirac ending). The coefficient discrepancy remains an unresolved audit. No paper is declared erroneous, no branch is silently reidentified, and no new spinor apex-extension theorem is released.

Reliability, finite coverage, and residual retention

Evidence classes

We follow the integrity and execution-scope distinction documented in XVI-O1-ST [25]. For this manuscript:

\texttt{ANALYTIC} A statement established by the displayed proof on the stated mathematical domain.

\texttt{SUPPLIED} A version-qualified source statement or historical computational report not independently executed here.

\texttt{UNCERTIFIED} A numerical approximation or verification claim without the complete artifacts and total error bounds required to interpret it as a certificate.

These labels do not replace proof hypotheses. In particular, an unavailable estimate is not made analytical by naming a theorem that would require it.

No numerical or symbolic program was executed for the results of this draft. The parent source's compilation and computation reports remain attributed historical reports, not new executions.

Residual-retention table

toprule Item Status here Exact residual or boundary

midrule endfirsthead toprule Item Status here Exact residual or boundary

midrule endhead bottomrule endfoot

All-angular Friedrichs tails ANALYTIC Lemma and Theorem ; no unproved uniform-order tail is used.

Compressed matrix positivity ANALYTIC Compression precedes inversion; Theorem .

Weighted boundary limit ANALYTIC Same-representation limit with –; positive orders only.

Wick quadratic radial limits ANALYTIC Common dense energy-cutoff domain $\mathcal{D}_c$, not arbitrary finite-particle vectors.

General bulk completion Wick algebra OPEN No unrestricted product or full apex two-point wavefront theorem is supplied.

Exterior diagnostic coefficient and contact concentration ANALYTIC Theorem ; zero angular exterior correction only.

Assembled contact transfer Conditional theorem Theorem is proved; its full physical-core hypotheses are unverified.

Interior normalization freedom ANALYTIC Proposition ; not a claim about a fixed physical state.

Physical stress scale shift and τ OPEN Requires the smooth-core stress and its complete finite subtraction limit.

First-order tadpole ANALYTIC Independent-cutoff C^k estimates and explicit finite contact reparametrization.

Numerical value of the sharp-scheme A_α Not evaluated Its finite reparametrization is proved; no numerical constant is reported.

Second-order interacting coefficient OPEN The relative-worldsheet primitive and residue are identified; no full coefficient is constructed.

Mellin resonant family ANALYTIC Theorem ; not a complete resonance classification.

KH uniqueness Restricted correction The polar-vector first-jet module is solved. Unrestricted contact classification remains open.

Dirac discrepancy Retained No physical resolution; journal metadata corrected.

Mosco/resolvent comparison Methodological Appendix ; no one-point convergence follows.

What finite testing can certify

Proposition (Class-relative finite coverage). *Let Z be a proved finite-dimensional residual space and $P : Z \to \mathbb{R}^k$ a declared linear probe map. Vanishing of every probe implies $z = 0$ for all $z \in Z$ if and only if P is injective. If Z is normed and P is injective, there is C_Z such that*

$$\|z\| \le C_Z \|\mathrm{Pz}\|_\infty, \qquad z \in Z.$$

Proof. The exact assertion is $\ker P = \{0\}$. For the estimate, the continuous function $z \mapsto \|\mathrm{Pz}\|_\infty$ has a strictly

positive minimum on the compact unit sphere of Z.

This is the coverage distinction documented in Oai282-A [26]. The present distributional identities were proved on arbitrary tests, not inferred from a finite sample. A finite number of vanishing pairings does not establish membership in Z, its dimension, or injectivity of P.

In particular, ordinary tests supported away from $\mathscr{W}$ annihilate every contact. They cannot normalize (c, τ, w), even if all such regular-part tests are available.

Mathematical release and unchanged programme boundaries

Theorem (I-B-VP-1-Ex2 release). *On the prescribed cone, auxiliary smooth completion, spectral conventions, product boundary relations, and subtraction prescriptions of this paper:*

1. *the Friedrichs worldsheet restriction has the quantitative rate and explicit tails of Theorem ;*

2. *every nonnegative compressed positive-order chart has the positive matrix measure and tail estimates of Theorem and Corollary ;*

3. *its weighted radial boundary field is an untruncated limit in the same spectral representation, with ;*

4. *the corresponding Wick quadratics exist and converge on $\mathcal{D}_c$, with –;*

5. *the normalized exterior zero-mode correction has the strong contact concentration and analytic double-logarithmic coefficient of Theorem ;*

6. *Theorem gives a sufficient moment-controlled assembly theorem, while Proposition proves the nonimplication from exterior data alone;*

7. *the first-order tadpole has independent-cutoff fixed-external-support estimates, the finite reparametrization , and the response ;*

8. *the next relative-worldsheet logarithm is ;*

9. *the boundary charts have the nonzero logarithms ;*

10. *the specified KH first-jet module is exactly , with the admissibility failures calculated in Theorem . The full physical core-contact limit, general resonance classification, unrestricted KH contact classification, Dirac discrepancy, and interacting completion are not closed by these results.*

Proof. Items (i)–(iv) are the results of Sections –. Items (v)–(vi) are Theorems , , and Proposition . Items (vii)–(viii) are Theorem , Proposition , and Proposition . Items (ix)–(x) are Theorems and . The final sentence records the separate hypotheses and constructions not supplied by those proofs.

No enlargement by analogy

The following are different objects:

• a spatial operator limit and a renormalized coincidence limit;

• a one-point finite part and a covariance;

• a bulk-derived boundary field and an independently prepared defect factor;

• a Wick quadratic and a Gaussian field;

• a first-order numerical insertion and an interacting time-ordered algebra;

• an off-apex Mellin logarithm and a supported Euler residue;

• a KH representative and a singular-metric Levi-Civita derivative of arbitrary contacts. No comparison manuscript is used to identify these objects.

Version-qualified characteristic stops

The supplied XIV-H record ends unresolved at its H2 gate. The later supplied XIV-I record constructs its named I2 internal amplitude but leaves the full joint characteristic map unresolved at I3. XIV-J constructs its supported-sector reference on its stated domain but leaves the full map unresolved at J2 [29]. These are different version-qualified statements. The I2 advance is not withdrawn.

This paper constructs none of:

1. the full characteristic map on tests crossing its locus;

2. its joint section and complete remainder;

3. its uniform transverse estimates;

4. compatibility with its permitted contacts and singular spectators;

5. its complete output wavefront theorem;

6. admission of its normalized quantum coefficient. A worldsheet field or tadpole on the prescribed cone does not discharge this chain. Technical XV manuscripts may exist while the downstream quantum-admission exit remains unopened.

Standing statuses

The supplied programme boundaries are retained as scope records, not proved anew as universal impossibility theorems:

• Problems 1, 2, and 8 remain unfinished.

• Problems 3–7 and 9–11 are not reached by the present results. Problem 6 is not the implication attempted here.

• Problem 12 remains unresolved.

• XVI-O's standing entry remains nolinkurl $O.1 = NOT_C LOSED$.

• XVI-N and Route B are not discharged.

• XV-B's downstream quantum-admission exit is not opened.

• The historical H2 ending and the unresolved I3/J2 obligations are not updated here.

The supplied XVI-Q record reports a separately scoped one-loop, order-κ^2, two-point closure in a different enlarged prescription. That documentary claim is not revoked, imported, or promoted to a statement about the present construction. It does not update XVI-O's prescription or supply bulk symmetry restoration [30].

No spacetime emergence, Einstein dynamics, backreaction, Page curve, de Sitter state, cosmological bounce, or ultraviolet completion is constructed. No inherited characteristic quantum coefficient is admitted. OpenAI catalogue entries, including Family 282, remain external records, not premises.

appendix

Mosco and resolvent convergence: the methodological boundary

Lemma (Infinite form energy is not a recovery gap). *If a closed nonnegative form Q is extended by $+\infty$ outside its domain, the recovery inequality*

$$\limsup_n Q_n[u_n] \le Q[u], \qquad u_n \to u,$$

places no energy restriction when $Q[u] = +\infty$. The substantive recovery obligation concerns $u \in \mathrm{Dom}\, Q$.

Proof. Every extended real number is at most $+\infty$.

Proposition (The variational resolvent implication). *Let Q_n, Q be closed nonnegative forms on one Hilbert space. Suppose:*

1. *u_n rightharpoonup u implies $Q[u] \le \liminf Q_n[u_n]$;*

2. *every $u \in \mathrm{Dom}\, Q$ has a strong recovery sequence $w_n \to u$ with $Q_n[w_n] \to Q[u]$. For their associated operators B_n, B and every $s > 0$,*

$$(B_n + s)^{-1} f \to (B + s)^{-1} f$$

strongly.

Proof. Minimize

$$J_n[w] = Q_n[w] + s\|w\|^2 - 2\mathrm{Re}\,\mathrm{ip}\, fw.$$

The minimizers v_n obey lVertv_$n\| \le \|f\|/s$. The lower bound and recovery sequences show that every weak cluster point minimizes the limiting strictly convex functional J, hence equals $v = (B + s)^{-1} f$. The minimum values

converge. For a recovery sequence w_n of v,

$$J_n[w_n] - J_n[v_n] = Q_n[w_n - v_n] + s]\mathrm{Vert}w_n - v_n\|^2.$$

The left side tends to zero. Thus $v_n - w_n \to 0$ strongly, and $v_n \to v$.

This is the standard variational mechanism associated with Mosco convergence [11]. It supplies no renormalized one-point limit.

Proposition (Strong resolvent convergence does not control inverse covariance weights). *There are strictly positive operators B_n converging strongly in the resolvent sense to I, and a fixed vector v, such that*

$$\langle v, B_n^{-1/2} v\rangle \to +\infty.$$

Proof. On $\ell^2(\mathbb{N})$, let B_n equal I except on the n-th basis vector, where its eigenvalue is 2^{-8n}. For fixed $s > 0$, the resolvent difference applied to f is supported on that basis vector and has norm at most $C_s|f_n| \to 0$. Thus $B_n \to I$ strongly in the resolvent sense.

Take $v_j = 2^{-j}$. Its n-th contribution to the displayed quadratic form is

$$|v_n|^2 (2^{-8n})^{-1/2} = 2^{2n},$$

which diverges. Every individual form value is finite, since $B_n^{-1/2}$ is bounded for each fixed n.

The example does not describe a physical core. It proves the missing logical implication is invalid without additional infrared control. The one-point subtraction and contact-normalization obligations are even further operations. This is the precise methodological separation from I-B-VP-5 [21].

Debugging summary and verification declarations

toprule Potential overstatement Correction used in this manuscript

midrule endfirsthead toprule Potential overstatement Correction used in this manuscript

midrule endhead bottomrule endfoot

A Sobolev or form-domain slogan proves the worldsheet restriction The creation-vector estimate and spectral tails prove the limit directly.

A Euclidean positive matrix is already a positive Lorentzian spectrum The cut jump and the Stieltjes representation are calculated.

Compression commutes with inversion The compressed matrix is inverted on V.

A compact spectral proof gives the untruncated boundary field The threshold and ultraviolet estimates – remove both cutoffs.

Finite particle number controls a Wick quadratic Quadratic statements use the common dense energy-cutoff domain $\mathcal{D}_{c}$.

A boundary square is Gaussian Its nonzero connected three-point contraction is retained.

The exterior radial coefficient is the full core contact A strong exterior limit is proved, but full assembly still requires the independent moment and remainder hypotheses.

An arbitrary interior bump is a fixed physical core state The bump is a nonimplication witness, not a state construction.

A scalar curvature normalization determines the stress contact The finite stress remainder, including derivative-curvature terms, remains a separate calculation.

Two tadpole cutoffs may be removed without a uniform estimate Equation gives an independent-cutoff C^{k} bound on regular external compact sets.

A finite constant is a physical prediction Its explicit contact-coordinate absorption is retained.

The second-order logarithm is an all-orders theorem Only its relative-worldsheet primitive and residue are evaluated.

Noninteger powers imply KH uniqueness The actual first-jet counterexample and its exact exclusions are

calculated.

Countably many exponent equations define a closed countable point set No such assertion is made. Actual coefficients and possible cancellations enter the criterion.

A corrected bibliography resolves the Dirac discrepancy Only the bibliographic entry is corrected; the physical audit remains unresolved.

Strong resolvent convergence controls inverse covariance weights Proposition gives a fixed-vector counterexample.

Source scope. The complete parent and Ex1 sources were supplied. The I-B and philosophy pages were consulted for the prescribed-host and interface background. Primary literature records and the indicated classical Bessel formulas were consulted for attribution. The large supplied retrieval inventory is not treated as a complete corpus audit or as a list of theorem premises.

The online Oai282-A page was not available through the consulted interface; that reference is used only through the supplied version-qualified coverage description. The coverage assertion needed here is proved in Proposition .

Execution scope. No successful pdfLaTeX execution, symbolic computation, numerical quadrature, repository build, proof-assistant verification, independent rebuild, independent referee review, or journal acceptance is asserted for this draft. The mathematical claims are the written theorems on their declared domains.

Source format. This is one pdfLaTeX source with an internal bibliography. No external data, figures, code, or bibliography file is required. The author name follows the supplied manuscript series. Submission requires the author's approval of the text, attribution, and stated scope.

Final distinction. The exterior concentration theorem, boundary operators, and first-order insertion are genuine results on specified objects. They do not become a complete physical defect theory by sharing an apex, a contact label, or an Enter report.

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[17] N. Cheung, *I-B-G: Intrinsic and holonomy-twisted Green operators on a prescribed conical spacetime*, version 2, supplied excerpts, Enter Calculus Companion. Form domains, positivity, deficiency indices, and supported Green-operator transfer are comparisons, not worldsheet trace premises. https://enter.grok.me/browse/I-B-G. Companion record; not a Scholar-indexed journal article.

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[22] N. Cheung, *I-B-VP-AB*, version 2, supplied companion description, Enter Calculus Companion. Regular coefficient and normalization comparison; no analytical premise is imported. Companion record; not a Scholar-indexed journal article.

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[27] N. Cheung, *VII-H* and *VII-I*, version 1, supplied excerpts, Enter Calculus Companion. Separate free-response and finite-width free-algebra benchmarks. Their products and ambiguity classes are not identified with the present worldsheet fields or tadpole. https://enter.grok.me/browse/VII-H. https://enter.grok.me/browse/VII-I. Companion

record; not a Scholar-indexed journal article.

[28] N. Cheung, *IX-A*, *IX-B*, *IX-C*, *IX-Free*, *IX-Free-2*, *IX-D*, *IX-E*, *IX-E-2*, and *IX-E-3*, version-qualified excerpts supplied with the problem, Enter Calculus Companion. Documentary product-domain and causal-extension comparisons only; none supplies an interacting product for this conical calculation. https://enter.grok.me/browse/IX-A. https://enter.grok.me/browse/IX-B. https://enter.grok.me/browse/IX-C. https://enter.grok.me/browse/IX-Free. https://enter.grok.me/browse/IX-Free-2. https://enter.grok.me/browse/IX-D. https://enter.grok.me/browse/IX-E. https://enter.grok.me/browse/IX-E-2. https://enter.grok.me/browse/IX-E-3. Companion record; not a Scholar-indexed journal article.

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