I-B-VP-5-Ex1-B Audited running, uniform core finite parts, and equal-radius non-universality

A self-contained extension and corrective audit of I-B-VP-5-Ex1 on the prescribed conical spacetime of Enter

Ngo Cheung

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Abstract

We analyze renormalized free scalar observables on smooth rotational curvature cores with a prescribed conical exterior. The heat-coefficient normalization is audited directly from the real-scalar determinant, inverse-metric variation, and Wick rotation. A lapse calculation and a constant-curvature spectral calculation agree on the stress running. An explicit all-order derivative bound supplies a bookkeeping table for longitudinal, radial, angular, and mixed exterior mode sums, including orders below one in the twisted sector.

The main additional analytical result is uniform boundedness and continuity of the complete scalar and conformal stress finite constants on a specified uniformly regular profile class. The proof combines a uniform local heat parametrix, a profile-uniform infrared estimate, and the exterior logarithmic tails. This continuity gives a stronger universality verdict than the nominal-radius argument alone: for every fixed core-adapted scale, finite-part universality also fails among smooth profiles whose *minimal curvature-support radius is exactly the same*. The proof uses smooth endpoint-anchored approximations and does not require unexecuted numerical comparisons of interior stresses.

For a supplied smooth magnetic flux, the complex-determinant normalization is independently checked against the constant-field heat spectrum. We distinguish physical lowering from auxiliary Cartesian lowering and obtain the corresponding current-moment conversion factor explicitly. We also construct the trace sector of the conformal

thin-core expansion through its finite second-derivative contact; this exhibits a power-divergent trace-bearing contact that projected stress moments cannot see.

The potential contribution is a controlled passage from exterior scattering bounds to stable interior finite parts, together with an equal-minimal-radius non-universality theorem. No priority is claimed for heat-kernel renormalization, cosmic-string core effects, or flux regularization. The complete stress-tensor limit, finite charged-current contact, interacting products, and gravitational response remain separate tasks. Enter supplies a finite record–residual interface, not a continuum acquisition or source law.

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Book expansion: equal-minimal-radius lead

Lead with the equal-minimal-radius theorem. Endpoint anchoring (nominal versus minimal radius) is the reason the profile subclass matters. Keep the uniformly regular profile class next to the continuity result. ``Finite-part non-universality'' is neither a numerical comparison nor a complete stress-tensor theorem. **Scope.** Uniform interior continuity = declared profile subclass; equal-minimal-radius non-universality = projected subtracted invariant; trace theorem ≠ completed stress tensor.

Prescribed spacetime, finite reporting, and analytical scope

The Enter distinction

The *Philosophy of Enter* separates destructive readout, surviving quantum output, an installed mathematical description, and a physical identification [10]. The spacetime construction I-B places a finite operational interface on a separately prescribed conical product [11]. The geometry, core profile, field equation, state, connection, and renormalization convention below are continuum inputs. They are not outputs acquired from a finite code.

For a calibrated orthonormal clock basis $\{|\tau_j\rangle\}$, let A contain all surviving finite registers and put

$$D_j = \langle \tau_j|_C \otimes I_A, \qquad M(X) = \sum_j \mathrm{lvert} j, w_j\rangle \mathrm{langle} j, w_j|_R \otimes D_j X D_j^\dagger.$$

The labels retain j, including when two installed words coincide. Failures and unused labels have explicit words.

Proposition (Complete finite report). *The displayed map is completely positive and trace preserving, and*

$$\mathrm{Tr}_R M(X) = \mathrm{Tr}_C X.$$

Both assertions hold after adjoining every untouched finite reference. The quantum output contains A, not a returned C-factor. On a branch of positive probability the surviving state is its actual normalized conditional

state; a zero branch has no assigned normalized state.

Proof. The recorded Kraus operators are lvertj, $w_j\rangle \otimes D_j$, and

$$\sum_j D_j^\dagger D_j = \sum_j |\tau_j\rangle\langle\tau_j|_C \otimes I_A = I_{CA}.$$

Writing $X = \sum_{jk} |\tau_j\rangle\langle\tau_k| \otimes X_{jk}$ shows that the output blocks are X_{jj}, whose sum is $\mathrm{Tr}_C X$. The same expansion includes a reference factor. Positivity gives the conditional-state statement.

This is the I-B/En09-A interface, not a new continuum measurement theorem [11, thm:instrument] [12, thm:main]. It neither selects a smooth core nor certifies a stress normalization. Consumption refers to the declared quantum interface, not physical annihilation of every implementation degree of freedom.

A conserved tensor is not thereby a source of the cone. I-B prop:not-source and X-A-ST prop:noeinstein remain in force [11, 13].

Sources and the actual extension

The complete supplied sources I-B-VP-5-Ex1-v2.tex and I-B-VP-5-Ex2-v2.tex are distinguished throughout [14, 15]. The first supplies the immediate moment problem; the second addresses scalar and Dirac operator selection, not one-point contact normalization.

I-B-G, I-B-VP, I-B-VP-AB, I-B-VP-0, and I-B-VP-1 identify comparison conventions [17, 19, 20, 21, 22]. The supplied I-B-G-S-v2 interface passage concerns spin-1 and spin-2 source-test compatibility, not a scalar interior heat-parametrix theorem [18]. No unavailable Bernoulli-jet, gravitational-core, or interacting-product theorem is reconstructed as a premise.

Standard smooth-background inputs are the local heat parametrix, smooth Wick renormalization, and the scalar heat coefficients [6, 3, 4, 5]. The estimates and continuity argument used here are stated with their quantitative hypotheses. Cosmic-string core effects and finite-flux calculations have substantial prior literature [1, 2]; no priority claim for those subjects is made.

Corrections and strengthened conclusions

The audit makes four points explicit:

1. A Euclidean coincidence derivative and its Lorentzian time-derivative continuation have opposite signs. The exterior stress formula must say which one it uses.

2. Uniform exterior tails do not themselves bound the interior finite constants. A uniformly regular profile subclass is needed for the interior theorem below.

3. Physical lowering of the rotational current test differs from auxiliary Cartesian lowering by α^2. An unconverted current coefficient cannot be imported.

4. The proposed equal-minimal-radius finite-part question need not remain entirely numerical. Continuity transports the nominal-radius discrepancy to profiles of exactly minimal radius one. Universality is thereby excluded, although specific finite constants and isolated coincidences are not numerically evaluated.

An unproved analytical assertion is marked open. It is not made into a theorem by appending an ``uncertified residual."

Core geometry, densities, and the fixed field prescription

Fix $0 < \alpha < 1$, and let P be smooth and nondecreasing, with

$$P(s) = \alpha \quad (0 \le s \le s_0), \qquad P(s) = 1 \quad (s \ge 1), \qquad 0 < s_0 < 1.$$

The Lorentzian metric and its Euclidean continuation are

$$g_{\varepsilon,P} = -\mathrm{d}t^2 + \mathrm{d}z^2 + P(r/\varepsilon)^2\mathrm{d}r^2 + \alpha^2 r^2 \mathrm{d}\varphi^2,$$

$$g^E_{\varepsilon,P} = \mathrm{d}\tau^2 + \mathrm{d}z^2 + P(r/\varepsilon)^2\mathrm{d}r^2 + \alpha^2 r^2 \mathrm{d}\varphi^2.$$

The transverse center is smooth in $x = (r\cos\varphi, r\sin\varphi)$. Use sphere-positive curvature and $\Delta = \nabla^a\nabla_a$ in Euclidean signature. Then

$$R_{\varepsilon,P}=\frac{2P^{'}(s)}{\varepsilon^{2}sP(s)^{3}},\qquad \mathrm{d}A_{\varepsilon,P}=\alpha rP(r/\varepsilon)\mathrm{d}r\mathrm{d}\varphi,\qquad K:=\int R_{\varepsilon,P}\mathrm{d}A_{\varepsilon,P}=4\pi(1-\alpha).$$

Here and below $s=r/\varepsilon$ when it is a radial profile variable.

The completion is $\mathbb{R}^{1,1}\times\mathbb{R}^{2}$, with worldsheet $\mathscr{W}=\mathbb{R}^{1,1}\times\{0\}$. Put

$$\mathrm{d}\Omega_{0}=\alpha\,\mathrm{d}^{2}y\,\mathrm{d}^{2}x,\qquad \langle\delta_{\mathscr{W}},f\rangle=\int f(y,0)\mathrm{d}^{2}y,\qquad \sigma_{\alpha}=2\pi\alpha.$$

A scalar core observable is pushed forward using its *physical* measure:

$$\langle F_{\varepsilon,P},f\rangle=\int F_{\varepsilon,P}(r)f(y,x)\mathrm{d}^{2}y\,\mathrm{d}A_{\varepsilon,P}.$$

Consequently its coefficient relative to $\mathrm{d}\Omega_{0}$ is $P(r/\varepsilon)F_{\varepsilon,P}$. The two density conventions are not interchangeable inside the core.

Except in Section , the field is one real massless scalar,

$$(-\mathrm{Box}+\xi R)\varphi=0,\qquad \xi\geq 0,\qquad q=\xi-\frac{1}{6}.$$

The state is the zero-mean quasifree spectral ground state of the nonnegative smooth-core spatial form. No uniqueness theorem for all algebraic ground states is asserted.

The Euclidean operator is

$$A=-\Delta+\xi R.$$

The metric is uniformly comparable with the Cartesian metric, and $\xi R\geq 0$. Transferring the Euclidean Nash inequality to its density gives

$$|K_{A}(u;X,X^{'})|\leq Cu^{-2}\quad(u>0)$$

on the diagonal and the corresponding Gaussian upper bound off the diagonal. The constant uses ellipticity and density comparability, not derivatives of P. One can obtain the on-diagonal bound directly from the Nash differential inequality for the heat semigroup; the standard Gaussian improvement is discussed in [7]. The large-u

bound is integrable for the massless four-dimensional Green function.

All finite renormalization constants used in the paper are fixed independently of P, ε, μ. Only dimensionless curvature counterterms are allowed. In particular, no separately adjustable dimensionful apex tension is inserted into the smooth-core prescription.

Stress-running audit

Heat coefficient, determinant, and scalar scale

Write the normalized heat expansion as

$$K_A(u; X, X) \sim (4\pi u)^{-2}(1 + a_1 u + a_2 u^2 + \cdots).$$

The Laplace-type convention is $A = -(\nabla^2 + E)$, so $E = -\xi R$. Hence

$$a_1 = -qR,$$

$$a_2 = \frac{\mathrm{Riem}^2 - \mathrm{Ric}^2}{180} + \frac{1}{2}q^2R^2 + \left(\frac{1}{30} - \frac{\xi}{6}\right)\Delta R.$$

These are the standard smooth scalar coefficients [5, Eq. (4.28)].

For example, fix the Wick-square scale convention by

$$F_\mu = \lim_{\delta \,\mathrm{downarrow}\, 0}\left[\int_\delta^\infty K_A(u; X, X)\mathrm{d}u - \frac{1}{16\pi^2\delta} + \frac{a_1}{16\pi^2}\log(\mu^2\delta)\right] + \gamma_R R,$$

with fixed γ_R. It follows directly that

$$\partial_{\log\mu} F_\mu = -\frac{qR}{8\pi^2}.$$

For one real scalar the determinant is

$$W = \frac{1}{2}\log\det(A/\mu^2).$$

For compactly supported variations its scale dependence is

$$\partial_{\log\mu} W = -\frac{1}{16\pi^2}\int a_2 \, \mathrm{d}V.$$

This follows either from the zeta determinant or from the logarithmic proper-time subtraction. On the noncompact product only the local variational statement is required. The integrated total derivative in has zero variation for compactly supported metric changes with the usual boundary terms retained.

Define

$$\mathrm{T}^{E}_{ab} = \frac{2}{\sqrt{g}}\frac{\delta W}{\delta g^{ab}},$$

with a compatible fixed local variational renormalization. The Lorentzian action has signature (− + + +). Under $t = -\mathrm{i}\tau$,

$$T^{L}_{tt} = -\mathrm{T}^{E}_{\tau\tau}, \qquad T^{L\,t}{}_{t} = \mathrm{T}^{E\,\tau}{}_{\tau} = -\rho.$$

Thus the plus sign and the factor 2 in the Euclidean inverse-metric definition are intentional.

toprule Audit item Convention and consequence

midrule Laplace-type potential $E = -\xi R$, therefore $a_1 = (1/6 - \xi)R = -qR$.

Real Gaussian determinant $W = \frac{1}{2}\log\det(A/\mu^2)$; $\partial_{\log\mu} W = -\int a_2/(16\pi^2)$.

Metric variable Variation is with respect to g^{ab}, not g_{ab}.

Euclidean stress factor $\mathrm{T}^{E} = 2(\sqrt{g})^{-1}\delta W/\delta g^{-1}$.

Lorentzian interpretation $T^{t}{}_{t} = \mathrm{T}^{E\,\tau}{}_{\tau} = -\rho$.

Complex charged scalar Its effective action is log det, twice the real determinant normalization.

bottomrule

Tensor variation and an independent lapse calculation

Define

$$H_{1,ab} = \frac{1}{\sqrt{g}}\frac{\delta}{\delta g^{ab}}\int R^2 \mathrm{d}V, \qquad H_{2,ab} = \frac{1}{\sqrt{g}}\frac{\delta}{\delta g^{ab}}\int \mathrm{Ric}^2 \mathrm{d}V, \qquad B_{ab} = H_{2,ab} - \frac{1}{3}H_{1,ab}.$$

This definition of B fixes its normalization; a different convention for the named Bach tensor must be converted.

Modulo the Euler density,

$$a_2 = \frac{1}{120}C^2 - \frac{1}{360}E_4 + \frac{1}{2}q^2R^2, \qquad \delta\int C^2 \mathrm{d}V = 2\int B_{ab}\delta g^{ab}\mathrm{d}V.$$

Theorem (Audited local stress running). *In the stated convention,*

$$\partial_{\log\mu} T^a{}_b = -\frac{1}{8\pi^2}\Big(\frac{1}{60}B^a{}_b + \frac{1}{2}q^2 H_1{}^a{}_b\Big).$$

On a product of a flat longitudinal plane and a transverse surface,

$$H_1{}^t{}_t = 2\Delta R - \frac{1}{2}R^2, \qquad B^t{}_t = -\frac{1}{6}\Delta R - \frac{1}{12}R^2.$$

The same coefficients follow by varying only g^{π}; no contact-improvement requirement is used to select them.

Proof. Apply $2(\sqrt{g})^{-1}\delta/\delta g^{ab}$ to . The Euler term has zero compactly supported variation in dimension four. This proves .

For a separate component calculation take $\delta g^{\pi} = \eta(x)$, with compact transverse support. At the product metric,

$$\delta\sqrt{g} = -\tfrac{1}{2}\eta\sqrt{g}, \qquad \delta R = \Delta\eta, \qquad \delta(\mathrm{Ric}^2) = \tfrac{R}{2}\Delta\eta, \qquad \delta(\mathrm{Riem}^2) = 0.$$

Here $\mathrm{Ric}^2 = R^2/2$ and $\mathrm{Riem}^2 = R^2$. Thus

$$\frac{1}{\sqrt{g}}\frac{\delta}{\delta g^{\pi}}\int a_2 \mathrm{d}V = -\frac{\Delta R}{360} - \frac{R^2}{720} + q^2\Big(\Delta R - \frac{R^2}{4}\Big).$$

Likewise the lapse variations of $\int R^2$ and $\int \mathrm{Ric}^2$ are

$$2\Delta R - \tfrac{1}{2}R^2, \qquad \tfrac{1}{2}\Delta R - \tfrac{1}{4}R^2.$$

Their combination gives , and the factor 2 in the stress definition reproduces .

par *Interpretive scope.* These are free-field quantities on a prescribed background, not Maxwell or Einstein source

equations [11, 13]. par

Proposition (Constant-curvature spectral cross-check). *On* $\mathbb{R}^2 \times S^2_a$, *at* $\xi = 1/6$,

$$a_2 = \frac{R^2}{360}, \qquad \partial_{\log \mu} T^t{}_t = \frac{R^2}{5760\pi^2}.$$

Proof. The transverse diagonal heat kernel is

$$\frac{1}{4\pi a^2} \sum_{\ell=0}^{\infty} (2\ell+1) e^{-v[\ell(\ell+1)+2\xi]}, \qquad v = u/a^2.$$

Euler–Maclaurin summation gives

$$\sum_{\ell \geq 0} (2\ell+1) e^{-v\ell(\ell+1)} = v^{-1} + \frac{1}{3} + \frac{v}{15} + O(v^2).$$

After multiplication by $e^{-v/3}$, the normalized four-dimensional u^2 coefficient is $1/(90a^4) = R^2/360$. A constant longitudinal inverse-lapse variation changes only the volume factor in this coefficient. Equations and then give $a_2/(16\pi^2)$.

par *Interpretive scope.* These are free-field quantities on a prescribed background, not Maxwell or Einstein source equations [11, 13]. par

Remark (What this audit does and does not check). The tensor calculation, lapse calculation, and spectral calculation agree analytically. No numerical discrepancy remains in these identities.

No independent numerical interior stress mode sum has been executed. The exterior reflection correction is μ -independent and cannot test running supported inside the core. That unexecuted corroboration is not a missing term in .

Exterior matching and explicit differentiated majorants

Reflection coefficients

For longitudinal momentum $p > 0$, put $x = p\varepsilon$ and $\nu_n = |n|/\alpha$. The regular interior radial solution satisfies

$$\left(\frac{s}{P}U_{n,x}^{'}\right)^{'} = \left[\frac{n^2P}{\alpha^2 s} + 2\xi\frac{P^{'}}{P^2} + x^2 sP\right]U_{n,x}.$$

It is positive and nondecreasing. Set $D_n(x) = U_{n,x}^{'}(1)/U_{n,x}(1) \geq 0$. Matching $I_{\nu_n}(pr) - b_n(x)K_{\nu_n}(pr)$ gives

$$b_n(x) = \frac{D_n(x)I_{\nu_n}(x) - xI_{\nu_n}^{'}(x)}{D_n(x)K_{\nu_n}(x) - xK_{\nu_n}^{'}(x)}.$$

Lemma (Undifferentiated reflection majorants). *For $\nu > 0$ and every nonnegative impedance,*

$$|b_\nu(x)| \leq \left(1 + \frac{x}{\nu}\right)\frac{I_\nu(x)}{K_\nu(x)}.$$

For the neutral zero channel,

$$|b_0(x)| \leq \frac{I_0(x)}{K_0(x)}.$$

Consequently, if $t, t^{'} > 1$,

$$|b_\nu(x)K_\nu(xt)K_\nu(xt^{'})| \leq B_\nu(x;t,t^{'})$$

$$:= \frac{1 + x/\nu}{2\nu}(tt^{'})^{-\nu/2}e^{-x(t+t^{'}-2)/2}, \qquad \nu > 0,$$

$$|b_0(x)K_0(xt)K_0(xt^{'})| \leq B_0(x;t,t^{'}) := I_0(x)K_0(x)e^{-x(t+t^{'}-2)}.$$

The constants do not use profile derivatives.

Proof. The quotient in is increasing as a function of $D \geq 0$, by the Wronskian $x(I_\nu^{'}K_\nu - I_\nu K_\nu^{'}) = 1$. Its endpoints are $-I_\nu^{'}/(-K_\nu^{'})$ and I_ν/K_ν. For $\nu > 0$, use

$$xI_\nu^{'} \leq (\nu + x)I_\nu, \qquad -xK_\nu^{'} \geq \nu K_\nu.$$

The first inequality also follows by comparison in the Riccati equation for $xI_\nu^{'}/I_\nu$. For $\nu = 0$, $I_1 \leq I_0$ and $K_1 \geq K_0$.

The Wronskian implies $I_\nu K_\nu \leq 1/(2\nu)$. Also

$$\frac{K_\nu(xt)}{K_\nu(x)} \leq t^{-\nu}, \qquad \frac{K_\nu(xt)}{K_\nu(x)} \leq e^{-x(t-1)}.$$

The first follows from $-uK_\nu'/K_\nu \ge \nu$; the second follows from the positive integral representation of K_ν. Taking the geometric mean of these bounds at each endpoint proves . Equation follows without taking the geometric mean. The standard Bessel identities are recorded in [8].

A derivative estimate valid also below order one

Lemma (All-order radial loss). *For every integer $m \ge 0$, all $u > 0$, and all $\nu \ge 0$,*

$$\left| u^m \frac{\mathrm{d}^m}{\mathrm{d}u^m} K_\nu(u) \right| \le C_m (1 + \nu + u)^m K_\nu(u).$$

In particular,

$$-uK_\nu'(u) \le (u + \nu + 1) K_\nu(u).$$

Proof. Normalize the positive measure $e^{-u\cosh v}\cosh(\nu v)\mathrm{d}v$ by $K_\nu(u)$, and let m_1 be its expectation of $\cosh v$. Integration of $\partial_v[\sinh v\, e^{-u\cosh v}\cosh(\nu v)]$ gives

$$u\,\mathbb{E}(\cosh^2 v - 1) = m_1 + \nu\mathbb{E}[\sinh v \tanh(\nu v)] \le (\nu + 1) m_1.$$

By Jensen, $u(m_1^2 - 1) \le (\nu + 1) m_1$, so $u m_1 \le u + \nu + 1$. Since $-K_\nu'/K_\nu = m_1$, this proves the first-order bound.

The recurrence then gives

$$K_{\nu+1}(u) = -K_\nu'(u) + \frac{\nu}{u} K_\nu(u) \le \left(1 + \frac{2\nu + 1}{u}\right) K_\nu(u).$$

Differentiating the integral representation m times bounds $|K_\nu^{(m)}|$ by $K_{\nu+m}$: expansion of $\cosh^m v \cosh(\nu v)$ has positive coefficients whose sum is one, and K_λ increases with $|\lambda|$. Iterating the recurrence yields

$$u^m K_{\nu+m}(u) \le \prod_{j=0}^{m-1} (u + 2\nu + 2j + 1) K_\nu(u),$$

which proves .

Visible bookkeeping

Use the full longitudinal Fourier representation before performing its angular integral. A longitudinal derivative therefore costs p, with no apparent singular factor at $y = y'$. In the exterior, an orthonormal angular derivative costs v/r.

Put

$$H = 1 + v + xt, \qquad H' = 1 + v + xt', \qquad B = B_v(x; t, t').$$

The following table lists upper losses for the radial reflection factor. Constants depending only on the indicated derivative orders are suppressed.

toprule Derivative class Multiplicative loss Majorant

midrule endfirsthead toprule Derivative class Multiplicative loss Majorant

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L longitudinal derivatives, at either endpoint $\varepsilon^{-L} x^L$ $\varepsilon^{-L} x^L B$

One radial derivative at r $(\varepsilon t)^{-1} H$ $(\varepsilon t)^{-1} HB$

Two radial derivatives at r $C(\varepsilon t)^{-2} H^2$ $C(\varepsilon t)^{-2} H^2 B$

m radial derivatives at r $C_m(\varepsilon t)^{-m} H^m$ $C_m(\varepsilon t)^{-m} H^m B$

a orthonormal angular derivatives at r $(\varepsilon t)^{-a} v^a$ $(\varepsilon t)^{-a} v^a B$

m, m' radial derivatives at the two endpoints $C(\varepsilon t)^{-m} (\varepsilon t')^{-m'} H^m (H')^{m'}$ The displayed loss times B

General mixed derivative: $L; m, m'; a, a'$ $C\varepsilon^{-N} x^L t^{-m-a} (t')^{-m'-a'} v^{a+a'} H^m (H')^{m'}$ The displayed loss times B, where $N = L + m + m' + a + a'$

For coordinate angular derivatives replace v^a/r^a by $|n|^a$, or by $|n + \beta|^a$ in a twisted parallel convention. Repeated

frame derivatives and covariant derivatives also differentiate frame coefficients. Their additional r^{-1} factors are already covered by the same total order bound. Neutral zero-channel angular derivatives vanish.

Including the measure $p\mathrm{d}p = \varepsilon^{-2} x \mathrm{d}x$, the general mixed majorant is

$$C\varepsilon^{-N-2} x^{L+1} t^{-m-a} (t')^{-m'-a'} v^{a+a'} H^{m} (H')^{m'} B_{\nu}(x; t, t') \mathrm{d}x.$$

For a neutral zero channel omit the angular factors and use B_0. At $x = 0$, its only extra singularity is $I_0 K_0 = O(1 + |\log x|)$; at infinity it is $O(x^{-1})$.

Theorem (Differentiated exterior mode theorem). *For exterior endpoints $r, r' > \varepsilon$, the Euclidean covariance difference from the Friedrichs cone is*

$$\delta G_{\varepsilon} = -\frac{1}{4\pi^2 \alpha} \sum_{n \in \mathbb{Z}} e^{\mathrm{i}n(\varphi - \varphi')} \int_0^{\infty} p\mathrm{d}p \, J_0(p|y - y'|) b_n(p\varepsilon) K_{\nu_n}(pr) K_{\nu_n}(pr').$$

Every fixed endpoint derivative has an absolutely convergent mode sum and integral, locally uniformly in the exterior. The estimates are profile-independent on

$$1 + \eta \le r/\varepsilon, r'/\varepsilon \le T$$

for fixed $\eta > 0, T < \infty$, with the dimensional powers of ε in retained. They justify all exterior scalar, current, and stress point-splitting derivatives of the smooth correction.

Proof. Fourier transform longitudinally and separate angular modes. The regular exterior radial solution is $I_{\nu} - b_{\nu} K_{\nu}$, and the decaying one is K_{ν}. Their Wronskian gives $I_{\nu}(pr_<) K_{\nu}(pr_>) - b_{\nu} K_{\nu}(pr) K_{\nu}(pr')$. The angular and longitudinal Fourier normalizations yield .

For positive modes, is a polynomial loss times

$$(tt')^{-\nu/2} e^{-x(t + t' - 2)/2}.$$

The angular orders grow linearly, so their polynomially weighted geometric sum converges. The momentum integral is integrable at zero and exponentially convergent at infinity. The neutral zero-channel bound has the same

property with a logarithm at zero. Dominated convergence applies to every fixed derivative.

The covariance difference is smooth across the exterior diagonal, where the two geometries have identical local subtractions. Thus the corresponding one-point differences are obtained by differentiating it.

par *Interpretive scope.* These are free-field quantities on a prescribed background, not Maxwell or Einstein source equations [11, 13]. par

Corollary (Far exterior tails). *For a unit neutral core,* $r \geq 2$,

$$|\delta F_{\neq 0}(r)| \leq Cr^{-2-1/\alpha}, \qquad |\delta T_{\neq 0}(r)| \leq C_{\xi} r^{-4-1/\alpha}.$$

For a supplied smooth flux with $0 < \beta < 1$, *set* $v_{*} = \min(\beta, 1-\beta)/\alpha$. *Then*

$$|\delta F(r)| \leq Cr^{-2-v_{*}}, \qquad |\delta T(r)| \leq C_{\xi} r^{-4-v_{*}}, \qquad |\delta J(r)| \leq Cr^{-3-v_{*}}.$$

These are sufficient, not asserted optimal, exponents.

Proof. At coincidence put $t = t' = r$ in the majorants and then $v = xr$. A derivative of total physical order N contributes r^{-N} times a polynomial in v, v. The remaining order factor is r^{-v}. Summing its polynomially weighted geometric tail proves the estimates. Lemma covers $0 < v < 1$ without an exceptional small-order gap.

par *Interpretive scope.* These are free-field quantities on a prescribed background, not Maxwell or Einstein source equations [11, 13]. par

These are Euclidean exterior assertions. They do not claim absolute convergence of arbitrary Lorentzian boundary values, nor uniformity at a singular limiting matching ring.

Matching, logarithmic tails, and complete finite constants

The zero-channel matching number

For $\xi > 0$, write $d_P = D_0(0)$ and $G = 2\xi(\alpha^{-1} - 1)$. The regular zero-energy solution gives

$$Gs_0^G \leq d_P \leq G, \qquad L_{\text{eff}} = \varepsilon e^{-1/d_P}, \qquad 0 \leq D_0(x) - d_P \leq x^2/2.$$

Proof. If $w = sU'/(PU)$, positivity and the integrated zero-energy equation give $0 \le w \le G$. Hence $U(s)/U(1) \ge s^{G}$, and

$$d_{P} = \int_{0}^{1} \frac{2\xi P'}{P^{2}} \frac{U(s)}{U(1)} \mathrm{d}s$$

gives the first bounds. Exterior zero-energy matching is $1 + d_{P} \log (r/\varepsilon)$, proving the length formula.

With boundary value $u(1) = 1$, the zero-channel Dirichlet-to-Neumann value is the minimum of

$$\int_{0}^{1} \left[\frac{s}{P} |u'|^{2} + \frac{2\xi P'}{P^{2}} |u|^{2} + x^{2} s P |u|^{2} \right] \mathrm{d}s.$$

The normalized zero-energy minimizer satisfies $0 < u \le 1$. Comparing the two minima proves the last inequality.

Accordingly,

$$b_{0}(x) = \frac{1}{\log (1/x) + d_{P}^{-1} + \log 2 - \gamma_{E}} + O\big(x^{2}(1 + |\log x|)^{2}\big).$$

The remainder is uniform when d_{P} is bounded above and away from zero.

The reference cone coefficients

The standard neutral real-scalar coefficients are

$$s = \frac{\alpha^{-2} - 1}{48\pi^{2}}, \qquad h = \frac{\alpha^{-4} - 1}{1440\pi^{2}}.$$

Here $F_{\mathrm{cone}} = s/r^{2}$, and, at conformal coupling, $T^{t}_{t,\mathrm{cone}} = h/r^{4}$.

For completeness these normalizations can be checked from the cone heat kernel. With $z = r^{2}/(2u)$, its transverse diagonal is

$$K_{\mathrm{cone},2} = \frac{e^{-z}}{4\pi\alpha u} \sum_{n \in \mathbb{Z}} I_{|n|/\alpha}(z).$$

The Laplace transform $\int_{0}^{\infty} e^{-z\cosh\eta} I_{\nu}(z)\mathrm{d}z = e^{-\nu\eta}/\sinh\eta$ gives the subtracted generating function

$$A(\eta) = \frac{1}{\alpha} \frac{\coth (\eta/(2\alpha))}{\sinh\eta} - \frac{1}{\cosh\eta - 1}.$$

Its expansion is

$$A(\eta) = \frac{\alpha^{-2} - 1}{6} + \frac{11 - 10\alpha^{-2} - \alpha^{-4}}{360}\eta^2 + O(\eta^4).$$

The scalar coefficient is $A(0)/(8\pi^2)$. The renormalized Euclidean derivative $C^E_{\tau\tau} = \langle \partial_\tau \varphi, \partial_\tau \varphi \rangle$ has coefficient

$$-\frac{1}{8\pi^2} \lim_{\eta \operatorname{downarrow} 0} \frac{A'(\eta)}{\sinh \eta} = \frac{\alpha^{-4} + 10\alpha^{-2} - 11}{1440\pi^2}.$$

Since

$$T^t_{\ t} = C^E_{\tau\tau} + \left(\xi - \frac{1}{4}\right)\Delta F \quad \text{in the flat exterior,}$$

the conformal value is precisely h. The subtracted heat integrals are convergent at a regular point; the displayed parameter is an Abel evaluation of those integrals, not an apex prescription. The Bessel transforms are standard [8]; the conventions agree with the supplied regular-part comparison records [19, 20].

Remark (The time-derivative debugging point). Equation uses the *Euclidean* derivative $C^E_{\tau\tau}$, with a plus sign. If one instead writes a continued Lorentzian C^L_{tt}, then $C^L_{tt} = -C^E_{\tau\tau}$, and the same equation has a minus sign in front of that object. The explicit zero-channel kernel below agrees with the audited Euclidean convention.

par *Interpretive scope.* These are free-field quantities on a prescribed background, not Maxwell or Einstein source equations [11, 13]. par

The centered Mellin calculation

Define

$$M_P(r) = \log r + 1 + d_P^{-1}, \qquad \mathcal{Q}(M) = \log M - \frac{1}{2M^2} + \frac{2 - \zeta(3)}{3M^3}.$$

The Mellin identity

$$f(t) = \int_0^\infty x^{1+t} K_0(x)^2 \mathrm{d}x = \frac{\sqrt{\pi}\,\Gamma(1 + t/2)^3}{4\Gamma(3/2 + t/2)}$$

gives

$$f(0) = \frac{1}{2}, \quad (\log f)'(0) = -(1 + \gamma_E - \log 2), \quad (\log f)''(0) = 1, \quad (\log f)'''(0) = \zeta(3) - 2.$$

Lemma (Uniform neutral logarithmic tail). *For a unit core and* $\xi > 0$,

$$\sigma_\alpha r^2 \delta F(r) = -\frac{1}{4\pi} \mathcal{Q}'(M_P(r)) + O(M_P(r)^{-5}) + O(r^{-1/\alpha}).$$

The remainders are uniform on profile families with $s_0 \geq s_* > 0$ *and fixed positive* ξ. *They are also uniform when* α, ξ *range over compact subsets of* $0 < \alpha < 1$, $0 < \xi < \infty$.

Proof. The zero-channel part is

$$\sigma_\alpha r^2 \delta F_0(r) = -\frac{1}{2\pi} \int_0^\infty x\, b_0(x/r) K_0(x)^2 \mathrm{d}x.$$

First restrict to $x \leq \sqrt{r}$; the omitted tail is exponentially small by the reflection bound. On the retained interval has an integrated error $O(r^{-2}(1 + \log r)^2)$. Its logarithmic denominator stays positive there for large r.

Normalize $2xK_0(x)^2 \mathrm{d}x$ as a probability measure and put $Z = \log x + 1 + \gamma_E - \log 2$. Then

$$\mathbb{E}Z = 0, \qquad \mathbb{E}Z^2 = 1, \qquad \mathbb{E}Z^3 = \zeta(3) - 2.$$

On $|Z| \leq M/2$, expand $(M - Z)^{-1}$ through its cubic term; the remainder is bounded by $C|Z|^4 M^{-5}$. The excluded low-x and high-x moment tails are exponentially small in M, or smaller. Thus

$$\sigma_\alpha r^2 \delta F_0(r) = -\frac{1}{4\pi}\left(M^{-1} + M^{-3} + (\zeta(3) - 2)M^{-4}\right) + O(M^{-5}).$$

No rational logarithmic approximation is integrated through its remote pole: it is used only in the truncated small-argument region. Add . The matching bounds make all choices uniform on the stated nondegenerate families.

Moments and their finite constants

For the reference finite part set

$$\mathrm{langleU_1}, f\rangle = \lim_{\eta \mathrm{downarrow}\, 0} \left[\int_{r > \eta} r^{-2} f \mathrm{d}\Omega_0 + \sigma_\alpha \log \eta \int f(y, 0) \mathrm{d}^2 y \right].$$

The transverse auxiliary Laplacian is $\Delta_0 = \partial_1^2 + \partial_2^2$. At conformal coupling use the reference stress $(h/4)\Delta_0 U_1$ for its longitudinal component. These are the fixed comparison conventions of the supplied apex-completion records [22, 23, 24].

For smooth radial χ, compactly supported and equal to one near zero, define per unit worldsheet volume

$$c_{P,\chi} = \int F_{\varepsilon,P,\mu}\chi \mathrm{d}A_{\varepsilon,P} - \mathrm{slangleU_1},\chi\rangle,$$

$$t_{P,\chi} = \int \frac{r^2}{4}\chi\, T_{\varepsilon,P,\mu}{}^{t}{}_{t}\mathrm{d}A_{\varepsilon,P} - \frac{h}{4}\left\langle \Delta_0 U_1, (r^2/4)\chi \right\rangle, \qquad \xi = \frac{1}{6}.$$

A superscript sharp means the radial indicator $r < 1$. It is an ordinary core integral with a canonical reference finite-part pairing: the indicator is smooth near the singular support and the distribution is ordinary near its jump. Direct integration gives

$$\mathrm{langleU_1},1_{r<1}\rangle = 0, \qquad \frac{h}{4}\left\langle \Delta_0 U_1, (r^2/4)1_{r<1} \right\rangle = -\frac{\sigma_\alpha h}{4}.$$

Theorem (Complete scalar moment and normalized finite constant). *For $\xi > 0$, let*

$$\ell = \log(1/\varepsilon), \qquad M = \ell + 1 + d_P^{-1}.$$

In fixed reference length units,

$$c_P^{\mathrm{sharp}}(\varepsilon;\mu) = \sigma_\alpha s\ell - \frac{qK}{8\pi^2}\log(\mu\varepsilon) - \frac{1}{4\pi}\mathscr{Q}(M) + C_P + O(M^{-4}).$$

The finite constant is

$$C_P = J_P(R_0) + \frac{1}{4\pi}\mathscr{Q}(M_0)$$

$$+ \int_{\log R_0}^{\infty} \left[\sigma_\alpha e^{2u}\delta F_P(e^u) + \frac{1}{4\pi}\mathscr{Q}'(u + 1 + d_P^{-1}) \right]\mathrm{d}u,$$

$$J_P(R_0) = \int_{r<R_0} F_{1,P,1}\mathrm{d}A_{1,P} - \sigma_\alpha \mathrm{slog}\, R_0, \qquad M_0 = \log R_0 + 1 + d_P^{-1}.$$

For $R_0 \geq 2$, the omitted integral has absolute value at most

$$CM_0^{-4} + CR_0^{-1/\alpha}.$$

These exterior error constants are uniform on the nondegenerate families of Lemma .

Proof. Constant scaling of the smooth heat prescription gives

$$F_{\varepsilon,P,\mu}(r) = \varepsilon^{-2}F_{1,P,1}(r/\varepsilon) - \frac{qR_{\varepsilon,P}(r)}{8\pi^2}\log(\mu\varepsilon).$$

The finite R-ambiguity scales homogeneously. For $R > 1$,

$$\frac{\mathrm{d}J_P(R)}{\mathrm{d}\log R} = \sigma_\alpha R^2 \delta F_P(R).$$

Integrating Lemma constructs C_p and proves –. Apply , use , and put $R = \varepsilon^{-1}$.

par *Interpretive scope.* These are free-field quantities on a prescribed background, not Maxwell or Einstein source equations [11, 13]. par

For a smooth cutoff define

$$m_j(\chi) = \int_0^\infty [\chi(u) - 1_{u<1}](\log u)^j \frac{\mathrm{d}u}{u}.$$

The same proof, on the fixed exterior support of the cutoff difference, gives

$$c_{P,\chi} = \sigma_\alpha s\ell - \frac{qK}{8\pi^2}\log(\mu\varepsilon) + C_P$$
$$-\frac{1}{4\pi}\left[\mathscr{Q}(M) + \frac{m_0(\chi)}{M} - \frac{m_1(\chi)}{M^2} + \frac{m_2(\chi)+m_0(\chi)}{M^3}\right] + O(M^{-4}).$$

Thus the finite constant is unchanged, but the inverse-M coefficients depend on the cutoff. In particular, absence of a $1/M$ term is a sharp-normalization statement.

The conformal stress tail

With the Euclidean derivative convention in , the unit-core zero-channel stress is

$$\delta T^t{}_{t,0}(r) = -\frac{1}{4\pi^2\alpha r^4}\int_0^\infty x^3 b_0(x/r)V_\xi(x)\mathrm{d}x, \qquad V_\xi = -\frac{1}{2}K_1^2 + \xi\Delta_x(K_0^2).$$

Indeed the Euclidean longitudinal derivative contributes $-[8\pi^2\alpha]^{-1}\int p^3 b_0 K_0^2 \mathrm{d}p$; combining it with

$\Delta K_0(pr)^2 = 2p^2(K_0^2 + K_1^2)$ gives the displayed expression.

The moments

$$\int_0^\infty x^3 K_1^2 \mathrm{d}x = \frac{2}{3}, \qquad \int_0^\infty x^3 \Delta_x(K_0^2)\mathrm{d}x = 2$$

follow respectively from the product Mellin integral and two integrations by parts. Hence

$$\int_0^\infty x^3 V_\xi \mathrm{d}x = 2q.$$

At $q = 0$, expansion of the logarithmic denominator therefore gives

$$\frac{\sigma_\alpha}{4} r^4 \delta T^t_{t,0}(r) = O(M_P(r)^{-2}).$$

All its logarithmic moments are absolutely integrable, so the proof is the same truncated expansion as in Lemma .

Together with , this constructs

$$C_{T,P} = \int_{r<R_0} \frac{r^2}{4} T_{1,P,1\,t}^{\,t} \mathrm{d}A_{1,P} - \frac{\sigma_\alpha h}{4} \log R_0 + \frac{\sigma_\alpha h}{4}$$

$$+ \frac{\sigma_\alpha}{4} \int_{R_0}^\infty r^3 \delta T_{P\,t}^{\,t}(r)\mathrm{d}r,$$

with tail bound

$$CM_0^{-1} + CR_0^{-1/\alpha}.$$

It is a complete finite constant, including the smooth renormalized interior, not an exterior-only surrogate.

par *Interpretive scope.* These are free-field quantities on a prescribed background, not Maxwell or Einstein source equations [11, 13]. par

Uniform interior control and continuity

A sufficient uniformly regular subclass

Definition (Uniformly regular unit-core class). Fix $0 < s_* < 1$, a finite L, and an integer $k \geq 20$. Let P_{k,L,s_*}

consist of smooth monotone profiles satisfying

$$P = \alpha \quad (s \le s_*), \qquad P = 1 \quad (s \ge 1), \qquad \max_{0 \le j \le k} \sup_{[s_*, 1]} |P^{(j)}| \le L.$$

All profiles are smooth across both endpoints. We fix α, the positive coupling ξ, the spectral state, and the finite renormalization constants.

The derivative order 20 is a sufficient reserve for the finite parametrix and derivative estimates used below; it is not asserted minimal.

The Cartesian transverse metric is

$$g_{ij} = \alpha^2 \delta_{ij} + (P(r)^2 - \alpha^2) \frac{x_i x_j}{r^2}.$$

It is constant for $r \le s_*$, has uniformly bounded derivatives through the required orders on the annulus, and has uniformly controlled decaying derivatives outside the annulus. Thus this class gives global ellipticity, density, and finite-order coefficient bounds on the *unit* cores.

It does not give uniform curvature bounds on the unrescaled cores as $\varepsilon \to 0$.

Lemma (Uniform renormalized local one-point bounds). *For every fixed $R_0 < \infty$, on P_{k, L, s_*},*

$$\sup_{r \le R_0} |F_{1, P, 1}(r)| \le C_{R_0}, \qquad \sup_{r \le R_0} |T_{1, P, 1}(r)| \le C_{R_0}.$$

The stress components are taken in the common Cartesian coordinates, or in uniformly equivalent smooth frames. At conformal coupling these bounds apply to the stress used in .

Proof. We give the ultraviolet and infrared arguments separately.

Small heat time. The quantitative local heat-parametrix construction for a uniformly elliptic scalar operator with bounded coefficients and finitely many bounded derivatives gives, on uniform small coordinate neighborhoods,

$$H_{2, P}(u; X, X') = (4\pi u)^{-2} e^{-d_P(X, X')^2/(4u)} \sum_{j=0}^{2} u^j u_{j, P}(X, X').$$

The coefficients, their required derivatives, and the neighborhood sizes are uniform in the class. For total endpoint

derivative order at most two,

$$|\partial_X^a \partial_{X'}^b (K_P - H_{2,P})(u; X, X')| \le C, \qquad 0 < u < u_0, \quad |a| + |b| \le 2,$$

on a smaller neighborhood of the diagonal. The same statement with a finite extra derivative is available for compactness arguments. This is the ordinary finite heat-parametrix estimate, not a parametrix on a singular cone [6]. Its construction uses transport equations for u_j and a Volterra correction; all constants depend on the uniform finite coefficient bounds supplied by .

Subtracting the integrated parametrix is precisely the local smooth-background subtraction for the Green function and its two-derivative observables, up to fixed local finite terms [3, 4]. Equation is integrable at zero. The finite local terms are uniformly bounded as well.

Large heat time. The Nash estimate is uniform. At a fixed heat time $v > 0$, local parabolic estimates applied to the Gaussian upper bound give uniform L^1 bounds in the second kernel variable for the first and second derivatives of $K_P(v; X, \cdot)$. The same holds with the endpoints reversed. The coefficient bounds in are sufficient for these classical parabolic estimates.

For $u \ge 1$, split the heat semigroup as

$$e^{-uA_P} = e^{-vA_P} e^{-(u-2v)A_P} e^{-vA_P}, \qquad 0 < v < \frac{1}{4}.$$

Differentiate only the two fixed-time endpoint kernels. Their uniform L^1 bounds and the central L^1-to-L^∞ Nash bound give

$$|\partial_X^a \partial_{X'}^b K_P(u; X, X')| \le C u^{-2}, \qquad |a| + |b| \le 2.$$

This is integrable at infinity. Intermediate heat times are controlled by the same fixed-time regularity. The subtracted Green function and its required coincidence derivatives are therefore uniformly bounded on compact sets. Applying the smooth local stress operator, including its fixed curvature terms, proves .

par *Interpretive scope.* These are free-field quantities on a prescribed background, not Maxwell or Einstein source

equations [11, 13]. par

Theorem (Uniform finite constants and continuity). *On* P_{k,L,s_*},

$$|C_P| + |C_{T,P}| + |I_P^\infty| \le C, \qquad I_P^\infty := hC_P - 4sC_{T,P},$$

where $C_{T,P}, I_P^\infty$ *are used at* $\xi = 1/6$. *Their exterior truncation errors are uniformly and .*

If P_j, P *belong to such a class and* $P_j \to P$ *in* $C^k([s_*, 1])$, *then*

$$d_{P_j} \to d_P, \qquad C_{P_j} \to C_P, \qquad C_{T,P_j} \to C_{T,P}, \qquad I_{P_j}^\infty \to I_P^\infty.$$

The same conclusion holds for any convergent smooth profile sequence contained in one such class.

Proof. The matching bounds with $s_0 \ge s_*$ place d_P in a fixed compact subset of $(0, \infty)$. At $R_0 = 2$, Lemma bounds the interior terms of and . Their exterior tails are already uniform, proving .

For continuity, the normalized radial variational problem or its regular ODE shows $d_{P_j} \to d_P$. For the interior kernels, identify all Hilbert spaces by multiplication by the square root of their densities. The transported quadratic forms have a common H^1 domain and uniformly coercive shifted forms. Their coefficients converge, and

$$|Q_j[u, v] - Q[u, v]| \le \eta_j \|u\|_{H^1} \|v\|_{H^1}, \qquad \eta_j \to 0.$$

The shifted variational resolvent identity therefore gives norm-resolvent convergence at a fixed negative spectral parameter. Functional calculus gives convergence of the heat operators for every fixed positive heat time.

Uniform local parabolic estimates give compactness of their kernels and of the endpoint derivatives required here on compact sets. Operator convergence identifies every such kernel limit uniquely. The local parametrices also converge, since their transport equations depend continuously on the convergent coefficients. Equations and then permit dominated convergence in the subtracted Green construction. Hence $F_{1,P_j,1}$ and the renormalized stress converge locally, including at the smooth center.

It follows that each fixed-R_0 interior term in and is continuous. Choose R_0 so large that their uniform tail errors are small, and then let $j \to \infty$. This proves continuity of the finite constants and their linear combination.

par *Interpretive scope.* These are free-field quantities on a prescribed background, not Maxwell or Einstein source equations [11, 13]. par

Remark (What is not proved on the unrestricted class). Without the derivative bounds, the exterior estimates still control the tails after the profile-dependent constants have been retained. They do not bound those constants. We do not assert, without a separate argument, that every unrestricted finite-constant family is unbounded; we assert that the exterior theorem alone does not supply its interior bound.

Likewise, fixed finite-order C^k bounds do not provide uniform estimates for every arbitrarily high differentiated interior observable.

Projected invariant and equal-minimal-radius non-universality

Audited shape-dependent running

Put

$$Q_2[P] = \int_0^1 sP'(s)^2 P(s)^{-5}\mathrm{d}s, \qquad A_\alpha = \alpha^{-2} - \alpha.$$

Direct substitution of gives

$$\int \frac{r^2}{4} R^2 \mathrm{d}A = 2\pi\alpha Q_2[P],$$

$$\int R\Delta(r^2/4)\mathrm{d}A = \frac{4\pi}{3}A_\alpha - 2\pi\alpha Q_2[P],$$

$$M_B[P] := \int \frac{r^2}{4} B^t{}_t \mathrm{d}A = -\frac{2\pi}{9}A_\alpha + \frac{\pi\alpha}{6}Q_2[P].$$

Indeed $\Delta(r^2/4) = P^{-2} - sP'/(2P^3)$; integration of $P'P^{-4}$ gives the endpoint term. Integration by parts transfers the Laplacian from R.

At $\xi = 1/6$, define

$$r_{T,P} = -\frac{M_B[P]}{480\pi^2}, \qquad \kappa_P = 4s r_{T,P} = -\frac{sM_B[P]}{120\pi^2}.$$

Thus the weighted stress running is $\partial_{\log \mu} t = r_{T,P}$.

The contact comparison has dilation direction

$$(c, \tau)\text{longmapsto}\,(c, \tau) + \sigma_{\alpha} \log \lambda\,(s, h/4),$$

and invariant $hc - 4s\tau$ [22, 14]. For smooth cores we use only the projected moment

$$I_{P,\chi} = hc_{P,\chi} - 4st_{P,\chi}.$$

This does not assume that the complete smooth stress lies in a traceless two-contact module.

Theorem (Projected asymptotics with audited running). *At conformal coupling, with all finite conventions fixed,*

$$t_P^{\text{sharp}}(\varepsilon; \mu) = \frac{\sigma_{\alpha} h}{4} \ell + r_{T,P} \log\,(\mu\varepsilon) + C_{T,P} + O(M^{-1}),$$

$$I_{P,\chi}(\varepsilon; \mu) = -\frac{h}{4\pi} \log \ell + \kappa_P \log \frac{1}{\mu\varepsilon} + I_P^{\infty} + o(1).$$

These remainders are uniform on a uniformly regular class. A fixed smooth cutoff changes neither finite constant.

At fixed μ, there is no finite limiting projected invariant. Two profiles with different Q_2 satisfy

$$I_{P,\chi} - I_{Q,\chi} = -\frac{s\alpha}{720\pi}(Q_2[P] - Q_2[Q]) \log \frac{1}{\mu\varepsilon} + O(1).$$

At $\mu = \kappa/\varepsilon$, $\kappa > 0$, the raw invariant still diverges. Its separately subtracted finite remainder is

$$_P(\kappa) : = \lim_{\varepsilon \text{downarrow} 0} \left[I_{P,\chi}(\varepsilon; \kappa/\varepsilon) + \frac{h}{4\pi} \log \ell \right] = I_P^{\infty} - \kappa_P \log \kappa.$$

Proof. Stress scaling is

$$T_{\varepsilon, P, \mu\,t}{}^{t}(r) = \varepsilon^{-4} T_{1, P, 1\,t}{}^{t}(r/\varepsilon) + \log\,(\mu\varepsilon)\, \partial_{\log \mu} T_{\varepsilon, P\,t}{}^{t}(r).$$

Pair this identity with the weighted sharp cutoff and use , , and the local running. This proves the first line of .

Insert ; the geometric single logarithms cancel in $hc - 4st$.

A smooth-cutoff scalar change is $O(\ell^{-1})$ by ; the conformal weighted-stress change is $O(\ell^{-2})$ by . Hence the finite

constant is cutoff-independent. Equation follows from and . Since $h > 0$, the double logarithm diverges even when $\kappa_P = 0$. Substituting the adapted scale gives .

par *Interpretive scope.* These are free-field quantities on a prescribed background, not Maxwell or Einstein source equations [11, 13]. par

Exactly minimal radius

Define

$$\rho(P) = \inf\{b > 0 : P(s) = 1 \text{for every} s \geq b\}, \qquad P_{\min} = \{P : \rho(P) = 1\}.$$

For a monotone profile in the present class, $\rho(P) = 1$ is equivalent to $P(s) < 1$ for every $s < 1$.

The matching number on this class is not fixed by the endpoint radius. It is the variational quantity

$$d_P = \min_{u(1)=1} \int_0^1 \left(\frac{s}{P}|u'|^2 + \frac{2\xi P'}{P^2}|u|^2 \right) \mathrm{d}s.$$

For a smooth admissible variation $P_\theta = P + \theta H$, where H vanishes near the endpoints, its normalized positive minimizer u_P gives

$$\left.\frac{\mathrm{d}d_{P_\theta}}{\mathrm{d}\theta}\right|_{\theta=0} = -\int_0^1 \frac{H}{P^2}(su_P'^2 + 4\xi u_P u_P')\mathrm{d}s.$$

To verify this, differentiate the minimizing quadratic form. The variation of its minimizer contributes zero by the Euler equation. Integrating the H'-term by parts cancels the two $P'H$-terms and gives . Thus matching remains genuinely profile-sensitive after minimal-radius normalization.

For $0 < a < 1$, put $P_a(s) = P(s/a)$. Its nominal radius may still be called one, but its minimal radius is $a\rho(P)$. Exact exterior matching and scaling give

$$d_{P_a}^{-1} = d_P^{-1} + \log(1/a), \qquad Q_2[P_a] = Q_2[P], \qquad \kappa_{P_a} = \kappa_P,$$

$$I_{P_a}^{\infty} = I_P^{\infty} - \kappa_P \log a, \qquad \mathfrak{I}_{P_a}(\kappa) - \mathfrak{I}_P(\kappa) = -\kappa_P \log a.$$

The first formula follows by continuing $1 + d_P \log(s/a)$ through the conical collar. For the second line use the exact identity $g_{\varepsilon, P_a} = g_{a\varepsilon, P}$ in . These identities alone do not compare equal-minimal-radius profiles.

Endpoint anchoring closes the universality question

There exist minimal-radius profiles with $\kappa_P \neq 0$. For an explicit family choose a smooth step S strictly between zero and one on $(0, 1)$, constant outside, and flat at its endpoints. Define

$$P_T(s)^{-3/2} = \alpha^{-3/2} - (\alpha^{-3/2} - 1) S\Big(\frac{\log s + T}{T}\Big).$$

Then $P_T = \alpha$ for $s \le e^{-T}$, $P_T(s) < 1$ for $s < 1$, and

$$Q_2[P_T] = \frac{4(\alpha^{-3/2} - 1)^2}{9T} \int_0^1 S'(v)^2 \mathrm{d}v.$$

This follows from the substitution $v = (\log s + T)/T$. The quantity ranges over all positive values. For example choose

$$Q_2[P_T] = \frac{1}{2} Q_*, \qquad Q_* = \frac{4}{3\alpha} A_\alpha.$$

Equation then gives $M_B \neq 0$.

Theorem (Equal-minimal-radius non-universality). *Fix $\alpha \in (0, 1)$, $\xi = 1/6$, and the renormalization convention of the paper.*

For every $\kappa > 0$, the function

$$P \mathrm{longmapsto} \Im_P(\kappa)$$

is not constant on $P_{\min}$. In particular, there is no single core-adapted scale $\mu = \kappa/\varepsilon$ making the separately subtracted projected finite remainder universal among profiles of exactly equal minimal curvature-support radius.

More quantitatively, for every compact $K \subset (0, \infty)$, there are two smooth profiles $P, Q \in P_{\min}$, contained in one uniformly regular class, and a number $c > 0$, such that

$$|\Im_Q(\kappa) - \Im_P(\kappa)| \ge c \qquad (\kappa \in K).$$

Proof. Choose $P \in P_{\min}$ with $\kappa_P \neq 0$, as above, and fix $0 < a < 1$. Although $P_a \notin P_{\min}$, define

$$P_\delta = (1-\delta)P_a + \delta P, \qquad 0 < \delta \leq 1.$$

Each P_δ is smooth and nondecreasing. It equals α on one common positive central interval. For every $s < 1$, its $\delta P(s)$ contribution implies $P_\delta(s) < 1$, while it equals one for $s \geq 1$. Thus $\rho(P_\delta) = 1$.

For any fixed finite k, the family $\{P, P_a, P_\delta : 0 \leq \delta \leq 1\}$ is contained in some P_{k,L,s_*}. Moreover $P_\delta \to P_a$ in C^k. Theorem and continuity of Q_2 yield

$$\mathfrak{I}_{P_\delta}(\kappa) - \mathfrak{I}_P(\kappa) \to -\kappa_P \log a \neq 0.$$

The convergence is uniform on compact κ-sets, because is affine in $\log \kappa$ and both coefficients converge. Choose $\delta > 0$ so small that the error is less than $\frac{1}{2}|\kappa_P \log a|$ on K. Take $Q = P_\delta$ and $c = \frac{1}{2}|\kappa_P \log a|$. This proves and the theorem.

par *Interpretive scope.* These are free-field quantities on a prescribed background, not Maxwell or Einstein source equations [11, 13]. par

Remark (The exact advance over the parent). The parent correctly observed that rescaling by itself does not compare equal-minimal-radius cores. The additional ingredient here is continuity of the *complete* finite constants, including the interior. The anchored family supplies a legal equal-radius comparison.

This theorem does not evaluate three preselected finite constants. It does not exclude an isolated coincidence between two particular profiles at one scale. Nor does it give universality or non-universality under additional constraints such as a fixed positive lower bound on curvature in a prescribed outer collar.

The conformal trace sector and the missing full stress tensor

At conformal coupling the smooth real-scalar trace has the state-independent form

$$\Theta_{\varepsilon,P} := T^{\mu}{}_{\mu} = \frac{\mathrm{Riem}^2 - \mathrm{Ric}^2}{2880\pi^2} + b_{\mathrm{Box}} \Delta R = \frac{R^2}{5760\pi^2} + b_{\mathrm{Box}} \Delta R,$$

where b_{Box} is fixed by the finite curvature renormalization. The curvature-square coefficient is universal; the total-derivative coefficient is not [4, 5].

Define

$$S_0[P] = \int_0^1 \frac{P'(s)^2}{sP(s)^5}\mathrm{d}s, \qquad D_P = \frac{4\pi}{3}A_\alpha - 2\pi\alpha Q_2[P].$$

The notation D_P here denotes the geometric moment, not the momentum-dependent radial impedance $D_n(x)$.

Theorem (Trace-sector concentration through the finite contact). *Push $\Theta_{\varepsilon,P}$ using the physical core measure. For every fixed smooth profile,*

$$_\varepsilon, P = \frac{\alpha S_0[P]}{720\pi\varepsilon^2}\delta_{\mathscr{W}} + \left[\frac{\alpha Q_2[P]}{2880\pi} + b_{\mathrm{Box}}D_P\right]\Delta_0\delta_{\mathscr{W}} + O_{\mathscr{D}'}(\varepsilon^2).$$

The remainder is uniform on bounded sets of compactly supported tests and on a uniformly regular profile class.

In particular, if $\chi = 1$ on the shrinking core,

$$\langle\Theta_{\varepsilon,P},\chi\rangle = \frac{\alpha S_0[P]}{720\pi\varepsilon^2} > 0.$$

Consequently the raw pushed mixed stress tensor has no finite distributional limit without additional apex subtractions.

Proof. For a smooth transverse test, its angular average is

$$\bar{f}(r) = f(0) + \frac{r^2}{4}\Delta_0 f(0) + O(r^4 p_4(f)).$$

A direct calculation gives

$$\int R^2\mathrm{d}A = \frac{8\pi\alpha}{\varepsilon^2}S_0[P], \qquad \int\frac{r^2}{4}R^2\mathrm{d}A = 2\pi\alpha Q_2[P].$$

These yield the two displayed coefficients from the curvature-square term in ; its fourth moment is $O(\varepsilon^2)$.

For the total derivative, integrate by parts:

$$\int (\Delta R) f \mathrm{d}A = \int R \Delta f \mathrm{d}A.$$

Its constant moment vanishes, and its quadratic moment is D_P by . The angular Taylor remainder, after the physical Laplacian, has integral $O(\varepsilon^2 p_4(f))$. The profile derivative bounds make this uniform in the stated class. The same estimates integrate over compact worldsheet support.

Equation follows because both derivative contacts annihilate a test constant near the core. Also $S_0[P] > 0$ when $\alpha < 1$. For a mixed tensor the trace is contraction with the fixed identity tensor, so distributional convergence of all its pushed components would imply convergence of this trace. Equation excludes it.

par *Interpretive scope.* These are free-field quantities on a prescribed background, not Maxwell or Einstein source equations [11, 13]. par

This constructs only the trace sector. It does not determine which full tensor counterterms implement the required subtraction while preserving the chosen Ward representative. The test $r^2\chi/4$ used for t annihilates a zeroth-order tension contact. Its finite value cannot replace that missing tensor analysis.

The remaining full-stress obligations are:

1. a complete, declared tensor contact module;

2. subtraction of its power-divergent trace-bearing sector and any longitudinal tension;

3. a trace prescription and a compatible Ward representative;

4. all transverse tensor coefficients;

5. convergence on arbitrary compact tensor tests. Theorem identifies a necessary subtraction; it is not a full stress-limit theorem.

Twisted scalar audit: magnetic running and lowering

The supplied smooth connection

Fix the actual flux lift $0 < \beta < 1$, unit charge, and

$$D = \nabla + \mathrm{i}A, \qquad A = \beta a(r/\varepsilon)\mathrm{d}\varphi.$$

Assume $a = 1$ for $s \geq 1$, $0 \leq a \leq 1$, and near zero

$$a(s) = s^2 b(s^2)$$

with smooth b. This condition, stronger than an unspecified $O(s^2)$, ensures Cartesian smoothness of the one-form. Its curvature is $F = \mathrm{d}A$, with $\int F = 2\pi\beta$.

The angular radial potential is

$$\frac{P(n + \beta a)^2}{\alpha^2 s}.$$

It is nonnegative, so the impedance remains nonnegative for $\xi \geq 0$. The exterior orders are $\nu_n = |n + \beta|/\alpha \geq \nu_* > 0$. Theorem and therefore hold. There is no zero-order logarithmic channel, but local geometric and magnetic running can still occur.

This is compatible with, but logically separate from, the twisted scalar Friedrichs operator selection in the supplied I-B-VP-5-Ex2 [15]. No Dirac conclusion is inferred from the present scalar heat coefficients.

Complex determinant and local variations

For one complex scalar take

$$F = \langle \Phi^\dagger \Phi \rangle, \qquad J^a = \mathrm{i}(\Phi^\dagger D^a \Phi - (D^a \Phi)^\dagger \Phi).$$

The Wick square is one covariance component. The complex stress and determinant have two real degrees of freedom.

The connection curvature in the Laplace-type formula is

$$\Omega_{ab} = [D_a, D_b] = \mathrm{i}F_{ab}.$$

Thus

$$a_{2,\mathrm{mag}} = \frac{1}{12}\Omega_{ab}\Omega^{ab} = -\frac{1}{12}F_{ab}F^{ab}.$$

For the complex determinant,

$$\partial_{\log\mu} W_{\mathbb{C}}\big|_{\mathrm{mag}} = c_{\mathrm{mag}}\int F^2 \mathrm{d}V, \qquad c_{\mathrm{mag}} = \frac{1}{96\pi^2}.$$

Variation of the positive Euclidean matter action gives

$$J^a = -\frac{1}{\sqrt{g}}\frac{\delta W_{\mathbb{C}}}{\delta A_a}, \qquad \mathrm{T}^E_{ab} = \frac{2}{\sqrt{g}}\frac{\delta W_{\mathbb{C}}}{\delta g^{ab}}.$$

The chosen connection sign is essential for the current formula. The general local current ambiguity is discussed in the charged Hadamard treatment [9].

Theorem (Audited local magnetic running). *In the convention ,*

$$\partial_{\log\mu} J^a\big|_{\mathrm{mag}} = \frac{1}{24\pi^2}\nabla_b F^{ba},$$

$$\partial_{\log\mu} T_{ab}\big|_{\mathrm{mag}} = \frac{1}{96\pi^2}\left(4F_{ac}F_b{}^c - g_{ab}F^2\right).$$

For the static transverse flux,

$$\partial_{\log\mu} T^t{}_t\big|_{\mathrm{mag}} = -\frac{F^2}{96\pi^2}.$$

The local running obeys the background-field Ward relation

$$\nabla^a(\partial_{\log\mu} T_{ab})_{\mathrm{mag}} = F_{ba}(\partial_{\log\mu} J^a)_{\mathrm{mag}}.$$

Proof. Gauge variation gives

$$\delta\int F^2 \mathrm{d}V = -4\int \nabla_b F^{ba}\delta A_a \mathrm{d}V.$$

Equations and give .

Metric variation at fixed covariant A gives

$$\frac{1}{\sqrt{g}}\frac{\delta}{\delta g^{ab}}\int F^2 \mathrm{d}V = 2F_{ac}F_b{}^c - \frac{1}{2}g_{ab}F^2,$$

proving . The longitudinal mixed component has no first term. Finally, the Bianchi identity gives

$$\nabla^a\left(F_{ac}F_b{}^c - \tfrac{1}{4}g_{ab}F^2\right) = F_{ba}\nabla_c F^{ca},$$

which proves .

par *Interpretive scope.* These are free-field quantities on a prescribed background, not Maxwell or Einstein source equations [11, 13]. par

Proposition (Independent constant-field spectral check). *For a unit-charge scalar in flat four-dimensional Euclidean space with constant transverse magnetic field B,*

$$K_B(u; X, X) = (4\pi u)^{-2}\frac{Bu}{\sinh(Bu)} = (4\pi u)^{-2}\left(1 - \frac{B^2u^2}{6} + O(u^4)\right).$$

Since $F^2 = 2B^2$*, this verifies* $a_{2,\mathrm{mag}} = -F^2/12$.

Proof. The transverse Landau levels are $(2m+1)|B|$, with degeneracy $|B|/(2\pi)$ per unit area. Their heat sum is $|B|/[4\pi\sinh(|B|u)]$. Multiplication by the two longitudinal free heat factors gives the formula. Its Taylor expansion proves the claim.

This is a spectral normalization check, not an executed smooth-core current computation.

Reference versus physical lowering

Let

$$R^i = \varepsilon^i{}_j x^j, \qquad \varepsilon = \begin{pmatrix} 0 & -1 \\ 1 & 0 \end{pmatrix}, \qquad R = \partial_\varphi.$$

Auxiliary Cartesian lowering gives the smooth one-form

$$\theta_0 = R_i^0 \mathrm{d}x^i = r^2 \mathrm{d}\varphi.$$

Physical core lowering gives

$$\theta_{g} = g_{ij} R^{j} \mathrm{d}x^{i} = \alpha^{2} r^{2} \mathrm{d}\varphi = \alpha^{2} \theta_{0}.$$

The factor is independent of P, because the rotational vector is tangential.

Define the reference-lowered first-jet projection, with its regular comparison part subtracted when appropriate, by

$$w_{0} = -\frac{1}{2} \int \theta_{0,i} J^{i} \chi \mathrm{d}A,$$

and the unrescaled physically lowered projection by $w_{g} = -\frac{1}{2} \int \theta_{g,i} J^{i} \chi \mathrm{d}A$. They satisfy $w_{g} = \alpha^{2} w_{0}$. For the supported vector contact $w \varepsilon^{i}{}_{j} \partial_{j} \delta_{\mathscr{W}}$,

$$-\frac{1}{2} \mathrm{langlew} \varepsilon^{i}{}_{j} \partial_{j} \delta_{\mathscr{W}}, \theta_{0,i} \chi \rangle = w.$$

Thus the physically lowered test must be divided by α^{2} if it is intended to extract that same vector contact coefficient.

Proposition (Normalized magnetic moment running). *If $\chi = 1$ on the flux support, then*

$$\partial_{\log \mu} w_{0} \big|_{\mathrm{mag}} = \frac{\beta}{12 \pi \alpha} \int_{0}^{1} \frac{a'(s)}{P(s)} \mathrm{d}s,$$

$$\partial_{\log \mu} w_{g} \big|_{\mathrm{mag}} = \alpha^{2} \partial_{\log \mu} w_{0} \big|_{\mathrm{mag}},$$

$$\partial_{\log \mu} t \big|_{\mathrm{mag}} = -\frac{\beta^{2}}{96 \pi \alpha} \int_{0}^{1} \frac{s\, a'(s)^{2}}{P(s)} \mathrm{d}s.$$

Here t uses the same $r^{2}/4$ weight as before, but is the complex-scalar longitudinal moment.

Proof. Antisymmetry and integration by parts give

$$-\frac{1}{2} \int \theta_{0,i} \nabla_{j} F^{ji} \mathrm{d}A = \frac{1}{4} \int (\mathrm{d}\theta_{0})_{ji} F^{ji} \mathrm{d}A = \int r F^{r\varphi} \mathrm{d}A.$$

Since

$$F^{r\varphi} = \frac{\beta a'(r/\varepsilon)}{\varepsilon P(r/\varepsilon)^{2} \alpha^{2} r^{2}},$$

the last integral is

$$\frac{2\pi\beta}{\alpha}\int_0^1 \frac{a'}{P}\mathrm{d}s.$$

Combining with proves ; proves .

Finally,

$$\int \frac{r^2}{4}F^2\mathrm{d}A = \frac{\pi\beta^2}{\alpha}\int_0^1 \frac{s\,a'\wedge 2}{P}\mathrm{d}s.$$

Pair the longitudinal component of with the weight to obtain .

par *Interpretive scope.* These are free-field quantities on a prescribed background, not Maxwell or Einstein source equations [11, 13]. par

Scope boundary (The retained charged residual). The local coefficient, its sign, determinant multiplicity, and lowering conversion are established in the conventions displayed here. The complete finite w has not been evaluated. Neither has a full charged three-contact stress/current completion or its invariant classification.

In particular, a regular current coefficient quoted as a physically lowered angular component in I-B-VP-0 must be converted before it is combined with w_0 [21]. The present calculation does not silently identify different current, density, or contact conventions. The scalar flux-core operator theorem of [15] supplies no missing interior one-point normalization.

Operator selection and programme boundaries

Operator convergence is not contact convergence

The supplied I-B-VP-5 record gives untwisted Friedrichs strong-resolvent selection in its stated monotone core class [16]. The complete supplied flux-core companion proves its twisted scalar selection by a magnetic-form argument and its Dirac selection by a separate first-order graph argument [15]. Neither statement normalizes the moments of this paper.

The logical gap cannot be filled by strong resolvent convergence alone. On $\ell^2(\mathbb{N})$, let $B_n = I$ except for eigenvalue

2^{-8n} in the n-th basis direction. Then $B_n \to I$ strongly in the resolvent sense. For $v_j = 2^{-j}$,

$$\text{langlev}, B_n^{-1/2} v\rangle \geq 2^{2n} \to \infty.$$

For fixed nonreal or negative resolvent parameter, the resolvent difference applied to f is bounded by $C|f_n| \to 0$, proving the first assertion. This is a logical counterexample, not a physical core model.

Thus operator selection, infrared covariance control, coincidence renormalization, and thin-core contact limits remain distinct statements. The interior theorem here supplies explicit additional estimates; it is not a consequence of the operator limit.

par *Interpretive scope.* These are free-field quantities on a prescribed background, not Maxwell or Einstein source equations [11, 13]. par

Characteristic products are not imported

The extensive supplied XIV material concerns a different nonlocal characteristic composition. Its retained target contains the complete sum $K_{47} + K_5 + K_6 + K_8 + K_9$. No exterior scalar mode estimate here defines its product with the grouped quantum cross-lines.

The version-qualified endings are kept distinct:

• XIV-H-v1 ends with its H2 amplitude obligation unresolved.

• XIV-I-v1 constructs its named I2 internal amplitude; this advance is not withdrawn. Its full I3 joint map is unresolved.

• XIV-J-v1 retains the unresolved J2 full-map obligation. Its supported-sector off-locus reference is not that map.

• The supplied technical XV-B-v2 interfaces do not supply a graph-specific full characteristic lift or quantum admission. Existence of that technical manuscript does not open the separate downstream quantum-admission exit. These are documentary boundaries of the supplied records [27, 28, 29, 30]; they are not premises of the heat-kernel or continuity proofs.

No claim is made to construct the missing joint map, joint section, transverse estimates, contact compatibility, output wavefront theorem, or admitted quantum coefficient.

Unchanged standing boundaries

The present release changes no unrelated programme status:

$$\text{nolinkurl}\, O.1 = NOT_C LOSED.$$

XVI-N and its undisposed Route B are not discharged [31]. XV-B is not opened as a downstream quantum-admission exit. Problems 1, 2, and 8 remain unfinished. No other problem number is advanced by the present calculation.

The historical H2 ending is not promoted to a current denial of XIV-I's supplied I2 construction. The unresolved I3/J2 full-map statements are retained. Separately scoped results elsewhere are neither verified nor revoked.

No spacetime emergence, Maxwell or Einstein backreaction, interacting Ward identity, Page curve, cosmological state, or ultraviolet completion is constructed. OpenAI catalogue entries are external records, not analytical premises.

Reliability ledger and mathematical release

Evidence and retained obligations

The XVI-O1-ST integrity distinction is used as a reporting discipline [25]:

\texttt{ANALYTIC} A displayed proof on its stated domain, with its identified standard smooth-background analytical inputs.

\texttt{SUPPLIED} A version-qualified source statement or historical report not independently executed here.

\texttt{UNCERTIFIED} A numerical approximation without the complete artifacts and justified total error bounds needed for certification.

\texttt{OPEN} An analytical construction or comparison not supplied. An open analytical assertion is not an

uncertified numerical term.

toprule Target Ending Exact scope or retained residual

midrule endfirsthead toprule Target Ending Exact scope or retained residual

midrule endhead bottomrule endfoot

Real-scalar stress-running normalization ANALYTIC Determinant sign, a_{1}, factor 2, inverse-lapse variation, and Lorentzian mixed component agree.

Constant-curvature normalization check ANALYTIC Explicit sphere heat spectrum; not a numerical core run.

Differentiated exterior sums ANALYTIC All fixed endpoint orders with the losses in ; compact exterior domain.

Small positive Bessel orders ANALYTIC Lemma covers twisted orders below one.

Complete scalar finite constant ANALYTIC Interior included; sharp and smooth cutoff conventions distinguished.

Interior uniformity ANALYTIC Only on the unit-core class $P_{k,L,s_{*}}$, $k \geq 20$, with fixed state and finite renormalization data.

Continuity of complete finite parts ANALYTIC Uniform parametrix, infrared bounds, and exterior tails; Theorem .

Equal-minimal-radius universality Excluded Endpoint anchoring and continuity prove Theorem ; no numerical interior values are claimed.

Three preselected finite constants OPEN No numerical evaluation; isolated profile coincidences are not classified.

Trace sector ANALYTIC Power contact and finite $\Delta_{0}\delta_{\mathscr{W}}$ term, with fixed b_{Box}.

Full renormalized tensor limit OPEN Complete tensor contact subtraction, tension, Ward representative, and arbitrary tensor tests remain.

Magnetic heat coefficient and running ANALYTIC Unit charge, $D = \nabla + \mathrm{i}A$, complex determinant; constant-field spectral check included.

Current lowering conversion ANALYTIC $w_g = \alpha^2 w_0$; unconverted regular-current coefficients are not imported.

Finite charged-current contact OPEN No full interior finite *w* or charged invariant classification.

Independent numerical interior stress/current check Not executed No raw data, truncation experiment, or numerical certificate is reported.

Other companion conclusions SUPPLIED Version-qualified comparisons only; no full-corpus audit or external quantum admission.

Finite tests and coverage

Proposition (Class-relative finite coverage). *If an independently proved finite-dimensional space Z contains every admissible residual for a claim, then a linear probe map* $E : Z \to \mathbb{R}^m$ *detects every residual if and only if it is injective. Without the containment and injectivity hypotheses, vanishing finite tests need not prove a distributional identity.*

Proof. Within *Z*, exact detection is $\ker E = \{0\}$. Neither selecting finitely many probes nor observing their vanishing proves that all admissible residuals lie in *Z*.

This is the restricted coverage distinction of Oai282-A [26, prop:coverage]. No truncated mode fit is used to establish the quantified exterior theorem or the profile continuity theorem.

Release theorem

Theorem (I-B-VP-5-Ex1-B-v2 release). *For the free spectral scalar prescription, density identification, and fixed renormalization data stated here:*

1. *the local neutral stress running is , with independently checked lapse and constant-curvature normalizations;*

2. *all fixed differentiated exterior mode sums have the explicit summable majorants of Section , including the twisted small-order sector;*

3. *the complete scalar and conformal weighted-stress finite constants have the convergent representations and ;*

4. *these constants are uniformly bounded and continuous on the uniformly regular unit-core subclass;*

5. *projected fixed-scale non-universality is retained, and no core-adapted scale makes the separately subtracted finite remainder universal even on the exactly equal-minimal-radius class;*

6. *the conformal trace sector has the expansion , which obstructs a raw full mixed-stress limit without further contact subtractions;*

7. *the magnetic local running, determinant multiplicity, and reference/physical lowering conversion are –. The full stress-tensor completion, finite charged-current contact, specific numerical interior constants, and all unrelated characteristic quantum products remain outside this release.*

Proof. Item (i) is Theorem and Proposition . Item (ii) is Lemmas and , the explicit table, and Theorem . Item (iii) is Theorem and –. Item (iv) is Lemma and Theorem . Item (v) is Theorems and . Item (vi) is Theorem . Item (vii) is Theorem and Proposition , with the spectral check in Proposition . The final sentence records constructions not supplied by those proofs.

par *Interpretive scope.* These are free-field quantities on a prescribed background, not Maxwell or Einstein source equations [11, 13]. par

Conclusion. The exterior theorem and the interior theorem have different hypotheses. Their combination supplies a stable finite-part functional and permits a genuine equal-minimal-radius non-universality argument. It does not produce a canonical apex observable.

The stress-running audit and the magnetic lowering conversion remove convention ambiguities inside the declared model. The trace expansion simultaneously explains why two projected moments cannot be promoted to the full tensor. None of these mathematical distinctions identifies a finite Enter report with a continuum acquisition, or a prescribed one-point function with a source of spacetime.

appendix

Correction log and verification declarations

Corrections relative to the supplied proposal

1. Euclidean time derivatives. A minus sign before a symbol named $C_{\tau\tau}$ is ambiguous unless that symbol is defined. For the actual Euclidean coincidence derivative the sign in is plus. The continued Lorentzian derivative has the opposite sign.

2. Uniform interiors. Exterior summability does not establish uniform local renormalized observables. The additional derivative-bounded unit-core class and both ultraviolet and infrared arguments are necessary for the theorem released here.

3. Equal-radius question. The nominal-radius rescaling identity is not itself an equal-radius proof. Continuity plus the endpoint-anchored family is the additional argument. Universal finite-part selection is excluded, while numerical comparisons and isolated coincidences remain unevaluated.

4. Magnetic lowering. The reference-lowered vector-contact projection and the unrescaled physically lowered projection differ by α^2. Both are legitimate declared moments; they are not the same normalization.

5. Smooth connection at the center. The condition $a(s) = s^2 b(s^2)$ near zero is used to ensure Cartesian smoothness. A bare asymptotic statement without the radial smoothness condition is insufficient.

6. Trace versus complete tensor. A trace-sector expansion is proved, not a full tensor completion. Its power divergence is not removed by the weighted longitudinal moment.

7. Residual evidence. No unexecuted numerical check is called a discrepancy, and no missing analytical proof is disguised as an uncertified error term.

8. Historical gates. H2 is an unresolved ending of XIV-H-v1. The supplied later I2 construction is retained; unresolved I3/J2 full maps and the unopened downstream admission exit are different statements.

Analytical inputs

The ordinary smooth heat-parametrix construction and its finite-order local estimates, local parabolic regularity, and smooth-background Wick renormalization are explicitly identified standard inputs. Their hypotheses are checked on P_{k,L,s_*}. No such construction is applied directly to a singular limiting metric.

The exterior estimates, matching variation, finite-constant continuity argument, endpoint anchoring, and lowering conversion are proved in the text. The choice of $k \geq 20$ is deliberately sufficient rather than optimal.

Source and execution scope

The philosophy page was consulted as introductory background. Direct retrieval of the I-B and I-B-G-S browse endpoints was unavailable through the consulted interface; their use rests on the supplied version-qualified material. The XVI-O1-ST record and the cited primary analytical sources were consulted for attribution and conventions. No exhaustive corpus search, priority audit, or independent verification of all Cheung manuscripts is asserted.

No numerical integration, symbolic-program run, independent rebuild, successful pdfLaTeX execution, proof-assistant verification, referee approval, or journal acceptance is asserted. The source is intended for two pdfLaTeX passes.

This file has an internal bibliography and no external figure, data, code, or bibliography-file dependency. The author name follows the supplied series. Submission requires the author's approval of the mathematics, attribution, and scope.

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[18] N. Cheung, *I-B-G-S: Intrinsic spin-1 and spin-2 Green operators on the prescribed conical spacetime*, version 2, supplied interface excerpts, Enter Calculus Companion. Source-test gauge compatibility and non-transfer boundaries only. No unavailable Bernoulli-jet or physical tensor-core theorem is a premise. https://enter.grok.me/browse/I-B-G-S. Companion record; not a Scholar-indexed journal article.

[19] N. Cheung, *I-B-VP: Vacuum polarization of the Friedrichs ground state on a prescribed conical spacetime*, version 3 and version-qualified predecessor passages, Enter Calculus Companion. Spectral, regular-part, and finite-reporting comparison; no one-point convergence follows merely from operator convergence. https://enter.grok.me/browse/I-B-VP. Companion record; not a Scholar-indexed journal article.

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