I-B-VP-4: Frozen-source Maxwell response on the prescribed conical Enter spacetime

Regular magnetic fields, a restricted apex completion, and a version-qualified source audit

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Abstract

We solve a declared classical Maxwell equation with the already computed charged Friedrichs vacuum current as a prescribed c-number source on the regular conical spacetime of I-B. The scalar connection, extension, and spectral state are not updated. With $F = \mathrm{d}a$, $B_z = F_{r\varphi}/(\alpha r)$, and $\nabla_\mu F^{\mu\nu} = \varsigma\kappa_{\mathrm{M}} j^\nu$, the complete stationary axial-magnetic family is

$$B_z = B_\infty - \frac{\varsigma\kappa_{\mathrm{M}}\mathfrak{c}}{2\alpha r^2}, \qquad a_\varphi = a_0 + \frac{\alpha B_\infty r^2}{2} - \frac{\varsigma\kappa_{\mathrm{M}}\mathfrak{c}}{2}\log(r/L), \qquad \mathfrak{c} = -\frac{e\beta(1-\beta)(1-2\beta)}{12\pi^2\alpha^2}.$$

Here the Maxwell sign is explicit: $\varsigma = +1$ gives the displayed divergence convention, whereas $\varsigma = -1$ matches the scalar action and charge convention specified below. Decay fixes B_∞, but not the flat circulation a_0; the latter is not an arbitrary removable global gauge constant.

For nonzero source the field has no locally integrable extension across the apex, although its logarithmic potential does. We go beyond this obstruction in a precisely restricted distributional sector. A determinant cancellation defines a smooth constitutive representative for transverse two-forms. One finite part then gives a strongly convergent Maxwell realization, with exactly one linked field–current contact parameter. Its relation to the circulation of a locally integrable potential is explicit. Large gauges on the punctured cone need not extend over the apex and can change that contact under the chosen completion.

A separate primary-source audit distinguishes inspected equations from metadata and inherited descriptions. It retains the printed Spinelly–Bezerra de Mello polynomial discrepancy, an additional version-specific Wick-square discrepancy, and a stress normalization difference rather than silently correcting them. The contribution is the complete convention, boundary, and restricted-extension analysis, not a new cosmic-string magnetic coefficient. No self-consistent scalar–Maxwell state, quantum Maxwell field, gravitational backreaction, or continuum acquisition by finite Enter reporting is constructed.

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Book expansion: sign/action ledger first

toprule Ledger item Distinction

midrule Action / equation inverse Fixed before any Maxwell solution

Magnetic field constant Distinct from flat circulation

Regular solution Smooth-host Maxwell data

Distributional completion Separate object class

Frozen-source compatibility Separate compatibility statement

Long source audit Documentary appendix; access grades retained

bottomrule

Purpose, prescribed inputs, and corrections to the target

There are two outputs in this paper.

The first is a Maxwell calculation with a prescribed source. The current is inherited from I-B-VP-v4 thm:current and from the complete regular-part calculation in I-B-VP-0 [20, 21]. We do not recompute that scalar expectation value.

The second is a version-qualified literature audit. Access to a formula, agreement after translation, and verification of an entire paper are different claims. The audit records each separately.

Induced scalar vacuum currents and magnetic fields around cosmic strings are established effects. In particular, Sitenko–Vlasii explicitly solve an induced-field problem, and Bezerra de Mello et al. analyze currents and magnetic fields for finite cores [2, 3]. The regular inverse-square magnetic coefficient below is not claimed as new.

The contribution within the companion framework. We supply:

1. the complete stationary magnetic solution family, including the distinct field and global-potential constants;

2. the exact locally integrable obstruction and its separation from distributional extendability;

3. a restricted distributional Maxwell completion, with its constitutive map, strong approximation, contact classification, and potential–contact dictionary proved explicitly;

4. a source-by-source audit with retained discrepancies and non-import boundaries. The restricted completion is additional data, not a canonical singular-metric Maxwell theory.

Four debugging corrections

The proposed target requires four qualifications.

1. The sign of Maxwell's equation must be declared. The same lower-index scalar current, together with $F = \mathrm{d}a$, can appear with either displayed divergence sign depending on the action convention. We retain a sign ς, and derive its action interpretation rather than infer it from the word ``Maxwell."

2. A potential has more freedom than its field. The radial equation leaves one magnetic constant. Integration to a potential leaves an additional flat circulation. On a punctured cone, a real multiple of $\mathrm{d}\varphi$ is not generally the differential of a globally single-valued real gauge function.

3. Failure of local integrability is not nonexistence of distributional extensions. We prove the former and construct a particular class of the latter. We do not multiply arbitrary apex distributions by the singular metric.

4. No integration constant repairs a nonzero source while keeping the connection flat. A flat connection has zero regular curvature and zero Maxwell divergence. It is compatible with the prescribed current only when that current vanishes.

Enter and the prescribed spacetime

I-B fixes the conical host and a separate finite operational interface [18]. Its metric has no state-dependent update rule. I-B-G chooses scalar propagation only after an extension has been supplied [19]; this does not supply a constrained Maxwell domain.

The philosophy of Enter distinguishes a destructive classical readout from a quantum-return map, a record from a decoder, and a mathematical comparison from a physical identification [17]. Those distinctions apply here: a finite report of a field formula is not the continuum field, its acquisition, or a source law.

The current, background connection, scalar state, cone, and Maxwell normalization are inputs. Conservation alone does not impose Maxwell or Einstein equations. The non-sourcing distinctions of I-B and X-A-ST remain unchanged [18, 23]. The philosophical discussion fixes interpretation and supplies no premise for the radial differential equation.

The host, scalar source, and signed Maxwell convention

Regular geometry and the fixed scalar source

Fix

$$0 < \alpha < 1, \qquad 0 \le \beta < 1, \qquad e > 0.$$

The regular spacetime is

$$M = \mathbb{R}_t \times (0,\infty)_r \times (\mathbb{R}/2\pi\mathbb{Z})_\varphi \times \mathbb{R}_z, \qquad g = -\mathrm{d}t^2 + \mathrm{d}r^2 + \alpha^2 r^2 \mathrm{d}\varphi^2 + \mathrm{d}z^2.$$

The removed defect is the worldsheet

$$\mathscr{W} = \mathbb{R}_t \times \{o\} \times \mathbb{R}_z.$$

We use $\hbar = c = 1$ and the spatial orientation $\mathrm{d}r \wedge \mathrm{d}\varphi \wedge \mathrm{d}z$.

The scalar is one charged complex field with the prescribed Friedrichs extension and distinguished spectral ground state. In a lifted parallel frame its character is

$$\Phi(\varphi + 2\pi) = e^{2\pi i\beta}\Phi(\varphi).$$

Equivalently, in a periodic representation one may use

$$D = \partial + \mathrm{i}eA^{(0)}, \qquad A^{(0)} = \frac{\beta}{e}\mathrm{d}\varphi.$$

This background connection is flat on M.

Assumption (Frozen scalar current). The source is the regular one-point expectation value

$$j^{\mathrm{flat}} = \frac{\mathfrak{c}}{r^2}\mathrm{d}\varphi, \qquad \mathfrak{c} = -\frac{e\beta(1-\beta)(1-2\beta)}{12\pi^2\alpha^2}.$$

It has the normalization

$$j_\mu = e\,\mathrm{i}\big(\Phi^\dagger D_\mu\Phi - (D_\mu\Phi)^\dagger\Phi\big)$$

before taking the renormalized expectation. No scalar field, extension, or state is recomputed after the Maxwell response is found.

This is the input identified with the scalar state by the supplied current theorems [20, 21]. Its relevant components are

$$j^\varphi = \frac{\mathfrak{c}}{\alpha^2 r^4}, \qquad j^{\wedge}_{\varphi} = \frac{\mathfrak{c}}{\alpha r^3}, \qquad j^t = j^r = j^z = 0.$$

Its regular conservation follows directly:

$$\nabla_\mu j^\mu = \frac{1}{\alpha r}\partial_\mu(\alpha r j^\mu) = 0.$$

For fixed $e > 0$, $\mathfrak{c} = 0$ precisely at $\beta = 0, 1/2$. The coefficient is odd under $\beta \mapsto 1-\beta$. The current is independent of the regular-host curvature coupling ξ, as recorded in the inherited theorem.

The Maxwell model is an additional declaration

Let $\kappa_{\mathrm{M}} > 0$ be a Maxwell normalization and $\varsigma \in \{+1, -1\}$. For a response potential a, distinct from $A^{(0)}$, impose

$$F = \mathrm{d}a, \qquad \nabla_\mu F^{\mu\nu} = \varsigma\kappa_{\mathrm{M}} j^\nu.$$

The homogeneous Maxwell equation is $\mathrm{d}F = 0$. The normalization $\kappa_{\mathrm{M}} = 1$ is the rationalized choice used in the comparisons below.

Equation is the Euler equation of the local Lagrangian

$$S_{\mathrm{ext}}[a;j] = -\frac{1}{4\kappa_{\mathrm{M}}}\int F_{\mu\nu}F^{\mu\nu}\mathrm{dvol}_g - \varsigma\int a_\mu j^\mu \mathrm{dvol}_g,$$

where the statement concerns compactly supported variations on M. A finite value of the action on all of the noncompact host is not assumed.

Proposition (Action and sign ledger). *For the scalar kinetic action*

$$S_{\mathrm{sc}}[\Phi;A] = -\int (D_\mu\Phi)^\dagger D^\mu\Phi\,\mathrm{dvol}_g, \qquad D = \partial + \mathrm{i}eA,$$

its connection variation has coefficient $+j^\mu$*:*

$$\delta_A S_{\mathrm{sc}} = \int j^\mu \delta A_\mu\,\mathrm{dvol}_g.$$

Thus the action-matched equation for this charge convention and $F = \mathrm{d}A$*, with the Maxwell kinetic sign in , is*

$$\nabla_\mu F^{\mu\nu} = -\kappa_{\mathrm{M}} j^\nu.$$

It corresponds to $\varsigma = -1$*. The schematic plus-divergence problem corresponds instead to* $\varsigma = +1$*.*

Proof. Use $\delta D_\mu\Phi = \mathrm{i}e\delta A_\mu\Phi$ and its adjoint. The variation of the negative kinetic term is

$$\mathrm{i}e\big(\Phi^\dagger D^\mu\Phi - (D^\mu\Phi)^\dagger\Phi\big)\delta A_\mu.$$

The Maxwell kinetic variation is $\kappa_{\mathrm{M}}^{-1}\nabla_\mu F^{\mu\nu}\delta A_\nu$ after integration by parts. Adding the coefficients gives the stated sign. The same variation of gives for either declared ς.

This ledger does not compute an effective action or vary the quantum state. It only fixes how the specified current normalization is compared with a classical Maxwell equation.

The complete regular magnetic solution

Definition (Stationary axial-magnetic sector). Restrict to real smooth potentials

$$a = a_\varphi(r)\mathrm{d}\varphi$$

on M, with no electric, radial, or longitudinal potential component. Define the oriented axial magnetic field by

$$B_z(r) := F_{r\varphi} = \frac{a_\varphi'(r)}{\alpha r}.$$

This is an ansatz for this paper, not a uniqueness class for all Maxwell fields on M.

Every such potential obeys the regular Lorenz condition:

$$\nabla_\mu a^\mu = \frac{1}{\alpha r}\partial_\varphi(\alpha r a^\varphi) = 0.$$

It is also in radial gauge $a_r = 0$.

Lemma (Radial equation). *Within Definition , all components of except the angular one vanish identically. The angular equation is equivalent to either of*

$$\frac{\mathrm{d}}{\mathrm{d}r}\left(\frac{a_\varphi'}{\alpha r}\right) = \frac{\varsigma\kappa_{\mathrm{M}}\mathfrak{c}}{\alpha r^3}, \qquad B_z' = \varsigma\kappa_{\mathrm{M}} j_\varphi^\wedge.$$

Proof. The only nonzero curvature component is $F_{r\varphi} = a_\varphi'$, so

$$F^{r\varphi} = \frac{a_\varphi'}{\alpha^2 r^2}.$$

Antisymmetry gives

$$\nabla_\mu F^{\mu\varphi} = \frac{1}{\alpha r}\partial_r\left(\frac{a_\varphi'}{\alpha r}\right).$$

Equating this to $\varsigma\kappa_{\mathrm{M}}\mathfrak{c}/(\alpha^2 r^4)$ proves . All remaining divergences are zero by the ansatz and its symmetries.

Theorem (Regular Maxwell solution and its constants). *For every fixed source coefficient $\mathfrak{c}$, every smooth solution in Definition is*

$$B_z(r) = B_\infty - \frac{\varsigma\kappa_{\mathrm{M}}\mathfrak{c}}{2\alpha r^2},$$

$$a_\varphi(r) = a_0 + \frac{\alpha B_\infty r^2}{2} - \frac{\varsigma\kappa_{\mathrm{M}}\mathfrak{c}}{2}\log(r/L), \qquad L > 0,$$

$$F = \left(\alpha B_\infty r - \frac{\varsigma\kappa_{\mathrm{M}}\mathfrak{c}}{2r}\right)\mathrm{d}r \wedge \mathrm{d}\varphi.$$

Conversely, these formulas solve both Maxwell equations and the stated gauge conditions on all of M.

The field has one real integration constant B_∞. *The potential has, in addition, the flat circulation constant* a_0, *whose global gauge meaning is treated below. For a fixed L, the condition* $B_z(r) \to 0$ *as* $r \to \infty$ *fixes* $B_\infty = 0$, *but does not fix* a_0.

Proof. Integrate once:

$$B_z = B_\infty - \frac{\varsigma \kappa_M \mathfrak{c}}{2\alpha r^2}.$$

Then integrate $a_\varphi' = \alpha r B_z$. Its second primitive constant is a_0. This proves both completeness and uniqueness at the indicated levels. The displayed functions are smooth for $r > 0$. Their curvature is exact, hence closed, and substitution in Lemma verifies the inhomogeneous equation. The gauge conditions were established before the lemma.

Two explicit sign specializations. For $B_\infty = 0$,

$$B_z = \frac{\varsigma \kappa_M e \beta (1-\beta)(1-2\beta)}{24\pi^2 \alpha^3 r^2}.$$

Thus $\varsigma = +1$ solves the plus-divergence equation in the question's schematic convention. For the action-matched convention $\varsigma = -1$, the right side has the opposite sign. Changing the orientation used to define B_z, rather than changing Maxwell's equation, also changes a reported axial component's sign. These operations must not be conflated.

Global gauge freedom and the flat mode

The form

$$\mathrm{d}\varphi = \frac{-x^2 \mathrm{d}x^1 + x^1 \mathrm{d}x^2}{r^2}, \qquad x^1 = r\cos\varphi, \quad x^2 = r\sin\varphi,$$

is global and smooth on the punctured transverse plane. It is closed there, but its meridian integral is 2π.

Proposition (The potential constant is not arbitrary gauge). *Two potentials in have the same regular field if*

and only if their B_∞ values agree. For fixed L, their difference is then

$$\Delta a = \Delta a_0 \, \mathrm{d}\varphi.$$

This difference is a globally exact real gauge transformation only when $\Delta a_0 = 0$.

For a $U(1)$ gauge group with elementary charge e, large regular gauge transformations permit

$$a_0 \mathrm{longmapsto}\, a_0 + \frac{m}{e}, \qquad m \in \mathbb{Z}.$$

The remaining datum can be recorded by

$$\exp(2\pi i e a_0).$$

For the decaying solution it is the response meridian phase at r = L.

Proof. Equal curvatures force equal B_∞, and the difference of the primitives is constant in a_φ. If $\Delta a = \mathrm{d}\chi$ with a globally single-valued real χ, its integral around a meridian is zero, so $2\pi\Delta a_0 = 0$. Conversely, the $U(1)$ transformation $u_m = e^{-im\varphi}$ is single valued and shifts the connection by $(m/e)\mathrm{d}\varphi$. The exponential is invariant under exactly these shifts. When $B_\infty = 0$, gives $a_\varphi(L) = a_0$.

A local gauge function proportional to φ can remove the constant on an angular chart. That local fact is not the global statement on M.

Changing the reference length does not create another physical constant:

$$L' = Le^{\ell}, \qquad a_0' = a_0 - \frac{\varsigma \kappa_{\mathrm{M}} \mathfrak{c}}{2}\ell$$

leaves the same potential in .

Flux, decay, and energy

For $0 < a < R < \infty$, the oriented flux through a transverse annulus is

$$\Phi(a, R) = \pi\alpha B_\infty (R^2 - a^2) - \pi\varsigma\kappa_{\mathrm{M}}\mathfrak{c}\log(R/a).$$

This also equals $2\pi[a_\varphi(R) - a_\varphi(a)]$. It does not involve a_0.

Corollary (Decay does not give finite total flux or apex energy). *Suppose* $\mathfrak{c} \neq 0$ *and* $B_\infty = 0$. *Then:*

1. $B_z \to 0$ *and the orthonormal magnitude* $|a_\varphi|/(\alpha r) \to 0$ *as* $r \to \infty$*;*

2. *the annular flux diverges logarithmically as either a*downarrow0 *or* $R \to \infty$*;*

3. *the regular electromagnetic energy per unit z-length is*

$$\mathscr{E}(a, R) = \int_a^R \frac{B_z^2}{2\kappa_{\mathrm{M}}} 2\pi\alpha r \, \mathrm{d}r = \frac{\pi\kappa_{\mathrm{M}}\mathfrak{c}^2}{8\alpha}(a^{-2} - R^{-2}).$$

No member of the magnetic family has finite energy on the entire transverse plane unless $\mathfrak{c} = 0$ *and* $B_\infty = 0$.

Proof. The first assertion follows from r^{-2} and $\log(r)/r$ decay. Equation proves the second. Substituting with $B_\infty = 0$ gives . A nonzero B_∞ instead gives a constant field at infinity and infinite exterior energy. For nonzero $\mathfrak{c}$, the r^{-2} apex term cannot be cancelled by any constant field.

In particular, a finite-total-flux condition on the whole zero-thickness massless model is not an available alternative normalization when $\mathfrak{c} \neq 0$. On a fixed annulus, a prescribed flux can fix B_∞; it still does not determine a_0.

The apex obstruction and the choice of a completion

Use the additional smooth completion of I-B-VP-1:

$$\overline{M} = \mathbb{R}^{1+1}_{t,z} \times \mathbb{R}^2_x, \qquad x = (r\cos\varphi, r\sin\varphi), \qquad \mathrm{d}\Omega = \alpha \, \mathrm{d}t \, \mathrm{d}z \, \mathrm{d}^2x.$$

The auxiliary smooth metric is

$$\bar{g} = -\mathrm{d}t^2 + \mathrm{d}z^2 + (\mathrm{d}x^1)^2 + (\mathrm{d}x^2)^2.$$

These are not developing coordinates. For quotient angles this is not automatically an orbifold test structure [22].

On the regular part, the transverse physical metric is

$$g_{ij} = \alpha^2 \delta_{ij} + (1 - \alpha^2)\frac{x_i x_j}{r^2}, \qquad \det(g_{ij}) = \alpha^2.$$

It does not have a direction-independent value at the origin when $\alpha \neq 1$. The density in , however, is smooth.

Let

$$\varepsilon = \begin{pmatrix} 0 & -1 \\ 1 & 0 \end{pmatrix}, \qquad R^i = \varepsilon^i{}_j x^j.$$

Then

$$j^i = \frac{\mathfrak{c}\, \varepsilon^i{}_j x^j}{\alpha^2 r^4}, \qquad F = f^\circ \, \mathrm{d}x^1 \wedge \mathrm{d}x^2, \qquad f^\circ = \alpha B_\infty - \frac{\varsigma \kappa_{\mathrm{M}} \mathfrak{c}}{2r^2}.$$

Theorem (Exact locally integrable obstruction). *If $\mathfrak{c} \neq 0$, the field in has no extension represented by a locally integrable two-form on $\overline{M}$. It also has no extension as a locally finite vector-valued Radon measure. The source has the corresponding obstruction as a locally integrable vector field.*

Nevertheless, the potential in , regarded as a Cartesian one-form, is locally integrable across $\mathscr{W}$.

Proof. For sufficiently small r, the coefficient f° in has magnitude bounded below by Cr^{-2} for some $C > 0$. Therefore its absolute transverse integral dominates

$$C' \int_0^\varepsilon \frac{\mathrm{d}r}{r} = \infty.$$

Any locally integrable extension agreeing off $\mathscr{W}$ would have the same coefficient almost everywhere there, which is impossible. The same lower bound makes the variation of any putative locally finite measure infinite in each apex neighbourhood.

The Cartesian magnitude of j is a nonzero constant times r^{-3}, whose absolute integral contains $\int_0^\varepsilon r^{-2}\mathrm{d}r$. For the potential,

$$\mathrm{lVerta}\|_{\bar{g},\,\mathrm{spatial}} = \frac{|a_\varphi(r)|}{r} \leq C\left(r + \frac{1 + |\log r|}{r}\right)$$

near the origin. Its transverse integral is finite because $\int_0^\varepsilon (1 + |\log r|)\mathrm{d}r < \infty$.

The theorem excludes the locally integrable class, not all distributions. The next section defines a restricted distributional realization. It does not assume that arbitrary products of the singular connection with delta derivatives exist.

A restricted distributional Maxwell realization

Why the magnetic sector admits a smooth representative

For a transverse two-form $F = f\,\mathrm{d}x^1 \wedge \mathrm{d}x^2$, ordinary index raising on the regular part gives

$$F^{12} = \frac{f}{\det(g_{ij})} = \alpha^{-2} f.$$

Thus the directional singularities of the individual inverse metric components cancel in this sector.

Definition (Completion constitutive map). For scalar distributions f on $\overline{M}$, define the antisymmetric contravariant tensor distribution

$$\mathscr{C}_\alpha[f]^{12} = \alpha^{-2} f, \qquad \mathscr{C}_\alpha[f]^{21} = -\alpha^{-2} f,$$

with all other components zero. Define its Maxwell representative by

$$\mathfrak{M}_\alpha[f]^\nu := \partial_\mu \mathscr{C}_\alpha[f]^{\mu\nu}.$$

Equivalently,

$$\mathfrak{M}_\alpha[f]^i = \alpha^{-2} \varepsilon^i{}_k \partial_k f, \qquad \mathfrak{M}_\alpha[f]^t = \mathfrak{M}_\alpha[f]^z = 0.$$

This is a smooth linear differential operation on the declared completion. It agrees exactly with the physical Maxwell divergence for regular transverse two-forms, because the volume density is constant in Cartesian coordinates and the Christoffel term contracted with an antisymmetric tensor vanishes. It does not define index raising for arbitrary tensor distributions.

For distributions independent of t, z,

$$\mathrm{d}(f\,\mathrm{d}x^{1}\wedge\mathrm{d}x^{2})=0$$

by the ordinary exterior derivative. Thus both Maxwell operations in this sector are defined without a product with the singular metric. The restriction is essential; general singular-metric operations require additional control [14].

The generating finite part

Normalize the worldsheet distribution by

$$\langle\delta_{\mathscr{W}},\psi\rangle=\int_{\mathbb{R}^{1+1}}\psi(t,z,0)\,\mathrm{d}t\,\mathrm{d}z.$$

Relative to $\mathrm{d}\Omega$,

$$\delta_{\mathscr{W}}=\alpha^{-1}\delta^{(2)}_{\mathrm{Leb}}(x),\qquad \sigma_{\alpha}=2\pi\alpha.$$

Fix $L>0$. Let $\rho\in C^{\infty}([0,\infty))$ be zero for $u\le 1$ and one for $u\ge 2$, with $0\le\rho\le 1$, and set

$$c_{\rho}=\int_{1}^{2}\frac{\rho(u)-1}{u}\,\mathrm{d}u.$$

Define

$$U^{\rho}_{L,\varepsilon}=\rho(r/\varepsilon)r^{-2}+\sigma_{\alpha}\Big(\log\frac{\varepsilon}{L}-c_{\rho}\Big)\delta_{\mathscr{W}}.$$

Lemma (Strong finite part). *The distributions converge strongly to a real, rotation-invariant distribution U_{L}, independent of ρ, with regular restriction r^{-2}. On every fixed compact test support,*

$$|\langle U_{L,\varepsilon}^{\rho}-U_{L},\psi\rangle|\le C\varepsilon^{2}p_{2}(\psi).$$

Its transverse scaling degree is 2, *and*

$$(E+2)U_{L}=\sigma_{\alpha}\delta_{\mathscr{W}},\qquad U_{L'}-U_{L}=-\sigma_{\alpha}\log(L'/L)\delta_{\mathscr{W}},\qquad E=x^{i}\partial_{i}.$$

Proof. The angular average of a smooth test obeys

$$\frac{1}{2\pi}\int_{0}^{2\pi}\psi(y,r\omega)\,\mathrm{d}\varphi=\psi(y,0)+O(r^{2}p_{2}(\psi)).$$

For a sharp cutoff, subtract the constant term and integrate the remainder against $\sigma_\alpha\, dr/r$. It is integrable, and the omitted remainder below ε is $O(\varepsilon^2 p_2)$. The transition from the sharp cutoff to ρ adds $\sigma_\alpha c_\rho \psi(y,0)$, which the subtraction in cancels. This proves profile independence and , uniformly on bounded test sets.

The Euler identity follows by pairing with a test and integrating its radial derivative; the inner boundary contributes $\sigma_\alpha \psi(y,0)$. The scale difference follows directly from the logarithmic subtraction. These identities give scaling degree 2. This is the scalar finite part of I-B-VP-1, with the normalization and proof recalled here [22].

A Maxwell extension and its complete contact freedom

For fixed B_∞, define

$$\hat{f}_L = \alpha B_\infty - \frac{\varsigma \kappa_M \mathfrak{c}}{2} U_L,$$

$$(\hat{j}_L)^i = -\frac{\mathfrak{c}}{2\alpha^2} \varepsilon^i{}_k \partial_k U_L, \qquad (\hat{j}_L)^t = (\hat{j}_L)^z = 0.$$

Theorem (Restricted distributional Maxwell solution). *The pair*

$$\hat{F}_L = \hat{f}_L\, dx^1 \wedge dx^2, \qquad \hat{j}_L$$

extends the regular solution and source. It satisfies

$$d\hat{F}_L = 0, \qquad \mathfrak{M}_\alpha[\hat{f}_L] = \varsigma \kappa_M \hat{j}_L, \qquad \partial_\mu (\hat{j}_L)^\mu = 0.$$

Replacing U_L by $U^\rho_{L,\varepsilon}$ gives exact identities at every regulator value. The field and source converge strongly, with errors bounded by $C\varepsilon^2 p_2$ and $C\varepsilon^2 p_3$, respectively. These approximants are regulated distributions, not a claimed smooth-core model.

Proof. On $r > 0$,

$$-\frac{1}{2}\partial_i r^{-2} = \frac{x_i}{r^4},$$

so the restrictions agree with . Applying to proves the inhomogeneous equation. Antisymmetry of ε makes the

source divergence zero. The exterior equation follows from translation invariance in t, z and transverse top degree. All statements hold before the limit because they are identities of linear differential operators. Lemma and transposition give the strong bounds.

The source extension is the electric-charge multiple of the derivative-compatible current extension in I-B-VP-1. No scalar state at the apex is thereby constructed.

Assumption (Restricted magnetic extension class). Consider real extensions with:

1. $F = f\,\mathrm{d}x^1 \wedge \mathrm{d}x^2$, independent of t, z, and rotation covariant, with the fixed regular restriction ;

2. transverse scaling degree at most 2 for $f - \alpha B_\infty$;

3. a translation-invariant, longitudinal-Lorentz-covariant current extension with transverse scaling degree at most 3, and $\partial_\mu j^\mu = 0$;

4. Maxwell's equation defined by Definition . When charge conjugation is imposed on the family, the induced parts $f - \alpha B_\infty$ and j are odd under $\beta \mapsto 1 - \beta$. The externally imposed uniform field is separated from this parity requirement.

Theorem (Complete contact classification in the magnetic sector). *Every pair in Assumption is uniquely*

$$f = \hat{f}_L + h\delta_{\mathscr{W}},$$

$$j^i = (\hat{j}_L)^i + \frac{h}{\varsigma\kappa_{\mathrm{M}}\alpha^2}\varepsilon^i{}_k\partial_k\delta_{\mathscr{W}}, \qquad j^t = j^z = 0,$$

for one real h.

Equivalently, if the current contact is written $w\varepsilon^i{}_k\partial_k\delta_{\mathscr{W}}$, Maxwell compatibility is

$$h = \varsigma\kappa_{\mathrm{M}}\alpha^2 w.$$

Thus a prescribed full current extension fixes the field contact; it is not an additional independently adjustable source-free Maxwell term.

For a charge-conjugation-compatible family, h, w are odd. At $\beta = 0, 1/2$ they vanish. For a generic sector the simultaneous contact space has dimension one, for fixed background parameters and fixed B_∞.

Proof. The difference $f - \hat{f}_L$ is supported on $\mathscr{W}$. The structure theorem for supported distributions, translation invariance, and the codimension-two scaling-degree bound imply that it is $h\delta_{\mathscr{W}}$: a transverse delta derivative of order m has scaling degree $2 + m$ [15, 16]. Applying the declared Maxwell operator then forces and proves uniqueness.

Independently, the current symmetries permit only radial and rotated first derivatives of $\delta_{\mathscr{W}}$ in a transverse supported difference. Longitudinal covariance excludes a fixed nonzero longitudinal vector. Conservation removes the radial derivative, since its divergence is a nonzero multiple of $\Delta\delta_{\mathscr{W}}$. The remaining contact is $w\varepsilon\partial\delta_{\mathscr{W}}$, and Maxwell's equation gives .

Conversely, the displayed pair has all the required restrictions, symmetries, degrees, and equations. Charge conjugation gives the last assertions.

The dimension is not the three-parameter simultaneous scalar/stress/current ambiguity of I-B-VP-1. Here the magnetic field is a different observable, and its contact is linked to the chosen current contact by Maxwell's equation.

Corollary (Scale dependence and nonremovable residues). *Writing $\ell = \log(L'/L)$, the same pair is represented at the new scale by*

$$h' = h - \frac{\varsigma\kappa_{\mathrm{M}}\mathfrak{c}\sigma_\alpha}{2}\ell.$$

Every extension in the class satisfies

$$(E + 2)(f - \alpha B_\infty) = -\frac{\varsigma\kappa_{\mathrm{M}}\mathfrak{c}\sigma_\alpha}{2}\delta_{\mathscr{W}},$$

$$(E + 3)j^i = -\frac{\mathfrak{c}\sigma_\alpha}{2\alpha^2}\varepsilon^i{}_k\partial_k\delta_{\mathscr{W}}.$$

The allowed contacts cannot remove these residues when $\mathfrak{c} \neq 0$.

Proof. Use in . For the current use $[E, \partial_i] = -\partial_i$. The contacts have exactly the relevant homogeneous degrees and are killed by the corresponding Euler operators.

These are extension scaling residues, not a calculation of a quantum electromagnetic trace anomaly.

Locally integrable potentials and the contact dictionary

The obstruction in Theorem concerns F, not its potential.

Lemma (Two distributional curl identities). *On the smooth completion, with locally integrable potentials,*

$$\mathrm{d}(\mathrm{d}\varphi) = \sigma_\alpha \delta_{\mathscr{M}}\, \mathrm{d}x^1 \wedge \mathrm{d}x^2,$$

$$\mathrm{d}(\log(r/L)\mathrm{d}\varphi) = U_L\, \mathrm{d}x^1 \wedge \mathrm{d}x^2.$$

The first $\mathrm{d}\varphi$ *in denotes the punctured angular one-form extended as a locally integrable Cartesian one-form; it is not the differential of a global single-valued angular function on the completion.*

Proof. For $a = A(r)\mathrm{d}\varphi$, test its transverse curl against a compact scalar ψ and integrate first over $r > \varepsilon$. The result is

$$\int_{r>\varepsilon} \frac{A'(r)}{r}\psi\, \mathrm{d}^2x + A(\varepsilon)\int_0^{2\pi} \psi(\varepsilon\omega)\mathrm{d}\varphi.$$

For $A = 1$, the boundary term tends to $2\pi\psi(0)$. For $A = \log(r/L)$, the expression is

$$\int_{r>\varepsilon} r^{-2}\psi\, \mathrm{d}^2x + 2\pi\log(\varepsilon/L)\psi(0) + o(1),$$

which is the finite part defining U_L. The normalization $\sigma_\alpha \delta_{\mathscr{M}} = 2\pi\delta^{(2)}_{\mathrm{Leb}}$ converts this to –. Longitudinal integration is passive.

Theorem (Potential–contact dictionary). *Extend as a locally integrable one-form* $\bar{a}$ *on* $\overline{M}$. *Its distributional curvature is*

$$\mathrm{d}\bar{a} = \left(\alpha B_\infty - \frac{\varsigma\kappa_{\mathrm{M}}\mathfrak{c}}{2}U_L + \sigma_\alpha a_0 \delta_{\mathscr{M}}\right)\mathrm{d}x^1 \wedge \mathrm{d}x^2.$$

Thus its Maxwell contact is

$$h = \sigma_{\alpha} a_0, \qquad w = \frac{\sigma_{\alpha} a_0}{\varsigma \kappa_{\mathrm{M}} \alpha^2}.$$

Every extension in Theorem is the curl of a potential of this locally integrable form.

Proof. The uniform-field potential is

$$\frac{\alpha B_{\infty}}{2}(x^1 \mathrm{d}x^2 - x^2 \mathrm{d}x^1),$$

whose curl is the smooth constant transverse form. Apply Lemma to the other two terms. Equation gives the source contact. Conversely, set $a_0 = h/\sigma_{\alpha}$.

For the derivative-compatible source $\hat{j}_L$, the selected contact is $h = 0$, hence the corresponding potential normalization is $a_0 = 0$ at the chosen L. This is a prescription, not a physical core-selection theorem.

Proposition (Large regular gauges need not be completion gauges). *The regular transformation* $a \mapsto a + (m/e)\mathrm{d}\varphi$ *leaves F unchanged on M. If both potentials are extended by the locally integrable rule of Theorem , their distributional curvatures differ by*

$$\frac{\sigma_{\alpha} m}{e} \delta_{\mathscr{M}} \mathrm{d}x^1 \wedge \mathrm{d}x^2.$$

There is no contradiction with ordinary gauge invariance: a gauge map of nonzero meridian winding on M does not extend to a smooth $U(1)$ *gauge map over the transverse disk.*

Proof. The first statement follows from $\mathrm{d}(\mathrm{d}\varphi) = 0$ on the punctured plane. The completion difference follows from . A smooth gauge map on a disk has winding zero on its boundary. Thus $e^{-\mathrm{i}m\varphi}$, $m \neq 0$, is not an allowed smooth completion gauge map.

For any smooth completion gauge function χ, ordinary distributional differentiation gives $\mathrm{d}(\bar{a} + \mathrm{d}\chi) = \mathrm{d}\bar{a}$. Gauge invariance under that declared group is preserved.

This result explains why regular gauge equivalence alone cannot choose an apex contact. The response circulation

and the completion gauge group must both be stated.

Scope of the completion. We have constructed a c-number magnetic-sector Maxwell realization and a compatible potential class. We have not constructed a general Maxwell Cauchy domain, a photon Hamiltonian, a gauge-constrained quantum algebra, or a smooth extension of the scalar background bundle. The determinant cancellation in Definition is not a rule for multiplying the singular conical metric by arbitrary supported tensors. This is compatible with, rather than a silent removal of, I-B-G's constrained-field boundary.

Flat-background compatibility is not self-consistency

Theorem (Exact flat-background compatibility test). *The prescribed flat connection $A^{(0)}$, or any connection gauge equivalent to it on M, satisfies the frozen-source Maxwell equation if and only if*

$$\mathfrak{c} = 0.$$

For $e > 0$, these are precisely $\beta = 0, 1/2$.

No choice of B_∞, a_0, or apex-supported contact can make a flat regular connection solve the equation when $\mathfrak{c} \neq 0$. When $\mathfrak{c} = 0$, the no-external-field response $B_\infty = 0$, $a_0 = 0$ is $a = 0$ and is a checked trivial frozen-source solution.

Proof. A flat connection has zero curvature on M, so its Maxwell divergence is zero. The right side of vanishes identically exactly when $\mathfrak{c} = 0$. Gauge transformations do not change curvature. An apex-supported contact has zero restriction to M and cannot change this regular equation. The integration constants of Theorem likewise cannot cancel the nonzero radial derivative required by . The zero-source case follows by substitution.

Proposition (What the calculation does not solve). *For nonzero source, the constructed response is not a solution of a self-consistent scalar–Maxwell state problem. Such a problem would require an independently defined functional $A \mapsto \omega_A(j[A])$, an extension/state prescription for the resulting nonflat connection, and an equation using that updated source. None is supplied by substituting into the fixed coefficient $\mathfrak{c}$.*

Proof. The inherited source is defined for the flat scalar connection . The response has nonzero curvature on M

when $\mathfrak{c} \neq 0$. Consequently $A^{(0)} + a$ is not gauge equivalent to the original flat connection and is not specified by a constant angular character alone. The original scalar theorem does not identify its expectation value with . The displayed calculation has used only that original value.

For the decaying response,

$$e[a_{\varphi}(R) - a_{\varphi}(a)] = -\frac{\varsigma\kappa_{\mathrm{M}} e\mathfrak{c}}{2}\log(R/a).$$

Even a small coefficient therefore does not provide a uniform all-scale bound on the change of circulation. This exact identity is not an estimate of an uncomputed self-consistent correction.

There is no scalar feedback and no metric feedback here. The classical response is not photon vacuum polarization, a quantum Maxwell field, a current–current product, or the constrained gauge-system construction of V7.

Source audit: access, translations, and retained discrepancies

Audit protocol

The access grades in this section mean:

\texttt{Ver} The indicated primary full-text equations were inspected. This is an access statement, not verification of the entire paper.

\texttt{Meta} Only a bibliographic record, abstract, or identified metadata was checked for the stated purpose; no formula agreement follows.

\texttt{Mem} An inherited description or unresolved source label only. It is not promoted to a primary-source check.

The equations below are pinned to the arXiv versions listed in the bibliography. In particular, equation numbers in an unversioned regenerated HTML rendering are not substituted for the inspected PDF numbering.

The complete I-B-VP-v4 source was not independently available in the supplied material; its current and observable excerpts are treated as the stated inputs [20]. Its previous audit grades are not reconstructed from memory. The external grades below record the access actually used for this draft.

For comparisons only, put

$$P_\beta = \alpha^{-2}[1 - 6\beta(1-\beta)] - 1, \qquad C_\beta = \alpha^{-4}[1 - 30\beta^2(1-\beta)^2] - 1,$$

$$D_1 = \mathrm{diag}\,(1, 1, -3, 1), \qquad D_2 = \mathrm{diag}\,(1, -1/2, 3/2, 1),$$

in the order (t, r, φ, z). The inherited normalization is

$$\langle \Phi^\dagger \Phi \rangle_{\mathrm{ren}} = \frac{P_\beta}{48\pi^2 r^2}, \qquad T_{\mathrm{complex}} = \frac{C_\beta}{720\pi^2 r^4} D_1 + \frac{(\xi - 1/6)P_\beta}{6\pi^2 r^4} D_2.$$

These formulas are not used to solve Maxwell's equation.

Direct current and magnetic-field comparisons

Sriramkumar 2001: inspected current and minimal stress. In [1], equations (2.10), (2.12)–(2.16), and (A2) were inspected. Set $\rho = r, \gamma = \beta$. Its lower angular current is exactly $\mathfrak{c}/r^2$. The symmetrized-kernel coefficient ie/2 agrees with the positive-frequency coefficient ie of I-B-VP-0.

Its metric has signature (+ − − −). For the displayed minimal mixed stress, its coefficients obey

$$\mathscr{A} = -C_\beta, \qquad \mathscr{B} = -20 P_\beta.$$

Thus its printed mixed tensor is the negative of at $\xi = 0$, as required when comparing the corresponding mixed energy/stress components after the signature change. Literal equality without this sign translation would be wrong. The fluctuation calculations are not audited here.

Sitenko–Vlasii 2009: a checked Maxwell comparison. Equations (29), (30), and (41) of [2] were inspected. Their notation uses $d = 3$, $v = \alpha^{-1}$, and fractional flux $F = \beta$. The printed massless formula gives

$$e j_\varphi^{\mathrm{SV}} = \frac{\mathfrak{c}}{r^2}.$$

Their current normalization and background coupling $\tilde{e}$ must not be silently identified with our operator convention or with their separate Maxwell charge e.

Their field relation (30), with that coefficient, is

$$B^{\mathrm{SV}}(r) = \int_r^\infty \frac{\nu}{s} e\, j_\varphi^{\mathrm{SV}}(s)\mathrm{d}s = \frac{\mathfrak{c}}{2\alpha r^2}.$$

This is exactly our independently derived $\varsigma = -1, \kappa_{\mathrm{M}} = 1, B_\infty = 0$ solution. It is not the same signed field as the plus-divergence problem. Their massive, higher-dimensional, and core conclusions are not imported.

Bezerra de Mello et al. 2015: component translation. The inspected version of [3] separates zero-thickness and core-induced terms in (4.1). Its ``physical" azimuthal component satisfies $\langle j_\varphi \rangle_0 = -\langle j_2 \rangle_0 / r$ in its $(+---)$ metric and angular coordinate of period $2\pi/q$. With $q = \alpha^{-1}$ and $\varphi_{\mathrm{source}} = \alpha\varphi$, the translation is

$$j_{\hat{\varphi}}^{\mathrm{ours}} = -\langle j_\varphi \rangle_0.$$

Equations (4.4)–(4.6) are image–integral representations. For $q = N, \beta = p/N, D = 3$, the integral weight vanishes and their finite sum gives

$$\langle j_\varphi \rangle_0 = \frac{e\, p(N-p)(N-2p)}{12\pi^2 r^3},$$

opposite to our lower orthonormal component, exactly as the component translation requires. This check uses the finite character identity already proved in I-B-VP-0 [21]. No independent all-parameter resummation, finite-core matching, or core-induced Maxwell response is claimed.

Spinelly–Bezerra de Mello: discrepancies retained

The inspected source is the PDF hep-th/0301169v1, specifically its equations (30), (36), (37), and (49)–(56) [4]. Only the regular zero-thickness terms are compared. The flux-even polynomials are unaffected by replacing γ with $1-\beta$ instead of β.

Three distinctions must remain visible.

1. Printed conformal polynomial. Equation (55) prints

$$u_1^{\mathrm{print}} = \frac{(\alpha^{-4} - 1) - 30\alpha^{-4}\gamma(1 - \gamma^2)}{1440\pi^2 r^4}.$$

The comparator polynomial contains $\gamma^2(1 - \gamma)^2$, not $\gamma(1 - \gamma^2)$. The discrepancy is retained. No author-approved correction or erratum is asserted.

2. Additional version-specific Wick-square discrepancy. The leading zero-thickness term printed in (37) is

$$\frac{1 - \alpha^2 - 6\gamma(1 - \gamma)}{48\pi^2\alpha^4 r^2}.$$

It differs by an extra α^{-2} from the leading terms in the same inspected version's (30) and (36), which agree with the Wick-square comparator in . Thus (37) is not recorded as an unqualified agreement.

3. Stress normalization. The point-splitting operator in (50) and the coefficients (54)–(56) have the one-real-field stress normalization used in the companion, while is the sum for two real components. After recording that factor of two, the improvement coefficient (56) agrees algebraically:

$$u_2 = \frac{P_\beta}{12\pi^2 r^4}.$$

This normalization translation does not repair SB1 or SB2 and is not an audit of every stress result in the source.

These are statements about specified printed formulas. They do not establish that every calculation in the paper is wrong, nor authorize silently replacing its equations in later work.

Other inspected scalar and boundary formulas

Guimarães 1995. In [5], the source's B is our α, $\gamma = \beta$, and its thermal parameter is an inverse temperature, not our fractional holonomy. The stated representation domain is $B > 1/2$. The zero-temperature formulas (19) and (30), with the adjacent ω_2, ω_4 polynomials, give

$$\omega_2 = \frac{P_\beta}{24\pi^2}, \qquad \omega_4 - \frac{1}{3}\omega_2 = \frac{C_\beta}{720\pi^2}.$$

Reordering (τ, z, ρ, φ) to (t, r, φ, z), the term $4(\xi - 1/6)\omega_2$ then gives the complex-field improvement in . Agreement is recorded only on the stated overlap. No all-angle extension is inferred from that paper.

Allen–Kay–Ottewill 1996. In [6], set $\kappa_{\mathrm{AKO}} = \alpha^{-1}$. Equations (3.6), (3.7), and (A13) give the logarithmic-channel coefficient

$$f_R(s) = \frac{1}{2\pi\alpha}\,[\log(s/q)]^{-1}, \qquad q = 2e^{-\gamma_E}/R.$$

The source's R is a logarithmic boundary length, not the response reference L of this paper. The stress integrands (4.13)–(4.15) and the regular neutral coefficient (4.16) were inspected, not numerically evaluated.

A principal-part Green function is not the nonnegative massless spectral covariance used here. Indeed $K_0(qr) \sim -\log(r/R)$ has transverse eigenvalue $-q^2$; adding longitudinal momentum produces $p_z^2 - q^2$. Thus a massless nonnegative square-root ground-state prescription does not follow from that principal-part formula. Core fitting and the long-range approximations are not imported.

Filgueiras–Moraes 2007. Equations (3), (6), and (8)–(10) of [7] were inspected. Their half-character $\lambda = 1/2$ gives

$$\nu = \frac{1}{2\alpha}$$

in the two angular channels $l = 0, -1$. The limit-circle condition $\nu < 1$ requires $\alpha > 1/2$; it is not an all-angle half-holonomy extension.

For a channel, a deficiency combination $K_\nu(k_- r) + e^{i\eta} K_\nu(k_+ r)$, where $k_\pm = k_0 e^{\pm i\pi/4}$, has coefficients $A_- r^{-\nu} + A_+ r^{\nu}$ satisfying

$$\frac{A_+}{A_-} = \frac{\Gamma(-\nu)}{\Gamma(\nu)}\left(\frac{k_0}{2}\right)^{2\nu} \frac{\cos(\eta/2 + \pi\nu/4)}{\cos(\eta/2 - \pi\nu/4)}$$

when the denominator is nonzero. This is the boundary-coordinate translation checked here, not a scalar vacuum-current calculation. The inherited eight-point numerical comparison has no supplied execution artifacts or

total error certificate in this draft and remains UNCERTIFIED.

One row per named comparison target

begingroup 1.13 toprule Source and access Equations and translation checked Verdict and explicit non-import

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midrule endhead bottomrule endfoot

Sriramkumar 2001 [1]; Ver (2.4), (2.10), (2.12)–(2.16), (A2). $\rho = r, \gamma = \beta$; symmetrized versus positive-frequency kernel; opposite-signature mixed stress. Current agrees. Minimal stress agrees after the stated signature translation. No fluctuation hierarchy, thermal analysis, or Maxwell response imported.

Spinelly–Bezerra de Mello 2003 [4]; Ver Inspected v1 (30), (36), (37), (49)–(56); flux-even identification and real/complex multiplicity. SB1 and SB2 retained; improvement matches at the stated normalization. No erratum, finite-core model, or blanket agreement asserted.

Bezerra de Mello–Bezerra–Saharian–Harutyunyan 2015 [3]; Ver (2.8)–(2.10), (4.1)–(4.6). $q = 1/\alpha$; source physical component is minus our lower orthonormal one. Exact finite-character check. Zero-thickness finite-image check agrees. General integral representation inspected, not independently resummed here. No finite-core response or matching imported.

Guimarães 1995 [5]; Ver (16), (19), adjacent coefficient polynomials, and (30). $B = \alpha, \gamma = \beta$; inverse-temperature notation and component order translated. Wick and zero-temperature complex stress agree on $\alpha > 1/2$. No all-angle or thermal-state result imported.

Allen–Kay–Ottewill 1996 [6]; Ver (3.6)–(3.7), (4.13)–(4.16), (A13). Logarithmic length and angular normalization translated. Coefficient and regular neutral stress checked. Integrand inspection is not integration or a positive-state theorem. No smooth-core fitting or principal-part ground state.

Filgueiras–Moraes 2007 [7]; Ver (3), (6), (8)–(10). Half-character, $\nu = 1/(2\alpha)$, and boundary-coefficient map. Boundary comparison only, with $\alpha > 1/2$ qualification. No relativistic vacuum current. Eight-point inherited report

remains UNCERTIFIED.

Sitenko–Vlasii 2009 [2]; Ver (29), (30), (41). $d = 3, \nu = 1/\alpha, F = \beta$; charge normalizations and Maxwell sign separated. Massless field relation agrees with $\varsigma = -1, \kappa_{M} = 1$. This advances a contextual citation to a formula check, not an import of massive, core, or higher-dimensional results.

de Sousa Gerbert 1989 [8]; Meta Primary publisher title/abstract checked; no numbered equation compared. Fermionic boundary problem, not the present scalar Friedrichs source. No extension parameter transferred.

Bordag–Voropaev 1993 [10]; Meta Primary arXiv abstract and bibliographic identification; no equation compared. Spin-1/2 magnetic-moment problem. No spinless scalar boundary condition or Maxwell normalization imported.

Beneventano–De Francia–Santangelo 1999 [9]; Meta Bibliographic identification of the previously unresolved label ``BdFS''. No numbered equation compared. Fermionic spectral-boundary reference. Not a confirmation of the scalar tensor or current. No theorem of this sheet.

Dowker 1977 [11]; Meta Bibliographic identification retained; no full-text equation audit in this draft. Cone-kernel context only. Not counted as confirmation of the charged current or Maxwell coefficient.

Linet 1987 [12]; Meta Primary publisher abstract on Euclidean cosmic-string Green functions; no numbered equation compared. Kernel context only. No current, flux, or stress normalization agreement claimed from metadata.

Frolov–Serebriany 1987 [13]; Meta Primary publisher abstract; no numbered equation compared. Vacuum-polarization context for different field species. No charged-current or present Maxwell comparison.

endgroup

``Not compared'' is a completed entry about the scope of this audit. It is not evidence for or against a formula in the source.

Search boundary and the non-Friedrichs report

The audit is a targeted inspection of the named primary sources, not a systematic survey of all cosmic-string literature. The label ``BdFS" is C. G. Beneventano, M. De Francia, and E. M. Santangelo, ``Dirac fields in the background of a magnetic flux string and spectral boundary conditions", *International Journal of Modern Physics A* **14** (1999), 4749–4762, arXiv:hep-th/9809081 [9]. The identification is metadata only. No equation is compared, and the paper is a fermionic reference, not a confirmation of the scalar tensor or current. I-B-VP version 3 already carried this expansion and did not use it as a scalar theorem.

The inherited report that no direct scalar computation was found for specified zero-thickness $\mathrm{E}_0, \mathrm{E}_\lambda, U(2)$ choices remains a limited-search report. This draft supplies neither an exhaustive negative search nor a novelty theorem for those families. In particular:

• the inspected logarithmic-channel resolvent is not a massless nonnegative spectral-state construction;

• the inspected half-character bound-state problem is not the present relativistic covariance;

• none of these observations excludes other literature outside the audited list.

No claim about a published erratum follows from not finding one in this targeted access.

Finite reports, class-relative coverage, and evidence

The complete supplied-report instrument

A report may install the finite description

$$(\text{source/version}, \alpha, \beta, e, \varsigma, \kappa_{\mathrm{M}}, L, B_\infty, a_0, h, \text{gauge group}, \text{audit row}, \text{evidence status}).$$

A finite word describes encodings and formula identifiers, not arbitrary exact real data without an encoding prescription.

Let a finite clock C have calibrated orthonormal basis $\{|\tau_j\rangle\}$, and let A contain every surviving finite register. Set

$$D_j = \langle \tau_j|_C \otimes I_A, \qquad \mathscr{M}(X) = \sum_j \text{lvertj}, w_j\rangle\text{langlej}, w_j|_R \otimes D_j X D_j^\dagger.$$

Proposition (Complete destructive reporting). *The map $\mathscr{M}$ is completely positive and trace preserving, and*

$$\mathrm{Tr}_R \circ \mathscr{M}(X) = \mathrm{Tr}_C X.$$

The identities remain valid with any untouched finite reference. On a positive-probability branch the returned quantum block is the corresponding normalized residual on A; no quantum clock factor is returned.

Proof. The recorded Kraus maps are $\text{lvertj}, w_j\rangle_R \otimes D_j$, whose squared moduli sum to I_{CA}. Writing

$$X = \sum_{j,k} |\tau_j\rangle\langle\tau_k| \otimes X_{jk}$$

shows that the output blocks are X_{jj}. Their sum is the partial trace. The same calculation includes an untouched reference.

This is the I-B/En09-A and XVI-O1-ST interface [18, 26, 27]. It installs descriptions. It does not acquire an unknown continuum field or independently verify a literature equation. The program has no scalar-field or Maxwell-field input port.

What finite tests can and cannot establish

Once the regular ansatz is proved, two values at distinct radii can determine its two potential constants for fixed source and L; one field value determines B_∞. That is finite identification inside a proved finite-dimensional solution family.

It does not establish the ansatz for arbitrary unknown fields. Likewise, testing finitely many formulas in a paper does not audit its uncompared equations.

For a distributional witness, choose compact antisymmetric tensors $H^{\mu\nu}$ in a regular flat chart and set

$$v^\nu = \partial_\mu H^{\mu\nu}.$$

Then $\partial_\nu v^\nu = 0$. Disjoint compact choices with nonzero v generate arbitrarily large linearly independent families.

Any fixed finite linear evaluation map therefore misses a nonzero conserved perturbation in this unrestricted class.

Conversely, an evaluation map injective on an independently proved finite-dimensional residual space does establish vanishing within that space. This is the coverage distinction stated in Oai282-A and I-B-G [25, 19]. The perturbations just described are not claimed to be alternative Friedrichs-state currents.

Evidence discipline

Access grades texttt *Ver*, texttt *Meta*, texttt *Mem* are independent of:

\texttt{ANALYTIC} A displayed mathematical argument, or an explicitly identified analytical input.

\texttt{SUPPLIED} An inherited computational report, not independently executed.

\texttt{UNCERTIFIED} A numerical or enclosure claim without the artifacts and total error bounds required for certification.

A source can be and still contain a retained discrepancy. An analytical comparison does not certify unrelated numerical figures. No numerical Maxwell run or apex simulation is claimed.

A future report must retain at least the source coefficient, component and sign convention, input encodings, radial interval, boundary data, completion/test structure if used, contact prescription, cutoff profile, raw subtraction terms, precision, residuals, and separate input, cutoff, and quadrature bounds. A successful regression that preserves a discrepancy does not remove it.

For the decaying field, input sensitivity is explicit, but remains separate from any numerical execution:

$$\partial_r B_z = -\frac{2}{r} B_z, \qquad \partial_\alpha B_z = -\frac{3}{\alpha} B_z$$

when $\beta, e, \varsigma, \kappa_{\mathrm{M}}$ are fixed. At a zero of the coefficient, relative residuals are not an appropriate universal diagnostic.

Ledger update, interfaces, and conclusion

begingroup 1.12 toprule Object Status here Exact boundary

midrule endfirsthead toprule Object Status here Exact boundary

midrule endhead bottomrule endfoot

Friedrichs scalar current Input retained Equation ; the scalar calculation is not reopened.

Regular Maxwell response Constructed The complete stationary axial-magnetic family, Theorem ; not a general Maxwell evolution theorem.

Decay and global potential freedom Separated Decay fixes B_∞; meridian phase fixes a_0 modulo the declared regular large gauges.

Ordinary apex integrability Obstructed for $\mathfrak{c} \neq 0$ No locally integrable or locally finite measure-valued field extension; the potential itself is locally integrable.

Restricted distributional magnetic completion Constructed and classified Definition and Theorems –; one linked field–current contact, not an unrestricted gauge domain.

Flat-background Maxwell compatibility Exactly decided Only the zero-current sectors satisfy the sourced equation while remaining flat; constants and apex contacts cannot repair a nonzero regular source.

Self-consistent scalar–Maxwell state **Open here** No functional $A \mapsto \omega_A(j[A])$, updated state, or feedback equation constructed.

Primary-source audit Completed for the declared rows Access and comparison scopes explicit; SB discrepancies retained; one source label unresolved; no exhaustive novelty claim.

Quantum Maxwell field and general constrained apex domain Not constructed V7 is not discharged by this classical magnetic-sector realization.

Massive or non-Friedrichs responses **Open here** No corresponding source/state calculation or feedback problem is supplied.

endgroup

The precise V4 advance. The frozen-source regular Maxwell calculation is complete in the declared magnetic sector. The locally integrable apex obstruction is proved, and a restricted distributional alternative is supplied with its complete contact space. The broader source-by-source audit is completed for the named targets with visible access limitations, not by declaring every source fully verified.

This does not close a canonical physical apex normalization, a finite-core matching problem, or a self-consistent induced connection problem. The scalar one-point extension work remains the separately qualified V1 result of I-B-VP-1 [22]. The other mass, state, and constrained-field ledger items retain their own scopes.

No imported gauge system. XI-B's flat de Donder free multiplet is a different system, as its supplied release makes explicit [24]. It is not a conical Maxwell gauge domain. Similarly, gauge passages in the ST sheets do not select B_∞, a_0, h or define a photon state. The present gauge and constitutive representatives were declared and checked directly.

Non-identifications. No Maxwell field equation is derived from a finite record. No Einstein equation, spacetime emergence, semiclassical metric solution, electromagnetic feedback on the scalar state, current–current product, interacting Ward theorem, or conical BRST construction is obtained.

The supplied OpenAI Vlasov–Maxwell and Kerr–Newman catalogue descriptions are documentary non-inputs. Neither their claims nor their proofs are used or verified here. They concern different models and provide no normalization or apex domain for this calculation.

No programme-wide status is updated. O.1 remains wherever it was not independently closed in its own version-specific scope. XVI-N is not discharged, the downstream XV-B quantum-admission exit is not opened, and is not promoted [27, 26]. These statements neither revoke an independently reported subsector result nor assert universal nonexistence.

Conclusion. The scalar current determines an inverse-square magnetic field only after Maxwell's equation and its sign are supplied. Decay fixes the regular field but leaves a global flat circulation. At the apex, that distinction becomes a contact distinction: the curl of a locally integrable angular potential contains a worldsheet term, and

Maxwell's equation links it to a supported current derivative. The exact regular solution, the obstruction, the restricted extension, and the literature audit therefore solve different questions. Keeping them separate is what makes the V4 result usable without overstating its physical or analytical scope.

appendix

Short acceptance and debugging checklist

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Using a lower angular source as an orthonormal current Use $j_{\varphi}^{\wedge} = j_{\varphi}/(\alpha r)$.

Losing the Maxwell sign Retain ς and the action calculation in Proposition .

Claiming one potential integration constant There is one field constant and an additional flat circulation constant.

Removing a_0 by a multivalued real gauge function Small global gauges do not remove it; regular large $U(1)$ gauges shift it only by m/e.

Using decay as a finite-total-flux condition The decaying massless solution still has logarithmically divergent total flux.

Inferring nonexistence of every distributional field from $F \notin L^1_{\mathrm{loc}}$ Theorem excludes only the stated class. A restricted finite-part realization is constructed separately.

Raising arbitrary apex tensors with the conical metric Only the determinant-cancelled transverse constitutive map is defined.

Adding an arbitrary magnetic delta while keeping the full current fixed Its curl-divergence creates a derivative current contact; equation must hold.

Treating punctured large gauges as smooth completion gauges Nonzero winding does not extend over the

transverse disk.

Calling the response self-consistent The current remains evaluated at the original flat scalar connection.

Calling an inspected source wholly verified records access to specified equations, not proof certification.

Silently replacing a printed SB equation SB1, SB2, and the normalization translation remain visible.

Treating the identified BdFS paper as a scalar theorem The expansion is the 1999 Beneventano–De Francia–Santangelo paper. It still contributes no scalar comparison and no theorem of this sheet.

Using finite report agreement as continuum coverage Coverage requires an independently proved residual class and a separating evaluation map.

endgroup

Declarations

Analytical status. The new field, integrability, completion, contact, and potential–contact statements are the displayed proofs. The scalar current is an explicitly inherited input.

Literature status. Equation numbers refer to the identified inspected versions. The audit does not certify unpublished companion records in full, does not assert an author-approved erratum, and does not claim an exhaustive search.

Novelty status. The magnetic vacuum-response coefficient is established in the cosmic-string literature. The claimed contribution is the present convention-complete solution and its restricted completion and audit within the specified companion problem.

Verification status. The source is a single pdfLaTeX document with an internal bibliography. No successful compiler run, numerical execution, proof-assistant verification, independent rebuild, referee approval, or journal acceptance is asserted.

References

[1] L. Sriramkumar, Fluctuations in the current and energy densities around a magnetic-flux-carrying cosmic string, *Classical and Quantum Gravity* **18** (2001), 1015–1025. DOI: 10.1088/0264-9381/18/6/304. Inspected version: https://arxiv.org/pdf/gr-qc/0011074v1. Audit locations: (2.4), (2.10), (2.12)–(2.16), (A2). The current and minimal-stress comparison does not audit the fluctuation calculations.

[2] Yu. A. Sitenko and N. D. Vlasii, Induced vacuum current and magnetic field in the background of a cosmic string, *Classical and Quantum Gravity* **26** (2009), 195009. DOI: 10.1088/0264-9381/26/19/195009. Inspected version: https://arxiv.org/pdf/0909.0405v2. Audit locations: (29), (30), (41). The Maxwell comparison is not an imported field solution.

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