# I-B-VP-1-Ex3 Uniform cut estimates, logarithmic concentration, and worldsheet subtraction

*Quantitative continuation and admissibility audit on a prescribed conical Enter spacetime*

**Ngo Cheung**

*18 October 2026*

---


## Abstract

We continue the domain-explicit apex analysis of I-B-VP-1-Ex2 on the prescribed conical product. For nonnegative compressed positive-order boundary charts, we replace an unspecified ultraviolet polynomial by explicit three-region bounds for the spectral density of the weighted radial-to-boundary difference. The region $\sqrt{\tau}\, r \geq 1$ is controlled uniformly in the nonnegative boundary matrix, and its smeared contribution has arbitrarily high algebraic order in $r$. This supplies a direct quantitative justification of the untruncated boundary limit and its Wick-quadratic consequence on the stated energy-cutoff finite-particle domain.

For exterior core diagnostics, we allow measurable reflection profiles with only weighted-integrable ultraviolet tails. Logarithmic-power behavior at zero produces a unified concentration law, including the coefficient $-1/(4\pi)$ in the double-logarithmic case, with explicit test-uniform and profile-remainder bounds. A moment expansion separates scalar contacts, dipoles, and quadrupoles, showing precisely why exterior information does not determine an assembled physical contact.

We also extend the relative-worldsheet logarithmic primitive against smooth parameter-dependent amplitudes, prove a fixed-external-support $C^k$ estimate for a switched bulk-to-worldsheet-to-bulk numerical sandwich, and calculate both Laurent coefficients at the infinite family's resonant Mellin pole. The logarithmic coefficient has an exact phase-sensitive cancellation criterion; its companion constant is given by gamma and digamma functions.

An explicit higher-rotation-weight KH contact supplements the polar-vector admissibility audit.

The potential usefulness lies in replacing formal apex assignments by quantitative domain, concentration, and normalization tests. No priority is claimed for Bessel extensions, generalized free fields, or local distributional renormalization. The complete physical core limit, unrestricted resonance and contact classifications, Dirac discrepancy, and interacting completion remain open.

### Bibliographic record


| | |
|---|---|
| **Title** | I-B-VP-1-Ex3 Uniform cut estimates, logarithmic concentration, and worldsheet subtraction |
| **Author** | Cheung, Ngo |
| **Publication date** | 2026/10/18 |
| **Institution** | Enter |
| **Report number** | I-B-VP-1-Ex3-v2 |
| **Publisher** | Cheung Ngo Medical Limited |
| **First page** | 1 |
| **Last page** | 50 |
| **Language** | en |


---

## Book edition note (t425u)

Standing Enter refusals, scope vocabulary (proved here/supplied input/conditional/ not constructed/not claimed), and Scholar bibliography policy are frozen in the companion chapter *I-B-VP Book Ledger* v1. O.1 remains NOT_CLOSED; XVI-N is not discharged; XV-B is not opened; no Host C, Page/island, or spacetime-emergence claim; wired=false; SOURCE_DATE_EPOCH=1792627200. This sheet is a companion manuscript record and is *not* claimed to be Google Scholar-indexed. The published Enter Calculus article is cited only for finite-instrument / operational interface discussions, never as a PDE premise.

## Book expansion: three-region bound first

**Reading order.** (1) three-region spectral bound and crossing-region estimate; (2) profile class and error bounds; (3) Laurent calculation; (4) KH examples. toprule Depends on Relation

midrule Ex2 boundary estimates Refines quantitative bounds (supplied input)

Ex1 corrections Scope stops carried forward

VP-1 parent Contact/completion context; not re-proved here

Open problems Named; not closed by the three-region bound

bottomrule

## Opening boundaries and the problem being continued

### The prescribed Enter framework

Cheung's philosophy of Enter distinguishes destructive readout, quantum return, an installed description, and physical identification [11]. The spacetime model of I-B prescribes a classical conical product and a separate finite operational interface [12]. Neither the philosophical essay nor the interface selects the continuum field objects studied below.

The finite interface can be recalled without importing a continuum measurement assertion. Let $C$ have a calibrated orthonormal basis $\{|\tau_j\rangle\}$, let $A$ contain all surviving finite registers, and retain the outcome label even when installed report words coincide. Put

$$D_j = \langle \tau_j|_C \otimes I_A, \qquad \mathscr{M}(X) = \sum_j \text{lvertj}, w_j\rangle\text{langlej}, w_j|_R \otimes D_j X D_j^\dagger.$$

Its Kraus completeness and discard identities are

$$\sum_j D_j^\dagger D_j = I_{CA}, \qquad \mathrm{Tr}_R \mathscr{M}(X) = \mathrm{Tr}_C X.$$

Expansion in clock matrix units proves these identities, including after adjoining an untouched finite reference.

The quantum output contains *A*, not a returned *C*. Positive-probability branches retain their actual conditional residual states; no normalized zero-probability branch is assigned.

This is the supplied-report interface of I-B and En09-A [12, thm:instrument] [13]. It does not acquire an unknown continuum worldsheet field, select a self-adjoint domain, prepare the spectral state, or normalize an apex contact. The geometric time coordinate is not consumed.

**Non-sourcing**

A distributional extension and a conserved contact are not source equations. No result below imposes

$$G_{\mu\nu} + \Lambda g_{\mu\nu} = \kappa T_{\mu\nu}.$$

The distinctions in I-B prop:not-source and X-A-ST prop:noeinstein are retained [12, 22]. In particular, a smooth representative of a Ward identity is not a definition of every singular-metric operation.

**Relationship to Ex2**

The complete supplied sources of I-B-VP-1, Ex1, and Ex2 are the principal records being continued [16, 17, 18]. The present work addresses five specific points:

**1.** the ultraviolet transition in compressed boundary charts;

**2.** exterior concentration and moment-controlled assembly;

**3.** the relative-worldsheet logarithmic subtraction with smooth external parameters;

**4.** resonant Laurent coefficients and KH admissibility;

**5.** reliability, retained residuals, and exact stopping conditions.

The new ultraviolet proof specifies the polynomial power and the crossing-region tail instead of invoking an unspecified polynomial. It also distinguishes the positive spectral density of the complete difference from the generally nonpositive jump of an individual resolvent-correction term.

The exterior theorem weakens ultraviolet profile assumptions and gives quantitative concentration. It does not

identify the diagnostic with the complete Wick square of a physical core. The worldsheet sandwich is a numerical kernel with one declared primitive extension, not an interacting operator product.

### Attribution and novelty scope

Bessel extensions, cone boundary conditions, generalized free fields, and scaling-degree renormalization have established foundations [1, 2, 3, 7, 8]. Core-sensitive long-range effects are also established [6]. The contribution here is narrower: explicit uniform estimates, a larger measurable profile class, moment-resolved transfer, and a complete resonant Laurent calculation for the stated companion problem.

The form-domain and supported-transfer statements of I-B-G, the infrared language of I-B-VP, and the operator-selection comparisons of I-B-VP-5 are kept separate from the boundary limits proved here [14, 15, 19]. I-B-VP-AB and I-B-VP-SAE are comparison records, not unquoted analytical premises [20, 21].

## Geometry, normalization, and domains

Fix

$$0 < \alpha < 1, \qquad 0 \le \beta < 1.$$

The regular host and its declared completion are

$$\begin{aligned} M &= \mathbb{R}_t \times (0,\infty)_r \times S^1_\varphi \times \mathbb{R}_z, \\ g &= -\mathrm{d}t^2 + \mathrm{d}r^2 + \alpha^2 r^2 \mathrm{d}\varphi^2 + \mathrm{d}z^2, \\ \overline{M} &= \mathbb{R}^{1+1}_y \times \mathbb{R}^2_x, \qquad y = (t,z), \qquad x = (r\cos\varphi, r\sin\varphi). \end{aligned}$$

The apex is the worldsheet $\mathscr{W} = \mathbb{R}^{1+1}_y \times \{0\}$. Use the auxiliary smooth metric and density

$$\overline{g} = -\mathrm{d}t^2 + \mathrm{d}z^2 + (\mathrm{d}x^1)^2 + (\mathrm{d}x^2)^2, \qquad \mathrm{d}\Omega = \alpha\, \mathrm{d}^2y\, \mathrm{d}^2x.$$

Define

$$\langle \delta_{\mathscr{W}}, f \rangle = \int f(y,0)\, \mathrm{d}^2y, \qquad \sigma_\alpha = 2\pi\alpha.$$

Thus $\delta_{\mathscr{W}} = \alpha^{-1}\delta^{(2)}(x)$ relative to $\mathrm{d}\Omega$.

These $x$-coordinates are not developing coordinates. No smooth value of the conical metric at $x = 0$ is supplied. No multiplication of a delta derivative by that metric is used.

For the charged field use the lifted holonomy convention

$$\Phi(\varphi + 2\pi) = e^{2\pi i\beta}\Phi(\varphi), \qquad q_n = n + \beta, \qquad \nu_n = |q_n|/\alpha.$$

Transverse comparisons can be made in the periodic gauge $e^{-i\beta\varphi}\Phi$. This is a declared punctured-plane gauge, not an extension of nontrivial flat holonomy through the apex.

**Spectral variables**

For a worldsheet test $h$, let

$$\widehat{h}(\omega, k) = \int e^{i\omega t - ikz} h(t, z)\, dt\, dz, \qquad V_+ = \{(\omega, k) : \omega > |k|\}.$$

The transverse spectral value will be denoted by $\tau > 0$, so that $\omega = \sqrt{k^2 + \tau}$. For mass squared $\tau$, set

$$\Delta_+^{(1+1)}(y; \tau) = \int_{\mathbb{R}} \frac{dk}{2\pi\, 2\sqrt{k^2 + \tau}}\, e^{-i\sqrt{k^2+\tau}(t - i0) + ikz}.$$

For vector-valued $h$, one convenient seminorm is

$$P_N(h) = \sup_{\omega, k} (1 + \omega^2 + k^2)^{(N+2)/2} \left\| \widehat{h}(\omega, k) \right\|.$$

Increasing $N$, or adjoining finitely many Fourier derivatives, gives the other Schwartz seminorms used below.

**Two Fock domains**

Let $\mathscr{H}_1$ be the one-particle space of the declared spectral state. For charged fields include the particle space and its conjugate antiparticle space. On the bosonic Fock representation, define

$$\mathcal{D}_{\mathrm{fin}} = \bigcup_{n < \infty} \mathrm{bigoplus}_{j=0}^{n} \mathscr{H}_1^{\otimes_s j}.$$

The linear estimate

$$\left\| (a(v) + a^*(v))\Psi \right\| \le 2\|v\| \left\| (N+1)^{1/2}\Psi \right\|$$

will be used for linear limits.

For quadratics use the smaller common dense domain

$$\mathcal{D}_{\mathrm{c}} = \bigcup_{\text{substack}\, n < \infty}$$

This is a domain, not an assertion of invariance under the quadratic operators. Particle number alone does not bound energy. No quadratic convergence theorem on all of $\mathcal{D}_{\mathrm{fin}}$ is asserted.

## Stage A: compressed resolvents and positive boundary spectrum

### The positive-order product boundary law

Consider a fixed finite set of deficient orders

$$0 < \nu_j < 1, \qquad j = 1, \ldots, d.$$

The logarithmic order $\nu = 0$ is excluded in this section. Let

$$L_j = -\partial_r^2 - r^{-1}\partial_r + \nu_j^2 r^{-2}$$

on $L^2((0,\infty), r\mathrm{d}r)$. Write the boundary expansion as

$$f_j(r) = a_j r^{-\nu_j} + b_j r^{\nu_j} + o(r^{\nu_j}).$$

Choose $V \subseteq \mathbb{C}^d$, its orthogonal projection $P_V$, and a Hermitian $H_V \geq 0$ on $V$. The boundary relation is

$$a \in V, \qquad P_V \mathrm{diag}\,(\nu_j) b = H_V a.$$

It is independent of longitudinal momentum.

The boundary Green form is, up to its orientation sign,

$$2\sum_j \nu_j (\overline{a}_j b_j' - \overline{b}_j a_j').$$

Relation is a maximal isotropic relation for this form. The endpoint at infinity is limit-point. It therefore specifies a self-adjoint radial realization. The standard one-channel setting is described in [1, 2]; the compressed

normalization needed here is calculated next.

Set

$$A_j = 2^{\nu_j - 1}\Gamma(\nu_j), \quad S_j = \frac{\pi}{2\sin(\pi\nu_j)}, \quad d_j = \frac{S_j}{A_j^2}, \quad a = \min_j \nu_j, \quad b = \max_j \nu_j.$$

Here $a, b$ denote exponent bounds, not boundary vectors. Let $\iota : V\hookrightarrow \mathbb{C}^d$ be inclusion and define

$$B_V(\zeta) = 2H_V + P_V \operatorname{diag}(d_j \zeta^{\nu_j})|_V, \qquad F(\zeta) = \iota B_V(\zeta)^{-1}\iota^{\dagger},$$

with principal powers on $\mathbb{C} \setminus (-\infty, 0]$. For $V = \{0\}$, set $F = 0$.

**Lemma (Normalized compressed resolvent).** *Put $E(\zeta) = \operatorname{diag}(A_j \zeta^{-\nu_j/2})$. Relative to $r'\mathrm{d}r'$, the resolvent kernel is*

$$\mathscr{R}_\zeta(r, r') = \operatorname{diag}\Big(I_{\nu_j}(\sqrt{\zeta}\, r_<) K_{\nu_j}(\sqrt{\zeta}\, r_>)\Big)$$

$$+\operatorname{diag}(K_{\nu_j}(\sqrt{\zeta}\, r)) E(\zeta)^{-1} F(\zeta) E(\zeta)^{-1} \operatorname{diag}(K_{\nu_j}(\sqrt{\zeta}\, r')).$$

*In a full chart,*

$$E^{-1} F E^{-1} = (S + 2EHE)^{-1}.$$

*The radial realization is nonnegative.*

*Proof.* The Wronskian of $I_\nu, K_\nu$ gives the first Green kernel. At zero,

$$I_\nu(pr) \sim I_\nu^0 p^\nu r^\nu, \qquad K_\nu(pr) \sim A_\nu p^{-\nu} r^{-\nu} + B_\nu p^\nu r^\nu,$$

where

$$I_\nu^0 = \frac{2^{-\nu}}{\Gamma(1+\nu)}, \qquad B_\nu = 2^{-\nu-1}\Gamma(-\nu).$$

The reflection identity gives

$$2\nu I_\nu^0 A_\nu = 1, \qquad I_\nu^0 S_\nu + B_\nu = 0.$$

Imposing on the Green kernel therefore gives . Compression occurs before inversion.

The resolvent correction has finite rank in the radial resolvent sense. Negative eigenvalues could occur only when the boundary matrix for a decaying $K_\nu$ solution is singular at $\zeta > 0$. But $B_V(\zeta) > 0$ there, unless $V = \{0\}$, which is Friedrichs. There is no negative spectrum. There is no square-integrable zero solution: the zero-energy powers are not square integrable at infinity for $0 < \nu_j < 1$.

**Uniform bounds before taking the cut**

For matrices use $\operatorname{Im} T = (T - T^\dagger)/(2\mathrm{i})$.

**Lemma (Uniform compressed inverse).** *For the fixed orders, uniformly in V and $H_V \geq 0$,*

$$\mathrm{lVertF}(\zeta)\| \leq C \begin{cases} |\zeta|^{-b}, & 0 < |\zeta| \leq 1, \\ |\zeta|^{-a}, & |\zeta| \geq 1. \end{cases}$$

*These estimates hold up to either shore of the cut.*

*If*

$$Y(\tau) = P_V \operatorname{diag}(d_j \sin(\pi\nu_j)\tau^{\nu_j})|_V,$$

*then*

$$-\operatorname{Im} B_V(-\tau + \mathrm{i}0)^{-1} = B_V(-\tau + \mathrm{i}0)^{-1} Y(\tau) B_V(-\tau + \mathrm{i}0)^{-*} \geq 0.$$

*Proof.* Let $\zeta = |\zeta| e^{\mathrm{i}\theta}$, $|\theta| \leq \pi$. After multiplying the quadratic form by $e^{-\mathrm{i}b\theta/2}$, its real part is at least

$$\cos(\pi b/2)\Big(2\mathrm{langleu}, H_V u\rangle + \sum_j d_j |\zeta|^{\nu_j} |(\iota u)_j|^2\Big).$$

The cosine is positive because $b < 1$. This proves invertibility and , uniformly in the compressed chart. Subtracting the inverse and its adjoint gives .

**Proposition (Boundary spectral density).** *The Euclidean boundary covariance*

$$\widetilde{W}_a(\zeta) = \frac{1}{2\pi\alpha} F(\zeta)$$

*has the Stieltjes representation*

$$\widetilde{W}_a(\zeta) = \int_0^\infty \frac{\rho_a(\tau)}{\zeta + \tau}\,\mathrm{d}\tau, \qquad \rho_a(\tau) = -\frac{1}{2\pi^2\alpha}\operatorname{Im} F(-\tau + \mathrm{i}0) \geq 0.$$

*There are no polynomial additions or zero-mass atoms. Moreover,*

$$\|\rho_a(\tau)\| \leq C \begin{cases} \tau^{-b}, & 0 < \tau \leq 1, \\ \tau^{-a}, & \tau \geq 1. \end{cases}$$

*Proof.* Equation gives positivity, and gives the bounds. A keyhole contour applied entrywise to $F$ yields . The small circle vanishes because $b < 1$, and the large circle vanishes because $a > 0$. Thus no extra polynomial or $1/\zeta$ term remains.

**Remark (What positivity does not mean).** The positive matrix $\rho_a$ is a cut jump, not the Euclidean value $F(p^2)$ renamed as a spectral density. The spectral generalized-free-field interpretation is the Gaussian one [3]. It does not make a nonzero Wick quadratic Gaussian.

## Stage A continued: an explicit ultraviolet crossing estimate

### The complete difference density

Define

$$D_r(\zeta) = \operatorname{diag}\left( \frac{(\sqrt{\zeta}\, r)^{\nu_j} K_{\nu_j}(\sqrt{\zeta}\, r)}{A_j} \right), \qquad T_r(\zeta) = D_r(\zeta) - I.$$

The weighted radial evaluations are

$$A_{r,j}(h) = r^{\nu_j} \frac{1}{2\pi} \int_0^{2\pi} e^{-\mathrm{i}q_j\varphi} \Phi(h, r, \varphi)\,\mathrm{d}\varphi.$$

On compact spectral intervals away from zero, the Bessel expansions in Lemma give a boundary evaluation as a limit in the spectral Gram kernel. The resolvent expression for the difference between a weighted radial evaluation and that boundary evaluation is

$$\mathscr{P}_r(\zeta) = \operatorname{diag}\left( r^{2\nu_j} I_{\nu_j}(\sqrt{\zeta}\, r) K_{\nu_j}(\sqrt{\zeta}\, r) \right) + T_r(\zeta) F(\zeta) T_r(\zeta).$$

Its spectral density, with angular normalization included, is

$$\rho_{e,r}(\tau) = -\frac{1}{2\pi^2\alpha}\operatorname{Im}\mathscr{P}_r(-\tau + \mathrm{i}0).$$

**Remark (Analytic products versus adjoints).** The analytic expression in is $T_r F T_r$, not $T_r F T_r^{\dagger}$. Its lower-shore value is the adjoint of its upper-shore value. The jump of this correction term need not be positive. Positivity belongs to the complete difference density $\rho_{e,r}$, obtained from the full resolvent Gram kernel. The estimates below bound the norm of each contribution; they do not assume termwise spectral positivity.

**Fixed-order Bessel bounds**

Put

$$\theta = \max\{b - \tfrac{1}{2}, 0\}, \qquad \mu = \max\{a, \theta - a\} < 1.$$

**Lemma (Bounds on the two cut regions).** *For $w > 0$, with constants depending only on the fixed finite set of orders,*

$$\max_j \left| \frac{(\mathrm{i}w)^{\nu_j} K_{\nu_j}(\mathrm{i}w)}{A_j} - 1 \right| \le C w^{2a}, \quad 0 < w \le 1,$$

$$\max_j \left| \frac{(\mathrm{i}w)^{\nu_j} K_{\nu_j}(\mathrm{i}w)}{A_j} - 1 \right| \le C w^{\theta}, \quad w \ge 1,$$

$$|J_{\nu_j}(w)| \le C w^{\nu_j}, \quad 0 < w \le 1,$$

$$|J_{\nu_j}(w)| \le C w^{-1/2}, \; w \ge 1.$$

*The same modulus bounds hold on the other shore.*

*Proof.* For $0 < \nu < 1$, the two small-argument series give

$$\frac{z^{\nu} K_{\nu}(z)}{A_{\nu}} = 1 + O(z^{2\nu}) + O(z^2).$$

This proves . The $J_{\nu}$ series proves .

On the imaginary axis, the connection with Hankel functions and their fixed-order large-argument expansion give

$$|K_\nu(\mathrm{i}w)| \le Cw^{-1/2}, \qquad |J_\nu(w)| \le Cw^{-1/2}.$$

Thus $w^\nu K_\nu(\mathrm{i}w)$ grows at most as $w^{\nu-1/2}$, giving . Continuity on the compact transition interval completes the bounds. These are the classical fixed-order estimates of [4, Sections 10.17, 10.27, 10.30, and 10.40]. No large-order angular limit is involved here.

**Theorem (Three-region difference bound).** *For* $0 < r \le 1$, *uniformly in* $V, H_V \ge 0$,

$$\|\rho_{e,r}(\tau)\| \le C \begin{cases} r^{4a}(\tau^a + \tau^{2a-b}), & 0 < \tau \le 1, \\ r^{4a}\tau^a, & 1 \le \tau \le r^{-2}, \\ r^{2a}(\sqrt{\tau}r)^{-1} + \tau^{-a}(\sqrt{\tau}r)^{2\theta}, & \tau \ge r^{-2}. \end{cases}$$

*In particular,*

$$\|\rho_{e,r}(\tau)\| \le Cr^{4a} \begin{cases} \tau^{-b}, & 0 < \tau \le 1, \\ \tau^{\mu}, & \tau \ge 1. \end{cases}$$

*All constants are independent of the nonnegative compressed boundary matrix. They are not asserted uniform as an order tends to* 0 *or* 1.

*Proof.* The free cut jump in is exactly

$$\frac{1}{4\pi\alpha}\mathrm{diag}\,(r^{2\nu_j}J_{\nu_j}(\sqrt{\tau}r)^2).$$

For the correction use

$$\|\mathrm{Im}\,(T_r F T_r)\| \le \mathrm{lVertT_}r\|^2\|F\|.$$

Set $w = \sqrt{\tau}r$.

When $w \le 1$,

$$r^{2\nu_j}|J_{\nu_j}(w)|^2 \le Cr^{4\nu_j}\tau^{\nu_j} \le Cr^{4a}\tau^a.$$

For the last inequality, if $\tau \ge 1$ use $r^4\tau \le r^2 \le 1$; if $\tau \le 1$ it is immediate. The correction is bounded by $Cr^{4a}\tau^{2a-b}$ below $\tau = 1$, and by $Cr^{4a}\tau^a$ above it. This proves the first two regions.

For $w \ge 1$, the free term is at most $Cr^{2a}w^{-1}$. Equations and bound the correction by $C\tau^{-a}w^{2\theta}$. This proves the third region.

It remains to check the explicit global power. The free crossing term is at most $Cr^{4a}\tau^{a}$, since $w^{-1} \le w^{2a}$. If $\theta \le 2a$, then

$$\tau^{-a}w^{2\theta} \le \tau^{-a}w^{4a} = r^{4a}\tau^{a}.$$

If $\theta > 2a$, then $r^{2\theta} \le r^{4a}$, and

$$\tau^{-a}w^{2\theta} \le r^{4a}\tau^{\theta-a}.$$

This proves the upper line of for $\tau \ge 1$. For $0 < \tau \le 1$, both terms in the first region are at most a constant times $r^{4a}\tau^{-b}$.

**The crossing region after worldsheet smearing**

Let

$$q_h(\tau) = \int_{\mathbb{R}} \frac{\left\|\widehat{h}(\sqrt{k^2+\tau},k)\right\|^2}{2\pi\, 2\sqrt{k^2+\tau}}\,\mathrm{d}k.$$

**Lemma (Longitudinal threshold and ultraviolet bounds).** *For sufficiently large $N$,*

$$q_h(\tau) \le C_N P_N(h)^2 \begin{cases} 1+|\log\tau|, & 0<\tau\le 1,\\ \tau^{-N}, & \tau \ge 1. \end{cases}$$

*Proof.* Below $\tau = 1$, split $|k| \le 1$ and its complement. The only threshold integral is

$$\int_0^1 (k^2+\tau)^{-1/2}\mathrm{d}k = O(1+|\log\tau|).$$

Above $\tau = 1$, substitute $k = \sqrt{\tau}\,v$ and use the definition of $P_N$. Increasing the Schwartz seminorm if necessary gives the displayed power.

**Corollary (Explicit crossing-region tail).** *If*

$$N > \max\{\tfrac{1}{2}, 1 + \theta - a\},$$

*then*

$$\int_{r^{-2}}^{\infty} \|\rho_{e,r}(\tau) \mathrm{rVertq}_h(\tau)\, d\tau \le C_N P_N(h)^2 r^{2N+2a-2}.$$

*Thus this region has arbitrarily high algebraic order after Schwartz smearing. There is no fixed spectral-cutoff residual.*

*Proof.* The two integrals to estimate are

$$r^{2a-1}\int_{r^{-2}}^{\infty} \tau^{-N-1/2} d\tau \quad \text{and} \quad r^{2\theta}\int_{r^{-2}}^{\infty} \tau^{-N-a+\theta} d\tau.$$

Both equal a finite constant times $r^{2N+2a-2}$ under the stated conditions.

**Theorem (Untruncated weighted boundary limit).** *For every boundary law and $h \in \mathcal{S}(\mathbb{R}^{1+1}; \mathbb{C}^d)$, the fields converge strongly on $\mathcal{D}_{\mathrm{fin}}$ to boundary fields $a(h)$ in the original spectral Fock representation. Their covariance is*

$$W_a(y) = \int_0^{\infty} \rho_a(\tau) \Delta_+^{(1+1)}(y;\tau) d\tau.$$

*For a sufficiently high Schwartz seminorm,*

$$\mathrm{lVertv}_{-r}(h) - v_a(h)\| \le C r^{2a} P_N(h).$$

*Linear combinations perpendicular to V have zero boundary value.*

*Proof.* On a compact interval $0 < \varepsilon \le \tau \le T < \infty$, both endpoint limits of the resolvent kernel are uniform. The corresponding spectral Gram blocks therefore have a boundary limit. This proves convergence of the truncated creation vectors inside the original one-particle representation.

The boundary spectral density obeys , and its integral against is finite. The truncated boundary vectors are thus Cauchy when both spectral endpoints are removed. Their limit remains in the original representation.

For the radial-to-boundary difference, use and . The threshold integral is finite because $2a - b > -1$, and the

ultraviolet integral is finite for $N > \mu + 1$. The squared norm is bounded by $Cr^{4a}P_N(h)^2$, proving . The more precise contribution from the crossing region is . Apply for the operator limit.

Finally, $F = \imath B_V^{-1}\imath^\dagger$ annihilates $V^\perp$, giving the stated boundary relations.

**Remark (Scope of the uniformity).** The result is uniform over nonnegative finite and compressed charts with the orders fixed. It is not uniform over logarithmic channels, negative boundary energies, momentum-dependent boundary laws, or an order approaching a deficiency endpoint. Those are different estimates, not numerical errors to be absorbed into a constant.

**Non-extraction.** These are limits of an already specified free field. They add no independently prepared defect factor and select no contact normalization. The finite instrument does not construct these domains or vectors.

## The Wick-quadratic consequence and the retained scalar obstruction

**Proposition (Quadratics on the energy-cutoff domain).** *For the boundary fields of Theorem , the normal-ordered quadratics* $:a_j a_k^\dagger:(h)$ *exist on* $\mathcal{D}_c$. *Hermitian combinations with real tests are symmetric and closable. Their radial versions converge on that domain, with*

$$\|(Q_r(h) - Q_a(h))\Psi\| \le C_{n,E} r^{2a} P_N(h)\|\Psi\|$$

*on each fixed particle-number and input-energy band.*

*Proof.* The boundary Fourier density is $2\pi 1_{V_+}(p)\rho_a(p_0^2 - p_1^2)$. Equations and give, for the fields and their difference features, bounds of the form

$$C(1 + p_0)^m (p_+ p_-)^{-b}, \qquad p_\pm = p_0 \pm p_1, \qquad b < 1,$$

with an additional factor $r^{4a}$ for the squared difference feature. The individual radial fields obey the same bound, either directly from , or from positivity of the joint radial–boundary Gram block.

For fixed total momentum $P$, positive energies give $0 < p_\pm < P_\pm$. The two beta integrals

$$\int_0^A u^{-b}(A-u)^{-b}\mathrm{d}u = A^{1-2b}\mathrm{B}(1-b,1-b)$$

bound the pair-creation kernel. Its growth in $P$ is polynomial and is absorbed by $\hat{h}(P)$.

For the number-preserving kernel, first restrict the input to $q_0 \le E$. That input measure has finite mass; Schwartz decay of $\hat{h}(p-q)$ makes the restricted kernel Hilbert–Schmidt. The finite-sector creation and annihilation estimates define the quadratic operator on $\mathcal{D}_{\mathrm{c}}$.

The identity

$$u_r \otimes v_r - u_0 \otimes v_0 = (u_r - u_0)\otimes v_r + u_0 \otimes (v_r - v_0)$$

reduces the quadratic difference to the same estimates with one difference feature. Cauchy–Schwarz and give . Symmetry on a dense domain gives closability.

The nonzero Wick square is not Gaussian. Its three-point Wick contraction is nonzero, as explained in Ex1 [17]. Nor does Proposition imply a general interacting boundary algebra or convergence on arbitrary finite-particle vectors.

For comparison with the one-point prescription, define

$$\mathrm{langleU_}L,f\rangle = \lim_{\varepsilon \mathrm{downarrow}\, 0}\left[\int_{r>\varepsilon}\frac{f}{r^2}\mathrm{d}\Omega + \sigma_\alpha \log(\varepsilon/L)\int f(y,0)\mathrm{d}^2y\right].$$

Angular Taylor averaging proves strong convergence and

$$(E_x+2)U_L = \sigma_\alpha \delta_{\mathscr{W}}, \qquad U_{L'} - U_L = -\sigma_\alpha \log(L'/L)\delta_{\mathscr{W}}.$$

These are the parent normalizations [16, 17].

For $\eta_\varepsilon(x) = \varepsilon^{-2}\eta(x/\varepsilon)$,

$$\mathrm{langleU_}L,\eta_\varepsilon\rangle = \varepsilon^{-2}(\mathrm{langleU_}L,\eta\rangle + \sigma_\alpha \log\varepsilon\,\eta(0)).$$

Consequently no nonzero $sU_L + c\delta_{\mathscr{W}}$ with $s \neq 0$ has an ordinary approximate-identity-independent worldsheet

restriction. A nonzero pure delta also fails to restrict. The identically zero distribution remains an exception.

**Stopping condition (Free-domain ending).** The ultraviolet residual identified in the positive-order compressed problem has been replaced by and . The following are not supplied:

• a logarithmic-channel version of this uniform theorem;

• every bulk Wick product on completion tests touching the apex;

• essential self-adjointness of every quadratic;

• an interacting or time-ordered defect algebra;

• restriction of the nonzero covariant c-number part. The VII-H free response and VII-I finite-width free algebra are separate constructions [25].

## Stage B: measurable logarithmic-power reflection profiles

### A larger explicit profile class

Let

$$A(u) = \int_u^\infty v K_0(v)^2 \mathrm{d}v.$$

The classical endpoint estimates are

$$A(0) = \frac{1}{2}, \qquad A(u) - \frac{1}{2} = O(u^2 \log^2 u), \qquad A(u) \operatorname{asymp} e^{-2u} \quad (u \geq 1).$$

The last comparison has positive fixed upper and lower constants. These follow from the modified-Bessel expansions and the integral $\int_0^\infty v K_0(v)^2 \mathrm{d}v = 1/2$ [4].

**Assumption (Measurable logarithmic-power class).** Fix $p > 0$ and $u_0 \in (0, 1)$. The profile $B : (0, \infty) \to \mathbb{C}$ is measurable and obeys

$$|B(u)| \leq C_0 (1 + \log (u_0/u))^{-p}, \quad 0 < u < u_0,$$

$$J_B := \int_{u_0}^{\infty} |B(u)|A(u)\frac{\mathrm{d}u}{u} < \infty.$$

No pointwise ultraviolet growth bound, smoothness, or monotonicity is required. For real one-point corrections take $B$ real.

This is larger in its ultraviolet allowance than Ex2's profile class. It permits weighted-integrable spikes of arbitrary pointwise height. For example, one can add smooth disjoint bumps near integers, with heights $e^{3n}$ and widths $e^{-3n}$; their contribution to is summable, although no fixed polynomial times $e^{2u}$ bounds their heights. This mathematical enlargement is not a claim that every such profile is realized by a physical core.

For $r > R$, define

$$f_R(r) = \frac{1}{4\pi^2\alpha}\int_0^{\infty} qB(qR)K_0(qr)^2\mathrm{d}q, \qquad Q_R(b_0) = 2\pi\alpha\int_R^{b_0} rf_R(r)\mathrm{d}r.$$

Here $b_0 > 0$ is a fixed exterior observation radius. Let $\mathscr{E}_R$ equal $f_R$ on $R < r < b_0$, zero elsewhere transversely, and constant in the worldsheet directions.

**Lemma (Existence and useful absolute bounds).** *Assumption makes $f_R(r)$ absolutely convergent for every $r > R$ and locally integrable on $R < r < b_0$. Moreover,*

$$Q_R(b_0) = \frac{1}{2\pi}\int_0^{\infty} B(u)[A(u) - A(Tu)]\frac{\mathrm{d}u}{u}, \qquad T = b_0/R.$$

*For $w \geq 2$,*

$$w^2\int_0^{\infty} u|B(u)|K_0(wu)^2\mathrm{d}u \leq \frac{C}{(1+\log w)^p} + C(1+J_B)e^{-c\sqrt{w}}.$$

*Also*

$$\int_R^{2R} r|f_R(r)|\mathrm{d}r \leq C$$

*with $C$ independent of $R$.*

*Proof.* For fixed $w > 1$, the ultraviolet ratio

$$\frac{w^2u^2K_0(wu)^2}{A(u)}$$

is bounded on $u \geq u_0$, because its large-$u$ behavior is a polynomial times $e^{-2(w-1)u}$. Near zero use and the logarithmic behavior of $K_0$. This proves absolute convergence. Fubini applied to absolute values gives local integrability and .

To prove , put $v = wu$. On $u < u_0$, split $v \leq \sqrt{w}$ and its complement. For sufficiently large $w$, the first part has

$$1 + \log(u_0/u) \geq c(1 + \log w).$$

The integral of $vK_0(v)^2$ is finite. In the second part use boundedness of $B$ near zero and the exponentially small Bessel tail.

On $u \geq u_0$, compare to . The supremum of the displayed ultraviolet ratio is at most $Cwe^{-cw}$ for sufficiently large $w$. The remaining bounded interval of $w$'s is covered by continuity of the Bessel bounds. This proves .

Finally,

$$2\pi\alpha\int_R^{2R} r|f_R(r)|\mathrm{d}r \leq \frac{1}{2\pi}\int_0^{\infty}|B(u)|[A(u) - A(2u)]\frac{\mathrm{d}u}{u}.$$

The last integral is finite. At zero its additional factor is $O(u^2\log^2 u)$; at infinity use $A(u)$.

## A quantitative concentration theorem

**Theorem (Test-uniform concentration with a logarithmic rate).** *Under Assumption , for $T = b_0/R \geq 4$,*

$$\int_R^{b_0} r^3|f_R(r)|\mathrm{d}r \leq Cb_0^2[(1 + \log T)^{-p} + T^{-1}].$$

*For every fixed compact test support in $r < b_0$,*

$$|\langle \mathscr{E}_R - Q_R(b_0)\delta_{\mathscr{W}}, f\rangle| \leq C_K b_0^2[(1 + \log T)^{-p} + T^{-1}]p_2(f).$$

*In particular, $\mathscr{E}_R - Q_R(b_0)\delta_{\mathscr{W}} \to 0$ strongly in $\mathcal{D}'$ on those test-support stages.*

*Proof.* The interval $R < r < 2R$ contributes $O(R^2)$ by . For $r \geq 2R$, changing variables in and applying gives

$$|f_R(r)| \leq \frac{C}{r^2}\Big[(1 + \log(r/R))^{-p} + (1 + J_B)e^{-c\sqrt{r/R}}\Big].$$

Split the remaining moment integral at $r = \sqrt{b_0}R$. The lower part is $O(b_0 R)$. On the upper part,

$1 + \log(r/R) \geq c(1 + \log T)$. The exponential term has integral $O(R^2)$. This proves .

For a test, let $\bar{f}(y, r)$ denote its angular average. Then

$$|\bar{f}(y, r) - f(y, 0)| \leq Cr^2 p_2(f).$$

Pairing with the radial correction and using its exact mass $Q_R(b_0)$ gives . The estimate is uniform on bounded sets of tests with fixed compact support.

**Mass asymptotics with a Dini remainder**

Assume in addition that, for some $\gamma \in \mathbb{C}$ and $\kappa > 0$,

$$B(u) = -\gamma[\log(\kappa/u)]^{-p} + e(u), \qquad \int_0^{u_0} |e(u)|\frac{\mathrm{d}u}{u} < \infty,$$

where $u_0$ has been decreased so $\log(\kappa/u_0) > 1$. Keep Assumption . For $T > e$, put

$$\Phi_p(T) = \int_1^{\log T} s^{-p}\mathrm{d}s = \begin{cases} \log\log T, & p = 1, \\ \dfrac{(\log T)^{1-p} - 1}{1 - p}, & p \neq 1. \end{cases}$$

**Theorem (Unified logarithmic-power mass law).** *There is a finite $C_B$ such that*

$$Q_R(b_0) + \frac{\gamma}{4\pi}\Phi_p(T) \to C_B.$$

*For sufficiently large T, the error is bounded by*

$$C(1 + \log T)^{-p} + \frac{1}{2\pi}\int_0^{\min\{u_0, T^{-1/2}\}} |e(u)|\frac{\mathrm{d}u}{u} + Ce^{-c\sqrt{T}}.$$

*Consequently,*

$$\mathscr{E}_R + \frac{\gamma}{4\pi}\Phi_p(T)\delta_{\mathscr{W}} \to C_B\delta_{\mathscr{W}}$$

*strongly, with the scalar error and the test error kept separate.*

*For* $p = 1, \gamma = 1$*, the divergent mass is* $-\frac{1}{4\pi}\log\log(b_0/R)$*. For* $p > 1$*, the mass has a finite limit even without a divergent subtraction.*

*Proof.* Use . The *e*-part has finite limit

$$\frac{1}{2\pi}\int_0^{u_0} e(u)A(u)\frac{\mathrm{d}u}{u}.$$

Its omitted $A(Tu)$ term is bounded by the Dini integral below $T^{-1/2}$ plus an exponentially small term above that point. The part $u \geq u_0$ is treated by ; the ratio $A(Tu)/A(u)$ decays exponentially uniformly there.

For the model term below $u_0$, replacing $A(u)$ by 1/2 changes only a convergent integral. The smooth lower cutoff $1/2 - A(Tu)$ can be replaced by $\frac{1}{2}1_{\{u > T^{-1}\}}$. After $v = Tu$, the discrepancy is bounded by the integrable kernel

$$\left|\frac{1}{2} - A(v) - \frac{1}{2}1_{\{v > 1\}}\right|\frac{\mathrm{d}v}{v}.$$

On $v \leq \sqrt{T}$, its logarithmic denominator is bounded below by $c\log T$. On the remaining range the kernel is exponentially small. The replacement error is $O((\log T)^{-p})$.

The only nonconstant large-$T$ contribution is therefore

$$-\frac{\gamma}{4\pi}\int_{T^{-1}}^{u_0}[\log(\kappa/u)]^{-p}\frac{\mathrm{d}u}{u}.$$

Substitution $s = \log(\kappa/u)$ gives $-\gamma\Phi_p(T)/(4\pi)$, a finite constant, and $O((\log T)^{-p})$. This proves –. Apply Theorem for .

**Proposition (Stability under integrable profile differences).** *Let* $B, \widetilde{B}$ *obey Theorem with the same* $p, \gamma$*, and suppose*

$$\int_0^{\infty}|\widetilde{B}(u) - B(u)|A(u)\frac{\mathrm{d}u}{u} < \infty.$$

*Their finite constants satisfy*

$$C_{\widetilde{B}} - C_B = \frac{1}{2\pi}\int_0^\infty [\widetilde{B}(u) - B(u)]A(u)\frac{du}{u}.$$

*Proof.* Subtract the exact mass identities and use dominated convergence.

**Physical comparison.** The flower-pot positive-coupling reflection profile of Ex1 and Ex2 has $p = 1, \gamma = 1$. Theorem includes that exterior diagnostic, but not a complete physical core contact. Allen–Kay–Ottewill provide the long-range exterior context [6]. No core state, stress subtraction, or Einstein source law is obtained by extending the profile class [12, 22].

## Stage B continued: moments, transfer, and invisible interior freedom

### A moment-resolved estimate

Let $\mu_R$ be a finite signed transverse measure, including the density convention $\alpha \mathrm{d}^2x$, supported in $r \le c_0 R$. Regard it as longitudinally constant. Define

$$m_R = \int \mathrm{d}\mu_R, \qquad d_R^i = \int x^i \mathrm{d}\mu_R, \qquad M_{2,R} = \int |x|^2 \mathrm{d}|\mu_R|.$$

**Theorem (Mass–dipole approximation).** *For every fixed compact test support,*

$$\left|\left\langle \mu_R - m_R\delta_{\mathscr{W}} + d_R^i\partial_i\delta_{\mathscr{W}}, f\right\rangle\right| \le C_K M_{2,R} p_2(f).$$

*Thus, if $m_R \to m$, $d_R \to d$, and $M_{2,R} \to 0$, then*

$$\mu_R \to m\delta_{\mathscr{W}} - d^i\partial_i\delta_{\mathscr{W}}$$

*strongly.*

*If $\mu_R$ is rotation invariant, put*

$$q_R = \int |x|^2 \mathrm{d}\mu_R, \qquad M_{4,R} = \int |x|^4 \mathrm{d}|\mu_R|.$$

*Then the sharper expansion is*

$$\left|\left\langle \mu_R - m_R\delta_{\mathscr{M}} - \frac{q_R}{4}\Delta\delta_{\mathscr{M}}, f\right\rangle\right| \le C_K M_{4,R} p_4(f),$$

*and* $M_{4,R} \le c_0^2 R^2 M_{2,R}$.

*Proof.* Taylor expansion gives

$$f(y,x) = f(y,0) + x^i\partial_i f(y,0) + O(|x|^2 p_2(f)).$$

Pairing yields , including the minus sign in the dipole distribution.

For a rotation-invariant measure, angular averaging gives

$$\bar{f}(y,r) = f(y,0) + \frac{r^2}{4}\Delta f(y,0) + O(r^4 p_4(f)).$$

This proves . The support condition gives the last inequality.

For positive measures, bounded masses imply $M_{2,R} \le c_0^2 R^2 m_R \to 0$. Renormalized one-point corrections are not generally positive measures. The absolute-moment hypothesis must therefore not be discarded in the signed problem.

**Quantitative assembly**

**Corollary (A sufficient assembled-contact criterion).** *Suppose a separately normalized core correction has*

$$\mathscr{T}_R = \mathscr{E}_R + \mu_R + \mathscr{F}_R$$

*in the same density and smooth structure. Assume* $\mu_R$ *is radial,* $m_R \to m$, $M_{2,R} \to 0$, *and* $\mathscr{F}_R \to \mathscr{F}_0$ *strongly. Under Theorem ,*

$$\mathscr{T}_R + \frac{\gamma}{4\pi}\Phi_p(T)\delta_{\mathscr{M}} \to \mathscr{F}_0 + (C_B + m)\delta_{\mathscr{M}}.$$

*For a fixed test-support stage, its error is bounded by the sum of:*

***1.*** *the exterior second-moment error in ;*

***2.*** *the scalar profile error in ;*

***3.*** $C_K(|m_R - m|p_0(f) + M_{2,R}p_2(f))$*;*

***4.*** *the actual* $\mathscr{F}_R - \mathscr{F}_0$ *pairing.*

*Proof.* Add Theorem and , with $d_R = 0$. No estimate for $\mathscr{F}_R$ is inferred from the other terms.

The criterion is sufficient. Convergence of an absolute second moment is not asserted necessary for every signed family with cancellations.

**What exterior agreement misses**

**Proposition (Scalar and quadrupole interior counterexamples).** *Choose a radial smooth* $\psi$ *supported in* $r < 1$ *with* $\int \psi \, \alpha \mathrm{d}^2x = 1$. *If* $R^2|a_R| \to 0$, *then*

$$j_R(x) = a_R R^{-2}\psi(x/R) \quad \text{satisfies} \quad j_R - a_R\delta_{\mathscr{W}} \to 0$$

*strongly. It changes no value for* $r > R$. *It can therefore alter a finite contact or a* $\log\log(b_0/R)$ *coefficient without changing the exterior diagnostic.*

*There also exists a radial smooth* $\chi$ *supported in* $r < 1$ *with*

$$\int \chi \, \alpha \mathrm{d}^2x = 0, \qquad q = \int |x|^2\chi(x)\alpha \mathrm{d}^2x \neq 0.$$

*For such a choice,*

$$R^{-4}\chi(x/R) \to \frac{q}{4}\Delta\delta_{\mathscr{W}}$$

*strongly, with an* $O(R^2p_4(f))$ *remainder.*

*Proof.* The first assertion is the radial second-moment estimate. Its error is at most $C_K|a_R|R^2p_2(f)$.

For the second, take the difference of two radial bumps with equal mass and different radial scales. The mass vanishes and the second moment does not. The scaled family has fixed signed second moment $q$ and fourth absolute moment $O(R^2)$. Apply .

**Remark (Admissibility matters).** The quadrupole limit lies outside the parent's scalar scaling-degree-2 ambiguity

class. It is not a new ambiguity inside that class. It shows that exterior agreement alone does not prove membership in the class.

Likewise, neither interior counterexample is asserted to be the Wick square of a fixed physical core state in a fixed subtraction scheme. It identifies information that a proposed physical assembly theorem must control.

**Stopping condition (Physical core transfer remains open).** The full interior prescription, nonzero angular-mode remainder, and stress-contact limit have not been calculated here for a named complete physical core. In particular, the finite traceless stress coordinate $\tau$ is not determined by a scalar moment calculation.

I-B-VP-5's spatial selection is not a proof of the hypotheses of Corollary [19]. A finite report does not measure these missing limits, and conservation does not supply Einstein dynamics [12, 13, 22].

## Stage C: a parameter-dependent logarithmic worldsheet primitive

This section is Euclidean. Let $u \in \mathbb{R}^2$ be a relative worldsheet coordinate and set

$$k_\alpha(u) = \frac{1}{4\pi^2 \alpha |u|^2}, \qquad \mathfrak{r}_\alpha = \frac{1}{2\pi\alpha}.$$

For $L_w > 0$, define

$$\langle \mathrm{P}_{L_w}, f \rangle = \lim_{\delta \downarrow 0} \left[ \int_{|u|>\delta} k_\alpha(u) f(u) \mathrm{d}^2 u + \mathfrak{r}_\alpha \log(\delta/L_w) f(0) \right].$$

This is a relative-worldsheet distribution. It is not the transverse generating distribution $U_L$.

**Lemma (Primitive residue and finite freedom).** *The limit exists strongly on fixed compact test-support stages and obeys*

$$(E_u + 2) P_{L_w} = \mathfrak{r}_\alpha \delta^{(2)}(u), \qquad P_{L_w'} - P_{L_w} = -\mathfrak{r}_\alpha \log(L_w'/L_w) \delta^{(2)}(u).$$

*Two extensions of $k_\alpha$ with scaling degree at most two differ by a scalar delta.*

*Proof.* Angular Taylor averaging removes the constant term and leaves an integrable radial remainder. The Euler and scale identities follow by radial integration and by subtracting the defining finite parts. A supported

distribution of scaling degree at most two in two dimensions has only a zeroth delta jet. This is also the elementary instance of the extension framework in [7, 8].

### Smooth external parameters

Let $\mathcal{O} \subseteq \mathbb{R}^m$ be an external parameter domain and $K$Subset $\mathcal{O}$. Let

$$H \in C^\infty(\mathcal{O}; C_c^\infty(\mathbb{R}_y^2 \times \mathbb{R}_u^2))$$

have common compact support in $(y, u)$. Put

$$J_\delta(X) = \int_{\mathbb{R}^2} \mathrm{d}^2 y \int_{|u| > \delta} k_\alpha(u) H(X; y, u) \mathrm{d}^2 u$$

$$+ \mathfrak{r}_\alpha \log(\delta / L_w) \int H(X; y, 0) \mathrm{d}^2 y.$$

Let $\mathcal{N}_{k,j}(H)$ denote the maximum of $|\partial_X^\lambda \partial_u^\eta H|$ on the common compact support and $X \in K$, for $|\lambda| \le k$, $|\eta| \le j$.

**Theorem (Fixed-parameter-support $C^k$ subtraction).** *The functions $J_\delta$ converge in $C^k(K)$, for every $k$, to a smooth $J_{L_w}$. For sufficiently small $\delta$,*

$$\mathrm{lVert} J_- \delta - J_{L_w} \|_{C^k(K)} \le C_{K,H} \delta^2 \mathcal{N}_{k,2}(H).$$

*More precisely,*

$$J_\delta(X) - J_{L_w}(X) = -\frac{\delta^2}{16\pi\alpha} \int \Delta_u H(X; y, 0) \mathrm{d}^2 y + \mathcal{R}_\delta(X),$$

$$\| \mathcal{R}_\delta \|_{C^k(K)} \le C \delta^4 \mathcal{N}_{k,4}(H).$$

*The constants depend on the fixed supports, not on $\delta$.*

*Proof.* For each external multiindex $\lambda$, angular Taylor averaging gives

$$\int_0^{2\pi} \partial_X^\lambda H(X; y, r\omega) \mathrm{d}\theta = 2\pi \partial_X^\lambda H(X; y, 0) + \frac{\pi r^2}{2} \Delta_u \partial_X^\lambda H(X; y, 0) + O(r^4 \mathcal{N}_{k,4}(H)).$$

After the constant logarithm is subtracted, the difference between the cutoff expression and its limit is the negative of the remaining integral over $0 < r < \delta$. Since the radial measure is $(4\pi^2\alpha)^{-1} \mathrm{d}r/r$, its second-order term is

$$-\frac{1}{4\pi^2\alpha}\frac{\pi\delta^2}{2}\frac{}{2}\Delta_u H = -\frac{\delta^2}{16\pi\alpha}\Delta_u H.$$

The fourth-order remainder integrates to $O(\delta^4)$. Using only the second-derivative Taylor remainder proves .

Every external derivative acts on $H$ on a fixed integration domain. Applying the same estimate to that differentiated amplitude proves convergence of the derivatives. No derivative of an error inequality is taken.

**Remark (The moving-singularity issue).** Theorem works because the relative coordinate $u$ is fixed before differentiation. If the singular locus itself depends on an external parameter, one must first make a controlled change to fixed relative coordinates and include its Jacobian and coefficient derivatives. A statement about one undifferentiated cutoff integral would not supply that argument.

## Stage C continued: a switched numerical sandwich

For a regular external Euclidean point $X = (y_X, r_X, \varphi_X)$, the neutral Friedrichs apex-endpoint kernel is

$$G_X(y) = G_F(X, (y, 0)) = \frac{1}{4\pi^2\alpha(r_X^2 + |y_X - y|^2)}.$$

This normalization is the one used in Ex1 and Ex2 [17, 18].

Fix $\chi \in C_c^\infty(\mathbb{R}^2)$. For regular external $X, Z$, define

$$H(X, Z; y, u) = \chi(y)\chi(y + u)G_X(y)G_Z(y + u)$$

and

$$D_\chi(X, Z) = \int \chi(y)^2 G_X(y)G_Z(y)\mathrm{d}^2 y.$$

For a scalar $g$, put

$$S_\delta^{[2]}(X, Z) = g^2[\int \mathrm{d}^2 y \int_{|u| > \delta} k_\alpha(u)H(X, Z; y, u)\mathrm{d}^2 u$$

$$+\mathrm{r}_\alpha \log(\delta/L_w)D_\chi(X, Z)].$$

**Corollary (Controlled sandwich and local normalization).** *Let $K$ be a compact external set of pairs $(X, Z)$ with*

$r_X, r_Z \geq r_0 > 0$. *Then* $S_\delta^{[2]}$ *converges in* $C^k(K)$ *to* $S_{L_w}^{[2]}$, *and*

$$\mathrm{lVertS}_{-}\delta^{[2]} - S_{L_w}^{[2]}\|_{C^k(K)} \leq C_{k,\chi,\alpha,K}|g|^2\delta^2.$$

*If* $0 < r_0 \leq 1$, *a conservative explicit radius dependence is*

$$C_{k,\chi,\alpha,K} \leq C'_{k,\chi,\alpha,K_y} r_0^{-6-k}.$$

*Its scale change is*

$$S_{L_w'}^{[2]} - S_{L_w}^{[2]} = -g^2 \mathrm{r}_\alpha \log(L_w'/L_w) D_\chi.$$

*For the declared relative-coordinate ambiguity* $P_{L_w} \mapsto P_{L_w} + \lambda_w \delta^{(2)}(u)$, *the numerical kernel changes by* $g^2\lambda_w D_\chi$.

*Proof.* The support of is common and compact in $y, u$. Both external endpoints stay a positive distance from the worldsheet, so the amplitude is smooth in all external variables. Apply Theorem .

A derivative of total order $j$ of $(r^2 + |y_X - y|^2)^{-1}$ is bounded by $C_j r_0^{-2-j}$. The two $u$-derivatives required by fall on $\chi(y+u)G_Z(y+u)$. The product estimate gives the stated conservative $r_0^{-6-k}$ dependence. Equation gives and the finite ambiguity.

This evaluates the local logarithmic primitive inside one smoothly switched bulk-to-worldsheet-to-bulk numerical kernel. It is more than merely displaying its logarithmic residue. It is not the complete second-order $\lambda\varphi^4$ correction, a time-ordered operator product, or a resummed point interaction.

**The first-order baseline is not changed**

For the real neutral convention $\lambda\varphi^4/4!$, the separate first-order apex and regular-diagonal subtractions use $L$ and $\mu$, respectively. They are at disjoint loci when the external point is regular [17, 18]. The contact-dependent term is checked directly from :

$$\int_{\mathbb{R}^2} G_X(y)^2 \mathrm{d}^2 y = \frac{1}{16\pi^3\alpha^2 r_X^2}, \qquad \frac{\partial}{\partial c}\delta_\lambda F(r) = -\frac{\lambda}{32\pi^3\alpha^2 r^2}.$$

The finite first-order constant is represented by the coordinate change

$$c_{\mathrm{eff}} = c - 32\pi^3\alpha^2 s A_\alpha, \qquad s = \frac{\alpha^{-2} - 1}{48\pi^2}.$$

This algebraic reparametrization is not a physical selection of $c$.

For $S_{\mathrm{def}} = g\int_{\mathscr{M}} \varphi^2/2$, comparison with gives

$$g = \frac{\lambda c}{2}, \qquad S_{\mathrm{def}} = \frac{\lambda c}{4}\int_{\mathscr{M}} \varphi^2.$$

The second-order primitive length $L_w$ is additional relative-worldsheet normalization data. It is not fixed by the transverse presentation length $L$ or by the regular-diagonal scale $\mu$.

**Stopping condition (Exact interaction boundary).** The numerical sandwich theorem does not construct an interacting apex algebra, an all-orders product, a renormalization-group flow, or an interacting Ward identity. It also does not prove a uniform limit as external endpoints approach the worldsheet: the displayed $r_0$-dependence records the loss of that estimate.

A finite logarithmic zero-channel point interaction still has the negative eigenvalue $-4e^{-2\gamma_E}/L_b^2$, obtained by matching the small-$r$ expansion of $K_0$ [1, 2]. That fact diagnoses one massless spectral realization, not every possible enlarged defect theory.

The separate VII–IX product domains are not imported [25, 26]. Neither the response nor the sandwich supplies an Einstein equation [12, 22].

## Stage D: the resonant family's full double-pole contribution

### Boundary data and exact off-diagonal entry

Assume

$$0 < \nu_0 < \nu_1 < 1, \qquad \sigma = \nu_0 + \nu_1 = \alpha^{-1} \in (1, 2), \qquad \delta = \nu_1 - \nu_0 > 0.$$

The holonomy is $\beta = \alpha\nu_0$. Fix a reference length for the boundary coefficients. All radial and momentum variables in this section are expressed in those fixed reference units; dimensions can be restored in the defining integral.

Take the infinite-energy rank-one boundary direction $H = tww^{\dagger}$, $t \to \infty$, with $w_0 w_1 \neq 0$. The rank-one inverse formula gives

$$C_{01}(q) = -\frac{K_w q^{-\delta}}{1 + d_w q^{-2\delta}},$$

where

$$K_w = S_1^{-1}\frac{A_1 \overline{w}_1}{A_0 \overline{w}_0}, \qquad d_w = \frac{S_1^{-1} A_1^2 |w_1|^2}{S_0^{-1} A_0^2 |w_0|^2} > 0.$$

Define

$$R_{01}(r) = \frac{1}{4\pi^2 \alpha}\int_0^{\infty} q C_{01}(q) K_{\nu_0}(qr) K_{\nu_1}(qr)\,\mathrm{d}q.$$

The real scalar off-diagonal part is $2\mathrm{Re}\,(e^{i\varphi} R_{01}(r))$.

For completeness, follows from

$$C = S^{-1} - \frac{S^{-1} u u^{\dagger} S^{-1}}{u^{\dagger} S^{-1} u}, \qquad u = \mathrm{diag}\,(A_j q^{-\nu_j}) w.$$

Dividing numerator and denominator by the smaller-order diagonal term gives . In particular, the boundary-data phase is not discarded.

**Mellin factors and the fundamental strip**

Let

$$M(s) = \frac{2^{s-3}}{\Gamma(s)} \prod_{\varepsilon_0, \varepsilon_1 = \pm 1} \Gamma\left(\frac{s + \varepsilon_0 \nu_0 + \varepsilon_1 \nu_1}{2}\right).$$

For $\mathrm{Re}\, s > \sigma$, it equals $\int_0^{\infty} u^{s-1} K_{\nu_0}(u) K_{\nu_1}(u)\,\mathrm{d}u$. One derivation inserts the two Schwinger representations of $K_\nu$, integrates in $u$, and evaluates the remaining beta integral. This is the classical Bessel Mellin formula [4, 5].

Direct substitution $q^{2\delta} = d_w v$ gives

$$\mathscr{C}(z) = -\frac{K_w \pi}{2\delta} d_w^{(z-\delta)/(2\delta)} \csc\left(\frac{\pi(z+\delta)}{2\delta}\right), \qquad -\delta < \mathrm{Re}\, z < \delta.$$

Since $\sigma + \delta = 2v_1 < 2$, the line

$$2 - \delta < c < 2 + \delta$$

is a common fundamental line, and

$$R_{01}(r) = \frac{1}{4\pi^2\alpha}\frac{1}{2\pi \mathrm{i}}\int_{\mathrm{Re}\, s = c} r^{-s} M(s)\mathscr{C}(2-s)\mathrm{d}s.$$

**The logarithmic coefficient and its companion constant**

Let $\psi = \Gamma'/\Gamma$ be the digamma function. For $m \geq 1$, impose the resonant family

$$1 < \sigma < 2, \qquad \delta = \frac{2-\sigma}{2m+1}, \qquad v_0 = \frac{\sigma - \delta}{2}, \qquad v_1 = \frac{\sigma + \delta}{2}.$$

Set

$$\eta_\ell = (2\ell + 1)\delta, \qquad c_\ell = -K_w(-d_w)^\ell,$$

and define

$$\begin{aligned}\mathscr{L}_m &= \frac{A_0 A_1 c_m}{4\pi^2\alpha} \\ &= \frac{(-1)^{m+1} S_1^{-1} A_1^2 \overline{w}_1}{4\pi^2\alpha\ \overline{w}_0} d_w^m, \\ \mathscr{B}_m &= \mathscr{L}_m\left[\log 2 + \frac{\psi(v_0) + \psi(v_1) - \psi(\sigma) - \gamma_E}{2} - \frac{\log d_w}{2\delta}\right].\end{aligned}$$

**Theorem (Complete resonant contribution with a controlled remainder).** *For any*

$$0 < \varepsilon < \min\{2\delta, 2v_0\},$$

*the expansion is*

$$\begin{aligned}R_{01}(r) = \frac{1}{4\pi^2\alpha}\sum_{\ell=0}^{m-1} c_\ell M(2-\eta_\ell) r^{-2+\eta_\ell} \\ + r^{-\sigma}\left[\mathscr{L}_m \log(1/r) + \mathscr{B}_m\right] + O(r^{-\sigma+\varepsilon}).\end{aligned}$$

*For every fixed $j \ge 0$, the remainder remains $O(r^{-\sigma+\varepsilon})$ after applying $(r\partial_r)^j$.*

*The coefficient $\mathscr{L}_m$ is nonzero for every mixed boundary direction $w_0 w_1 \neq 0$.*

*Proof.* The simple ultraviolet poles of $\mathscr{C}(2-s)$ occur at $s = 2 - \eta_\ell$. For $\ell < m$, these lie strictly above $\sigma$ and contribute the first line of .

Write $s = \sigma + \zeta$. Expanding gives

$$M(\sigma+\zeta) = \frac{A_0 A_1}{\zeta} + A_0 A_1 \left[ \log 2 + \frac{\psi(v_0) + \psi(v_1) - \psi(\sigma) - \gamma_E}{2} \right] + O(\zeta).$$

Indeed the singular gamma factor is $\Gamma(\zeta/2) = 2/\zeta - \gamma_E + O(\zeta)$; the three other gamma factors are regular and nonzero.

Since $2 - \sigma = (2m+1)\delta$, equation gives

$$\mathscr{C}(2-\sigma-\zeta) = \frac{c_m}{\zeta} - \frac{c_m \log d_w}{2\delta} + O(\zeta).$$

The product has a double pole. Multiplication by $r^{-\sigma-\zeta} = r^{-\sigma}(1 - \zeta \log r + O(\zeta^2))$ shows that its residue is exactly the second line of , with –.

The next possible leftward matrix pole is $\sigma - 2\delta$. The next Bessel pole is at $\delta$, whose distance below $\sigma$ is $2v_0$. Thus the line $\operatorname{Re} s = \sigma - \varepsilon$ crosses no further pole.

Stirling's formula makes $M(s)$ decay exponentially on vertical lines, up to a polynomial. The cosecant factor in supplies additional exponential decay. Contour displacement is therefore justified and the new line contributes $O(r^{-\sigma+\varepsilon})$. Multiplication of the integrand by $(-s)^j$ proves the differentiated remainder without differentiating an undifferentiated error bound.

All factors in are nonzero in the mixed chart, proving the last assertion.

**Proposition (Exact cancellation conditions).** *For the single mixed rank-one chart, the logarithmic scalar coefficient is*

$$2\mathrm{Re}\,(e^{i\varphi}\mathscr{L}_m).$$

*It vanishes identically as an angular function if and only if the off-diagonal chart is unmixed,* $w_0 w_1 = 0$. *For a mixed chart it can vanish at particular angles or under rotation averaging, but not as the complete angular distribution.*

*More generally, in a specified sum of contributions with the same exponent and the same angular harmonic, the logarithm cancels exactly when the sum of their complex coefficients* $\mathscr{L}_m$*, after the actual tensor or observable projection, is zero.*

*Proof.* For a complex number $z$, $\mathrm{Re}\,(e^{i\varphi}z)$ vanishes for every $\varphi$ exactly when $z = 0$. Equation excludes this in a mixed chart. For an unmixed direction the original inverse formula has $C_{01} = 0$. The sum statement is linear independence of the specified power, logarithm, and angular harmonic after equal terms have been collected.

No apex-supported contact can remove a nonzero off-apex logarithm: it has zero restriction to every regular open set, whereas does not.

**Stopping condition (Resonance ending).** Theorem evaluates the complete double-pole contribution for the family . It does not classify every compressed chart, every coefficient cancellation, or every differentiated Ward flux. The actual-pole criterion of Ex1 and Ex2 remains a criterion on boundary data, not a closed countable set of points in the $(\alpha, \beta)$ plane.

The off-apex logarithm is not automatically a supported dilation residue. No unrestricted stress/current extension or uniqueness theorem is released.

## Stage D concluded: KH elimination conditions and a higher-weight residual

Let

$$R^i = \varepsilon^i{}_j x^j, \qquad \varepsilon = \begin{pmatrix} 0 & -1 \\ 1 & 0 \end{pmatrix}.$$

The smooth Killing–homothety operations are

$$\mathrm{W1}_A(Q) = \partial_\mu Q^\mu{}_A,$$

$$\mathrm{W2}(Q) = \partial_i(Q^i{}_j R^j),$$

$$\mathrm{W3}(Q) = \partial_i(Q^i{}_j x^j) - Q^i{}_i.$$

They are operations on the auxiliary completion, not a singular-metric index-lowering rule.

**The polar-vector submodule**

For a fixed real $v \neq 0$, put $D_v = v^k \partial_k \delta_{\mathscr{M}}$. Consider precisely

$$Q^A{}_B = u\,\delta^A_B D_v, \qquad Q^i{}_j = a_0 \delta^i_j D_v + b_0 v^i \partial_j \delta_{\mathscr{M}} + c_0 v_j \partial^i \delta_{\mathscr{M}},$$

with zero mixed blocks. This is the polar-vector, parity-even equivariant submodule, not the unrestricted first-jet space.

**Proposition (Exact polar-vector calculation).** *The simultaneous KH equations and zero trace in have exactly the solutions*

$$Q^A{}_B = -\frac{2a_0}{3}\delta^A_B D_v,$$

$$Q^i{}_j = a_0\Big[\delta^i_j D_v - \frac{5}{3} v^i \partial_j \delta_{\mathscr{M}} + v_j \partial^i \delta_{\mathscr{M}}\Big].$$

*For $a_0 \neq 0$, each of the following additional conditions excludes this submodule:*

***1.*** *reference-lowered symmetry;*

***2.*** *full reference divergence $\partial_i Q^i{}_j = 0$;*

***3.*** *degree-$-4$ almost homogeneity $(E_x + 4)^2 Q = 0$.*

*Proof.* Using $x^\ell \partial_k \delta_{\mathscr{M}} = -\delta^\ell_k \delta_{\mathscr{M}}$, W2 gives $c_0 = a_0$, and W3 gives

$$3a_0 + 3b_0 + 2c_0 = 0.$$

The trace gives $2u + 2a_0 + b_0 + c_0 = 0$. Thus $b_0 = -5a_0/3$ and $u = -2a_0/3$. W1 vanishes by longitudinal translation

invariance.

The antisymmetric transverse part is

$$Q_{ij} - Q_{ji} = -\frac{8a_0}{3}(v_i\partial_j - v_j\partial_i)\delta_{\mathscr{W}},$$

and the full reference divergence is

$$\partial_i Q^i{}_j = a_0\Big[v_j\Delta - \frac{2}{3}v^k\partial_j\partial_k\Big]\delta_{\mathscr{W}}.$$

Both are nonzero for $a_0 \neq 0$. Finally, every first normal delta jet obeys $(E_x + 3)Q = 0$, hence $(E_x + 4)^2 Q = Q$.

**Remark (A notational correction).** Equation writes the constant vector coefficient explicitly. The ambiguous typography $\partial_j v^k \partial_k$ in Ex2 must not be read as differentiating the constant $v^k$. The intended operator is $v^k\partial_j\partial_k$.

**Why reference symmetry alone is not unrestricted uniqueness**

**Proposition (A symmetric higher-rotation-weight first jet).** *Let $S_{ijk}$ be a nonzero real, completely symmetric, trace-free rank-three tensor on $\mathbb{R}^2$. Define*

$$Q^i{}_j = S_{ijk}\partial_k\delta_{\mathscr{W}}, \qquad Q^A{}_B = Q^A{}_i = Q^i{}_A = 0.$$

*Then Q is reference symmetric and traceless and satisfies W1–W3. It has transverse scaling degree three. It generally fails full reference conservation and degree-−4 almost homogeneity.*

*An explicit example is*

$$(Q^i{}_j) = \begin{pmatrix} \partial_1 & -\partial_2 \\ -\partial_2 & -\partial_1 \end{pmatrix}\delta_{\mathscr{W}}.$$

*Proof.* Trace-freeness gives $Q^i{}_i = 0$. Symmetry in $j, k$ and contraction with $\varepsilon^j{}_k$ give

$$Q^i{}_j R^j = 0.$$

Contraction with $x^j$ gives

$$Q^{i}{}_{j}x^{j} = -S_{ijj}\delta_{\mathscr{W}} = 0.$$

Thus W2 and W3 vanish, and W1 is automatic. The explicit tensor corresponds to $S_{111} = 1$ and $S_{122} = S_{212} = S_{221} = -1$.

For , the reference divergence is

$$(\partial_1^2 - \partial_2^2, \; -2\partial_1\partial_2)\delta_{\mathscr{W}},$$

which is not zero. The Euler calculation is the same as in Proposition .

The example has rotation weight three and is outside the polar-vector submodule. It is not automatically admissible for an off-diagonal state whose declared parameter dependence allows only a specific weight-one coefficient. That admissibility question must be stated, not inferred.

**Stopping condition (Exact KH scope).** The polar-vector module and its three exclusions are completely calculated. The higher-weight example proves that removal of that module, or reference symmetry alone, is not a proof of unrestricted KH uniqueness. No complete supported contact classification is claimed.

Regular $g$-symmetry does not authorize lowering supported tensors with the singular conical metric. These contact calculations neither normalize a physical core nor supply Einstein dynamics [16, 17, 18, 12, 22].

## Dirac audit and the retained comparison boundary

No Dirac formula is used in the preceding proofs. The disclosed comparison remains unresolved [16, 17, 18]. For the supplied disputed brackets,

$$P_{\pm} = (\alpha^2 - 1)(17\alpha^2 + 7) + 120\beta^2(\alpha^2 - 2\beta^2 \pm 1),$$

the exact difference is

$$P_{+} - P_{-} = 240\beta^2.$$

An overall multiplicity or sign fixed at zero flux does not remove this flux-dependent difference. That observation

does not identify its physical origin.

The primary record of Spinelly–Bezerra de Mello concerns a massless left-handed field and finite magnetic-flux-tube models [10]. Its journal reference is *International Journal of Modern Physics D* **13** (2004), 607–624, as corrected in Ex2. This bibliographic correction is not a same-branch, same-chirality, same-flux, same-subtraction calculation.

**Stopping condition (Dirac ending).** The coefficient discrepancy is not resolved. Neither paper is declared erroneous, and no new spinor domain, apex extension, or physical branch identification is asserted. The separately described Dirac-domain and scalar thermal projects in I-B-VP-7 and I-B-VP-8 remain non-identifications [27].

## Reliability, coverage, and the release ledger

### Evidence and execution

We retain the integrity discipline documented in XVI-O1-ST [23]:

**\texttt{ANALYTIC}** A result established by the displayed proof on the stated domain, including its identified classical analytical inputs.

**\texttt{SUPPLIED}** A version-qualified source statement or historical computational report not independently executed here.

**\texttt{UNCERTIFIED}** A numerical approximation or certification claim lacking the artifacts and total error bounds required for the stronger interpretation.

An unproved mathematical implication is not converted into a numerical residual by attaching to it. It remains an open proof obligation. Conversely, the explicit crossing estimate is analytical, not a numerical fit.

No symbolic program, quadrature, compiler, repository build, or proof assistant was executed for this draft. Historical reports in the parent source remain attributed reports. The source-level calculations here do not independently rerun them.

**Updated residual table**

toprule Object Status Exact boundary or retained residual

midrule endfirsthead toprule Object Status Exact boundary or retained residual

midrule endhead bottomrule endfoot

Compressed inverse and matrix boundary spectrum ANALYTIC Fixed positive orders; compression before inversion; no logarithmic channel.

Ultraviolet transition ANALYTIC Three regions in ; explicit global power $\mu < 1$.

Crossing-region spectral tail ANALYTIC Equation ; no fixed spectral-cutoff residual remains.

Weighted linear boundary limit ANALYTIC Same representation, strong on $\mathcal{D}_{\mathrm{fin}}$, rate $r^{2a}$.

Wick-quadratic consequence ANALYTIC Strong on $\mathcal{D}_{\mathrm{c}}$; not all finite-particle vectors.

General bulk apex Wick algebra OPEN No unrestricted product or complete interacting domain.

Measurable-profile exterior concentration ANALYTIC Weighted-integrable ultraviolet tails; test error and scalar mass error separated.

Logarithmic-power mass law ANALYTIC Dini remainder ; explicit error .

Full physical core-contact limit OPEN Interior, nonzero-mode, and local subtraction hypotheses not verified for a complete physical prescription.

Moment-controlled assembly Conditional theorem Corollary ; actual remainder $\mathscr{F}_R - \mathscr{F}_0$ retained.

Physical stress scale shift and $\tau$ OPEN A scalar moment does not calculate the stress remainder.

Second-order relative-worldsheet primitive ANALYTIC Parameter-dependent $C^k$ estimate and leading $\delta^2$ cutoff error.

Switched numerical sandwich ANALYTIC Regular external compact sets only; not the full second-order

interacting coefficient.

All-orders interaction OPEN No time-ordering, complete graph inventory, or interacting Ward theorem.

Resonant Laurent calculation ANALYTIC Exact logarithm, companion constant, and shifted-contour remainder for the stated family.

General resonance classification OPEN Other charts, actual cancellations, and differentiated Ward fluxes remain separate.

KH contacts Restricted calculation Polar-vector module solved; explicit higher-weight residual; no unrestricted classification.

Dirac discrepancy Retained audit No physical resolution or new spinor theorem.

**Finite probes need coverage**

**Proposition (Finite-probe criterion).** *Let $Z$ be a proved finite-dimensional residual space and $P : Z \to \mathbb{R}^k$ a declared linear probe map. Vanishing of every probe implies $z = 0$ for every $z \in Z$ exactly when $P$ is injective. In a fixed norm on $Z$, injectivity implies*

$$\|z\| \le C_Z \|Pz\|_\infty.$$

*Proof.* The first statement is $\operatorname{Ker} P = \{0\}$. For the estimate, $\|Pz\|_\infty$ has a positive minimum on the compact unit sphere of $Z$.

This is the finite-test distinction documented in Oai282-A [24]. The required elementary criterion is proved here. No external catalogue theorem is a premise.

Every identity released in this paper is proved on arbitrary tests in its stated class. Finite samples establish neither class membership nor coverage. In particular, tests supported away from $\mathscr{W}$ annihilate all apex contacts and cannot select $(c, \tau, w)$.

## Release theorem and unchanged programme boundaries

**Theorem (I-B-VP-1-Ex3 mathematical release).** *On the host, completion, density, positive-order product boundary laws, and numerical subtraction prescriptions specified above, the following are established:*

***1.*** *the complete difference spectral density obeys and ;*

***2.*** *the ultraviolet crossing contribution obeys , and the weighted boundary fields have the untruncated same-representation limit ;*

***3.*** *their Wick quadratics have the common-domain consequence ;*

***4.*** *measurable profiles satisfying Assumption have the quantitative exterior concentration ;*

***5.*** *with the Dini expansion , their mass and contact laws are –;*

***6.*** *interior contributions have the moment bounds –, the sufficient assembly , and the explicit nonimplication witnesses of Proposition ;*

***7.*** *the relative-worldsheet primitive has the parameter-dependent estimates –, and its switched regular-external sandwich has –;*

***8.*** *the resonant family has the full double-pole contribution , with the exact cancellation conditions of Proposition ;*

***9.*** *the polar-vector KH module and its exclusions are as in Proposition , and the higher-weight residual satisfies the stated KH equations. The full physical core-contact problem, unrestricted resonance and contact classifications, Dirac discrepancy, and interacting completion remain unclosed.*

*Proof.* Items (i)–(iii) are Theorem , Corollary , Theorem , and Proposition . Items (iv)–(vi) are Theorems , , , Corollary , and Proposition . Item (vii) is Theorem and Corollary . Items (viii)–(ix) are Theorem and Propositions , , and . The final sentence records constructions not supplied by these results.

### Separations preserved by the release

The following implications are not asserted:

• strong resolvent convergence ⇒ inverse-covariance or one-point convergence;

• exterior concentration ⇒ a complete normalized physical contact;

• a linear boundary field ⇒ a Gaussian quadratic field;

• a numerical primitive extension ⇒ an interacting time-ordered algebra;

• a Mellin logarithm ⇒ a supported dilation anomaly;

• KH compatibility ⇒ singular-metric Levi-Civita differentiation;

• finite reporting ⇒ field acquisition or source dynamics.

The global wavefront, threshold, Maxwell, spatial-selection, non-product, Dirac-domain, and thermal companion projects retain their own domains [15, 19, 27]. No statement about one of them is obtained by renaming a kernel in the present proof.

### Characteristic-composition boundaries

The supplied historical XIV-H record has its H2 stop. The supplied XIV-I record constructs its named I2 internal amplitude while leaving the joint map unresolved at I3. The supplied XIV-J record constructs a supported-sector reference on its stated domain while leaving the full map unresolved at J2 [28]. These version-qualified statements are not collapsed into one verdict; the I2 advance is not withdrawn.

Nothing here constructs:

**1.** the full joint characteristic map on tests crossing its characteristic locus;

**2.** its joint section and complete remainder;

**3.** its uniform transverse estimates;

**4.** compatibility with its partial contacts and singular spectators;

**5.** its complete output wavefront theorem after pushforward;

**6.** normalized admission in its stated quantum output space. The constructed worldsheet primitive is a different distribution on a different space. Technical XV manuscripts may exist without opening the downstream quantum-admission exit.

**Standing programme records**

In the requested programme reading:

• Problems 1, 2, and 8 remain unfinished.

• Problems 3–7 and 9–11 are not reached. Problem 6 is not the implication attempted here.

• Problem 12 remains unresolved.

• XVI-O retains nolinkurl $O.1 = NOT_C LOSED$ and nolinkurl $bulk_s ymmetry_r estored = false$.

• XVI-N and Route B are not discharged.

• XV-B's downstream quantum-admission exit is not opened.

• The historical H2 ending and unresolved I3/J2 obligations are not updated by this paper.

The supplied XVI-Q Q11 record reports a separately scoped two-point result in a different prescription. That report is neither erased nor imported here; it does not update XVI-O or restore bulk symmetry [29].

No spacetime emergence, Einstein dynamics, Maxwell or Einstein backreaction, Page curve, de Sitter state, cosmological bounce, or ultraviolet completion is constructed. No inherited characteristic quantum coefficient is admitted. OpenAI catalogue entries, including Family 282, are external records and not premises.

appendix

## The resolvent methodological boundary

For completeness, the distinction used in the core comparison has a simple fixed-vector witness.

**Proposition (Strong resolvent convergence does not control inverse weights).** *There exist positive bounded operators $B_n$, each with bounded inverse, such that $B_n \to I$ strongly in the resolvent sense, but for a fixed $v \in \ell^2(\mathbb{N})$*

,

$$\text{langlev}, B_n^{-1/2} v\rangle \to +\infty.$$

*Proof.* Let $B_n$ equal $I$ except on the $n$-th basis vector, where its eigenvalue is $2^{-8n}$. For $s > 0$, the resolvent difference applied to $f$ has norm at most $C_s|f_n| \to 0$. Take $v_j = 2^{-j}$. Its $n$-th contribution to the inverse-weight form is

$$2^{-2n}(2^{-8n})^{-1/2} = 2^{2n}.$$

Every individual form value is finite, but the sequence diverges.

The standard Mosco lower-bound and recovery conditions do imply strong resolvent convergence by the minimization of

$$Q_n[u] + s\|u\|^2 - 2\text{Re ip}\, fu.$$

The recovery inequality imposes no energy condition at points where the limiting form is $+\infty$. These variational facts, recalled in Ex1 and Ex2, do not supply the additional inverse-weight, coincidence, or renormalized contact estimates [9, 17, 18, 19]. No new physical-core conclusion is inferred here.

## Debugging, source scope, and submission declarations

toprule Potential gap or overstatement Treatment in Ex3

midrule endfirsthead toprule Potential gap or overstatement Treatment in Ex3

midrule endhead bottomrule endfoot

An unspecified ultraviolet polynomial hides the crossing $\sqrt{\tau r} \geq 1$ The three-region bound and explicit crossing integral give its power and smeared error.

The jump of every resolvent correction is positive Only the complete spectral Gram difference is positive; correction terms are bounded in norm.

Compression can be moved through inversion The inverse is taken on $V$ after compression.

A boundary estimate is uniform through $\nu = 0$ or $\nu = 1$ Orders are fixed in a compact positive deficient range; endpoint uniformity is not claimed.

Quadratic convergence holds on all finite-particle vectors The common domain remains $\mathcal{D}_{\mathrm{c}}$.

A reflection profile must have pointwise controlled ultraviolet growth Weighted integrability suffices for the larger measurable class.

A scalar mass asymptotic is already a distributional limit The independent second-moment estimate is supplied.

A signed interior family has no higher jets because its support shrinks Mass, dipole, and quadrupole terms are separated explicitly.

Exterior agreement proves membership in the scalar contact class The quadrupole witness shows why a separate admissibility estimate is needed.

External differentiation commutes with a moving singular cutoff automatically The relative variable is fixed before differentiation; the derivative estimate is proved on the resulting amplitude.

A renormalized numerical sandwich is an interacting product Only that switched kernel and its local normalization are released.

The Mellin logarithm is known only up to a proportionality constant Both Laurent coefficients and a shifted-contour remainder are evaluated.

Vanishing at one angle removes an off-diagonal logarithm The complete angular coefficient must vanish; a mixed

chart does not satisfy that condition.

Removing the polar-vector KH contact proves unrestricted uniqueness The explicit symmetric weight-three contact remains outside that module.

A bibliographic correction resolves the spinor discrepancy The physical Dirac audit remains unresolved.

**Supplied-source scope.** The complete I-B-VP-1-v4, Ex1-v1, and Ex2-v1 sources were supplied. Other Cheung records are used only at the scope specified by their supplied excerpts or consulted introductory pages. The large retrieval inventory is not treated as a complete corpus audit or as a list of mathematical premises. The unavailable online coverage page is not used as a proof input; Proposition supplies the criterion required here.

**Literature scope.** Classical Bessel formulas, primary article records, and the cited extension literature are used for attribution and the explicitly identified analytical inputs. No full audit of every cited paper is asserted. No unquoted theorem from I-B-VP-AB or I-B-VP-SAE is needed to derive the boundary inverse used here.

**Execution scope.** No successful pdfLaTeX run, numerical experiment, symbolic algebra execution, proof-assistant check, independent rebuild, independent referee review, or journal acceptance is asserted. The proofs are the written calculations on the declared domains.

**Source format.** This is a single-file pdfLaTeX source with an internal bibliography. No external code, data, figure, or bibliography file is required. Two compiler runs are intended. The author name follows the supplied series; submission requires the author's approval of the text, attribution, and scope.

**Final distinction.** The uniform boundary estimate, measurable-profile concentration theorem, and parameter-dependent primitive are concrete mathematical advances. They do not become a complete physical defect theory by sharing a worldsheet, a contact coordinate, or an Enter report.

## References

**[1]** B. S. Kay and U. M. Studer, Boundary conditions for quantum mechanics on cones and fields around cosmic strings, *Communications in Mathematical Physics* **139** (1991), 103–139. DOI: 10.1007/BF02102731.

**[2]** J. Dereziński and S. Richard, On Schrödinger operators with inverse square potentials on the half-line, *Annales Henri Poincaré* **18** (2017), 869–928. https://arxiv.org/abs/1604.03340. Bessel-extension spectral context. The compressed boundary normalization is calculated here.

**[3]** M. Dütsch and K.-H. Rehren, Generalized free fields and the AdS-CFT correspondence, *Annales Henri Poincaré* **4** (2003), 613–635. https://arxiv.org/abs/math-ph/0209035. Spectral generalized-free-field context; no AdS interpretation is imported.

**[4]** F. W. J. Olver et al., editors, *NIST Digital Library of Mathematical Functions*, Chapter 10. https://dlmf.nist.gov/10. Especially Sections 10.17, 10.27, 10.30, and 10.40 for fixed-order cut estimates, connection formulas, and endpoint expansions.

**[5]** G. N. Watson, *A Treatise on the Theory of Bessel Functions*, second edition, Cambridge University Press, 1944. Classical integral and Mellin-transform context.

**[6]** B. Allen, B. S. Kay, and A. C. Ottewill, Long range effects of cosmic string structure, *Physical Review D* **53** (1996), 6829–6846. https://arxiv.org/abs/gr-qc/9510058. Exterior zero-mode and core-sensitive context; not a complete contact-normalization premise.

**[7]** R. Brunetti and K. Fredenhagen, Microlocal analysis and interacting quantum field theories: Renormalization on physical backgrounds, *Communications in Mathematical Physics* **208** (2000), 623–661. https://arxiv.org/abs/math-ph/9903028. Scaling-degree extension framework. No interacting singular-background product is imported.

**[8]** N. V. Dang, The extension of distributions on manifolds, a microlocal approach, *Annales Henri Poincaré* **17** (2016), 819–859. https://arxiv.org/abs/1412.2808. Supported normal-jet and extension context.

**[9]** U. Mosco, Convergence of convex sets and of solutions of variational inequalities, *Advances in Mathematics* **3** (1969), 510–585. Variational-convergence background only; not a one-point convergence theorem.

**[10]** J. Spinelly and E. R. Bezerra de Mello, Vacuum polarization of a charged massless fermionic field by a

magnetic flux in the cosmic string spacetime, *International Journal of Modern Physics D* **13** (2004), 607–624. DOI: 10.1142/S0218271804004591. https://arxiv.org/abs/hep-th/0306103. Model-scope and unresolved coefficient comparison only.

*Cheung source and companion records.* Complete supplied sources, supplied excerpts, and consulted online introductory records are distinguished below. Citation does not assert independent verification of an entire manuscript or corpus.

**[11]** N. Cheung, *The Philosophy of Enter: Destructive Readout, Quantum Recovery, and the Refusal of Identification*, Cheung Ngo Medical Limited (2026). Framing essay; not a theorem premise. Programme note: https://enter.grok.me/enter.

**[12]** N. Cheung, *I-B-v1: A Finite Enter Spacetime: Calibrated Destructive Time Readout on a Prescribed Conical Product*, version 1, Enter Calculus Companion. https://enter.grok.me/browse/I-B. Prescribed host; thm:instrument and prop:not-source. Companion record; not a Scholar-indexed journal article.

**[13]** N. Cheung, *En09-A*, version 1, supplied destructive-clock and complete record–residual interface excerpts, Enter Calculus Companion. https://enter.grok.me/browse/En09-A. Non-extraction and downstream-admission boundaries. Companion record; not a Scholar-indexed journal article.

**[14]** N. Cheung, *I-B-G: Intrinsic and holonomy-twisted Green operators on a prescribed conical spacetime*, version 2, supplied excerpts, Enter Calculus Companion. Spatial form domains, deficiency indices, positivity, and supported-transfer comparisons only. No Green operator is transferred by the present boundary proof. https://enter.grok.me/browse/I-B-G. Companion record; not a Scholar-indexed journal article.

**[15]** N. Cheung, *I-B-VP: Vacuum polarization of the Friedrichs ground state on a prescribed conical spacetime: Local Hadamard subtraction, holonomy, and the renormalized stress tensor*, version 3 excerpts and version-qualified source descriptions, Enter Calculus Companion. Infrared, spectral-state, regular-host, and finite-coverage comparisons. The positive-order boundary estimates used here are calculated explicitly. https://enter.grok.me/browse/I-B-VP. Companion record; not a Scholar-indexed journal article.

**[16]** N. Cheung, *I-B-VP-1: Prescribed apex finite parts on a conical Enter spacetime: Wick square, stress tensor, and charged current: strong extension, Ward identities, and classified contact freedom*, version 3, October 2026. Complete supplied source I-B-VP-1-v4.tex, Enter Calculus Companion. Parent finite-part and contact conventions. Historical scripts, logs, and compiler reports were not independently executed here. https://enter.grok.me/browse/I-B-VP-1. Companion record; not a Scholar-indexed journal article.

**[17]** N. Cheung, *I-B-VP-1-Ex1: Worldsheet domains, core diagnostics, and first-order apex dependence*, version 1, 18 October 2026. Complete supplied source I-B-VP-1-Ex1-v2.tex, Enter Calculus Companion. Boundary-domain, exterior diagnostic, first-order subtraction, resonance, and KH corrective analysis. https://enter.grok.me/browse/I-B-VP-1-Ex1. Companion record; not a Scholar-indexed journal article.

**[18]** N. Cheung, *I-B-VP-1-Ex2: Spectral tails, exterior contact limits, and admissibility on a prescribed conical spacetime*, version 1, 18 October 2026. Complete supplied source I-B-VP-1-Ex2-v2.tex, Enter Calculus Companion. Immediate predecessor. The present paper makes the ultraviolet crossing estimate explicit and strengthens the profile, moment, primitive, and resonant-coefficient analysis. https://enter.grok.me/browse/I-B-VP-1-Ex2. Companion record; not a Scholar-indexed journal article.

**[19]** N. Cheung, *I-B-VP-5*, version 1, supplied excerpts, Enter Calculus Companion. Especially thm:selection, prop:contactseparation, and thm:release. Spatial operator selection is not a renormalized coincidence or contact limit. https://enter.grok.me/browse/I-B-VP-5. Companion record; not a Scholar-indexed journal article.

**[20]** N. Cheung, *I-B-VP-AB*, version 2, supplied companion description, Enter Calculus Companion. Regular coefficient comparison only; not an analytical premise. Companion record; not a Scholar-indexed journal article.

**[21]** N. Cheung, *I-B-VP-SAE*, version 3, supplied companion description, Enter Calculus Companion. Extension-ledger and normalization comparison only. No unavailable Krein or spectral theorem is imported. Companion record; not a Scholar-indexed journal article.

**[22]** N. Cheung, *X-A-ST*, version 1, supplied excerpts, Enter Calculus Companion. https://enter.grok.me/browse/X-A-ST. prop:noeinstein and the separation of finite reports from supplied

continuum dynamics. Companion record; not a Scholar-indexed journal article.

**[23]** N. Cheung, *XVI-O1-ST*, version 5, supplied integrity descriptions and consulted companion record, Enter Calculus Companion. https://enter.grok.me/browse/XVI-O1-ST. Evidence classes, residual retention, complete reporting, and execution-scope discipline. Companion record; not a Scholar-indexed journal article.

**[24]** N. Cheung, *Oai282-A*, version 1, supplied descriptions of prop:coverage and prop:invisible, Enter Calculus Companion. https://enter.grok.me/browse/Oai282-A. Class-relative finite-test coverage only. The elementary criterion needed in this paper is proved in Proposition . No OpenAI catalogue theorem is a premise. Companion record; not a Scholar-indexed journal article.

**[25]** N. Cheung, *VII-H* and *VII-I*, version 1, supplied excerpts, Enter Calculus Companion. Separate free-response and finite-width free-algebra benchmarks, not the worldsheet fields or numerical sandwich constructed here. https://enter.grok.me/browse/VII-H. https://enter.grok.me/browse/VII-I. Companion record; not a Scholar-indexed journal article.

**[26]** N. Cheung, *IX-A* through *IX-E-3*, including the version-qualified *IX-Free*, *IX-Free-2*, and *IX-D* records, supplied excerpts, Enter Calculus Companion. Documentary free-product, causal-product, and continuity boundaries only. No product theorem is transferred to the cone. https://enter.grok.me/browse/IX-A. https://enter.grok.me/browse/IX-E-3. https://enter.grok.me/browse/IX-Free. https://enter.grok.me/browse/IX-Free-2. https://enter.grok.me/browse/IX-D. Companion record; not a Scholar-indexed journal article.

**[27]** N. Cheung, *I-B-VP-6*, *I-B-VP-7*, and *I-B-VP-8*, version 1, supplied scope excerpts, Enter Calculus Companion. Non-product, Dirac-domain, thermal, and standing-status non-identifications only. No theorem of those different problems is used here. https://enter.grok.me/browse/I-B-VP-6. https://enter.grok.me/browse/I-B-VP-7. https://enter.grok.me/browse/I-B-VP-8. Companion record; not a Scholar-indexed journal article.

**[28]** N. Cheung, *XIV-H*, *XIV-I*, *XIV-J*, and *XV-B*, version-qualified excerpts supplied with the problem, Enter

Calculus Companion. Historical H2 stop, named I2 internal construction, unresolved I3/J2 full map, and distinct unopened downstream quantum-admission exit. No characteristic coefficient is constructed here. https://enter.grok.me/browse/XIV-H. https://enter.grok.me/browse/XIV-I. https://enter.grok.me/browse/XIV-J. https://enter.grok.me/browse/XV-B. Companion record; not a Scholar-indexed journal article.

**[29]** N. Cheung, *XVI-N*, version r12; *XVI-O*, version o70-sync; and *XVI-Q*, release Q11, supplied version-qualified scope and status excerpts, Enter Calculus Companion. Standing boundaries and the separately scoped two-point record only. The present paper does not verify or change those results. https://enter.grok.me/browse/XVI-N. https://enter.grok.me/browse/XVI-O. https://enter.grok.me/browse/XVI-Q. Companion record; not a Scholar-indexed journal article.

**[30]** N. Cheung, *Black-Hole Information as an Access Problem: Enter, Quantum Recovery, and Complete Residuals*, figshare (2026). DOI: 10.6084/m9.figshare.34335024.v1. https://doi.org/10.6084/m9.figshare.34335024.v1.

**[31]** N. Cheung, *Operational Wormhole Certificates—Existence, Dimension-Dependent Scaling, Operational Functions, and Composition*, figshare (2026). DOI: 10.6084/m9.figshare.34549575.v1. https://doi.org/10.6084/m9.figshare.34549575.v1.

**[32]** N. Cheung, *Enter, Quantum Magic, and Quantum Gravity: Readout, Recovery, and the Limits of Geometric Response*, figshare (2026). DOI: 10.6084/m9.figshare.34335582.v1. https://doi.org/10.6084/m9.figshare.34335582.v1.

**[33]** N. Cheung, *Operational Consistency Does Not Select Lorentzian Geometry*, figshare (2026). DOI: 10.6084/m9.figshare.34441758.v1. https://doi.org/10.6084/m9.figshare.34441758.v1.

**[34]** N. Cheung, *Realization of Relational Time by Destructive Enter on a Clock Register*, figshare (2026). DOI: 10.6084/m9.figshare.34442907.v1. https://doi.org/10.6084/m9.figshare.34442907.v1.