I-B-VP-5-Ex1 Smooth curvature cores, convergent exterior mode sums, and renormalized apex moments

A proof-level improvement and corrective audit of I-B-VP-CORE within the prescribed conical Enter framework

Ngo Cheung

18 October 2026

Abstract

We study free scalar fields on smooth, rotationally symmetric curvature cores joined to a prescribed conical exterior. The principal analytical advance is a profile-independent reflection estimate that proves absolute convergence of the exterior Euclidean mode sum and every fixed differentiated sum on compact exterior sets. This removes the exterior mode-representation hypothesis of I-B-VP-CORE for the smooth nonnegative-curvature class.

For positive curvature coupling, we derive the complete smooth-core scalar moment, including its interior contribution, and strengthen the logarithmic-channel expansion to a uniform $O(M^{-4})$ remainder on explicitly nondegenerate profile families:

$$c_P^{\mathrm{sharp}}(\varepsilon;\mu) = 2\pi\alpha \log(1/\varepsilon) - \frac{(\xi - \frac{1}{6})K}{8\pi^2}\log(\mu\varepsilon) - \frac{1}{4\pi}\left[\log M - \frac{1}{2M^2} + \frac{2-\zeta(3)}{3M^3}\right] + C_P + O(M^{-4}), \qquad M = \log(1/\varepsilon) + 1 + d_P^{-1}.$$

Here $K = 4\pi(1-\alpha)$, and the superscript specifies a sharp radial moment. Smooth test cutoffs produce additional, explicit inverse-M terms; thus the absence of a $1/M$ term is not cutoff-independent.

An independent heat-coefficient and lapse-variation calculation fixes the conformal stress running and its shape dependence. The resulting dilation-invariant *projected moment* diverges at fixed renormalization scale. A core-adapted scale does not remove its universal log log divergence. After that common divergence is subtracted,

an exact profile-rescaling identity obstructs universality on the stated nominal-radius class, while the equal-minimal-radius finite-part problem remains open. We give convergent representations and truncation bounds for the finite constants rather than unexecuted numerical values.

The potential significance is a controlled separation of operator selection, exterior scattering, full scalar contact concentration, and stress-moment normalization. No priority is claimed for cosmic-string core effects, Bessel matching, or heat-kernel renormalization. No full stress-distribution limit, interacting defect theory, or gravitational source equation is constructed.

Bibliographic record

Title	I-B-VP-5-Ex1 Smooth curvature cores, convergent exterior mode sums, and renormalized apex moments
Author	Cheung, Ngo
Publication date	2026/10/18
Institution	Enter
Report number	I-B-VP-5-Ex1-v2
Publisher	Cheung Ngo Medical Limited
First page	1
Last page	43
Language	en

Book edition note (t425u)

Standing Enter refusals, scope vocabulary (proved here/supplied input/conditional/ not constructed/not claimed), and Scholar bibliography policy are frozen in the companion chapter *I-B-VP Book Ledger* v1. O.1 remains NOT_CLOSED; XVI-N is not discharged; XV-B is not opened; no Host C, Page/island, or spacetime-emergence claim; wired=false; SOURCE_DATE_EPOCH=1792627200. This sheet is a companion manuscript record and is

not claimed to be Google Scholar-indexed. The published Enter Calculus article is cited only for finite-instrument / operational interface discussions, never as a PDE premise.

Book expansion: reflection before applications

Put the exterior reflection estimate before applications. Distinguish sharp moments, smooth-cutoff moments, and full interior finite constants. Wherever a finite stress constant appears: it is a **projected moment**, not a full stress-tensor limit.

Framework, attribution, and the corrections being made

A prescribed spacetime with a separate finite interface

The philosophy of Enter distinguishes destructive readout, a surviving quantum residual, an installed description, and a physical identification [9]. I-B prescribes a conical spacetime and equips it with a separate finite operational interface [10]. The metric, field equation, core profile, state prescription, and renormalization data used below are supplied continuum inputs. They are not extracted from a finite record.

For a calibrated clock basis $\{|\tau_j\rangle\}$, with A containing every surviving finite register, put

$$D_j = \langle \tau_j|_C \otimes I_A, \qquad \mathscr{M}(X) = \sum_j \text{lvertj}, w_j\rangle\text{langlej}, w_j|_R \otimes D_j X D_j^\dagger.$$

The words w_j include explicit failure and unused labels. The identities

$$\sum_j D_j^\dagger D_j = I_{CA}, \qquad \mathrm{Tr}_R \mathscr{M}(X) = \mathrm{Tr}_C X$$

follow by expansion in clock matrix units and remain valid with every untouched finite reference. A positive-probability branch has its actual conditional A-state; a zero branch is not assigned a normalized state. The output contains A, not a returned quantum C-factor. This is the I-B/En09-A supplied-report interface [10, thm:instrument] [11]. It neither measures an unknown continuum contact nor selects a profile, self-adjoint domain, or scale.

A conserved stress tensor is not thereby a source of the cone. The non-sourcing distinctions of I-B prop:not-source and X-A-ST prop:noeinstein are retained [10, 12]. No Einstein equation, backreaction, or spacetime emergence is inferred.

Source scope

The immediate source being improved is the complete supplied I-B-VP-CORE.tex [13]. The complete supplied I-B-VP-1 version 3, Ex1, and Ex2 are used as version-qualified comparison records [15, 16, 17]. I-B-VP-5 supplies the operator-selection and contact-separation context [14]. The zero-channel matching statements required here are derived again.

I-B-G, I-B-VP, I-B-VP-AB, and I-B-VP-0 identify the regular-host spectral and holonomy conventions [18, 20, 21, 22]. The identifier I-B-G-S was named in the plan, but no complete version-qualified theorem from it was supplied. No Bernoulli-jet or contraction theorem is attributed to it as an analytical premise [19].

The published core-effect and renormalization literature provides context and standard smooth-background inputs [1, 2, 3, 5, 6, 7]. Every exterior mode estimate used below is proved explicitly.

Essential debugging decisions

The following distinctions change the formulation of CORE:

1. The full scalar moment has single-logarithmic geometric and local-running terms as well as the nonlocal double logarithm. It is not simply a pure $\log\log$ divergence.

2. The inverse-M coefficients depend on the radial test cutoff. CORE's displayed cancellation of $1/M$ holds for a particular sharp radial normalization.

3. The stress quantities computed here are projected moments. A full smooth-core stress has a tension contribution and a trace anomaly and need not lie in the traceless contact class of I-B-VP-1.

4. An exterior mode sum is independent of μ and cannot independently measure a running term supported inside the core.

5. At $\mu = \kappa/\varepsilon$, the universal log log term still diverges. Finite-part universality can be asked only after that divergence is separately subtracted.

6. A profile with an unused conical collar has a nominal radius different from its minimal support radius. Rescaling it changes the matching number at the nominal boundary.

The results below are statements about these corrected objects, not an unqualified ratification of every claim or historical numerical report in CORE.

Smooth cores, density conventions, and renormalization

Geometry

Fix $0 < \alpha < 1$. A profile P is smooth, nondecreasing, satisfies

$$P(s) = \alpha \quad (0 \le s \le s_0), \qquad P(s) = 1 \quad (s \ge 1), \qquad 0 < s_0 < 1,$$

and defines

$$g_{\varepsilon,P} = -\mathrm{d}t^2 + \mathrm{d}z^2 + P(r/\varepsilon)^2 \mathrm{d}r^2 + \alpha^2 r^2 \mathrm{d}\varphi^2.$$

The center is smooth: there the transverse metric is $\alpha^2(\mathrm{d}r^2 + r^2\mathrm{d}\varphi^2)$. The exterior $r \ge \varepsilon$ is the exact cone.

With sphere-positive curvature convention,

$$R_{\varepsilon,P} = \frac{2P'(s)}{\varepsilon^2 s P(s)^3}, \qquad \mathrm{d}A_{\varepsilon,P} = \alpha r P(r/\varepsilon)\,\mathrm{d}r\,\mathrm{d}\varphi, \qquad s = r/\varepsilon.$$

Consequently

$$K := \int R_{\varepsilon,P}\,\mathrm{d}A_{\varepsilon,P} = 4\pi(1-\alpha).$$

The completion has Cartesian smooth structure $x = (r\cos\varphi, r\sin\varphi)$ and $\mathscr{W} = \mathbb{R}^{1,1}_{t,z} \times \{0\}$. For comparison cone distributions use

$$\mathrm{d}\Omega_0 = \alpha\,\mathrm{d}^2y\,\mathrm{d}^2x, \qquad \langle \delta_{\mathscr{W}}, f \rangle = \int f(y,0)\,\mathrm{d}^2y, \qquad \sigma_\alpha = 2\pi\alpha.$$

A smooth-core scalar is pushed to this completion by

$$\langle \mathscr{F}_{\varepsilon,P}, f\rangle = \int F_{\varepsilon,P}(y,x) f(y,x)\, \mathrm{d}^2 y\, \mathrm{d}A_{\varepsilon,P}.$$

Thus its generalized-function coefficient relative to $\mathrm{d}\Omega_0$ is $P(r/\varepsilon)F_{\varepsilon,P}$. One must not interchange the physical core density and the cone density at the center.

Field and state

Except in Section , the field is one real massless scalar with $\xi \geq 0$,

$$(-\mathrm{Box} + \xi R)\varphi = 0, \qquad q = \xi - \frac{1}{6}.$$

The spatial operator is the nonnegative closure of its smooth-core quadratic form. The state is the zero-mean quasifree *spectral* ground-state prescription. No classification of every algebraic ground state is asserted.

For each fixed smooth core the metric is uniformly elliptic in the Cartesian identification, its density is bounded above and below, and $\xi R \geq 0$. The Euclidean heat kernel has the usual large-time Gaussian bound. One direct route is the Euclidean Nash inequality, transferred by these uniform ellipticity and density bounds, followed by the standard semigroup argument; adding the nonnegative potential does not worsen the diagonal upper bound. In four Euclidean dimensions this gives $O(u^{-2})$ at large heat time. It makes the massless covariance infrared finite on compact tests. Local parabolic regularity gives the corresponding fixed-core derivative estimates on compact sets.

We use the standard local smooth-background heat/Hadamard renormalization construction [5, 6, 7]. It supplies smooth renormalized one-point functions on each fixed smooth core. It is not being applied to a singular limiting metric.

A fixed compatible scale convention

Write the local Euclidean heat expansion as

$$K_A(u; X, X) \sim \frac{1}{(4\pi u)^2}\left[1 + a_1 u + a_2 u^2 + \cdots\right], \qquad a_1 = -qR.$$

For example, the scalar convention can be fixed by

$$F_{\mu}(X) = \lim_{\delta \,\mathrm{downarrow}\, 0} \left[\int_{\delta}^{\infty} K_{A}(u; X, X)\, \mathrm{d}u - \frac{1}{16\pi^{2}\delta} + \frac{a_{1}(X)}{16\pi^{2}} \log\left(\mu^{2}\delta\right) \right] + \gamma R(X),$$

with γ fixed once and for all. The stress is defined in a compatible local variational scheme, with its finite dimension-four curvature counterterms also fixed. The corresponding relative Euclidean effective action has

$$\frac{\partial W}{\partial \log \mu} = -\frac{1}{16\pi^{2}} \int a_{2}\, \mathrm{d}V.$$

Only compactly supported metric variations are needed in this equation on the noncompact geometry.

Our mixed longitudinal stress convention is the analytic continuation of $2(\sqrt{g})^{-1}\delta W/\delta g^{\tau\tau}$. It agrees with $T^{t}_{\ t} = -\rho$ in Lorentzian signature.

No new dimensionful finite counterterms are introduced. Different fixed admissible finite curvature constants change the finite constants below, not the logarithmic running coefficients.

Radial matching and profile-independent reflection bounds

Let $x = p\varepsilon$, $p > 0$, and $\nu_{n} = |n|/\alpha$. The regular interior radial solution satisfies

$$\left(\frac{s}{P} U'_{n,x} \right)' = \left[\frac{n^{2} P}{\alpha^{2} s} + 2\xi \frac{P'}{P^{2}} + x^{2} s P \right] U_{n,x}.$$

It is proportional to $s^{|n|}$ near zero for $n \neq 0$, and is normalized by $U_{0,x}(0) = 1$ for $n = 0$. Put

$$D_{n}(x) = \frac{U'_{n,x}(1)}{U_{n,x}(1)}.$$

The regular solution is positive and increasing, so $D_{n}(x) \geq 0$.

Matching the exterior regular solution $I_{\nu_{n}}(pr) - b_{n}(x) K_{\nu_{n}}(pr)$ gives

$$b_{n}(x) = \frac{D_{n}(x) I_{\nu_{n}}(x) - x I'_{\nu_{n}}(x)}{D_{n}(x) K_{\nu_{n}}(x) - x K'_{\nu_{n}}(x)}.$$

The denominator is strictly positive.

Lemma (Reflection bounds). *For every smooth profile in Section , every* $\xi \ge 0$, *and* $x > 0$,

$$-\frac{I_1(x)}{K_1(x)} \le b_0(x) \le \frac{I_0(x)}{K_0(x)}, \qquad |b_0(x)| \le \frac{I_0(x)}{K_0(x)},$$

$$|b_n(x)| \le \left(1 + \frac{x}{\nu_n}\right)\frac{I_{\nu_n}(x)}{K_{\nu_n}(x)}, \qquad n \ne 0.$$

These bounds do not involve derivatives of P.

Proof. As a function of $D \ge 0$, the quotient in is increasing: its derivative has the positive Wronskian numerator

$$x(I_\nu' K_\nu - I_\nu K_\nu') = 1.$$

Its endpoints are $-I_\nu'/(-K_\nu')$ and I_ν/K_ν. For $\nu = 0$, use $I_1 \le I_0$ and $K_1 \ge K_0$. For $\nu > 0$,

$$xI_\nu' \le (\nu + x)I_\nu, \qquad -xK_\nu' \ge \nu K_\nu,$$

which proves . The recurrence and positivity properties used here are classical [8, Sections 10.29 and 10.37].

Lemma (A summable exterior majorant). *For* $\nu > 0$, $x > 0$, $t, t' > 1$,

$$|b_\nu(x)K_\nu(xt)K_\nu(xt')| \le \frac{1 + x/\nu}{2\nu}(tt')^{-\nu/2}e^{-x(t + t' - 2)/2},$$

whenever b_ν *satisfies . For* $\nu = 0$,

$$|b_0(x)K_0(xt)K_0(xt')| \le I_0(x)K_0(x)e^{-x(t + t' - 2)}.$$

Here $I_0(x)K_0(x) = O(1 + |\log x|)$ *at zero and* $O(x^{-1})$ *at infinity. Every fixed number of exterior radial or angular derivatives is bounded by the same majorants times a fixed polynomial in* ν, x, t, t', *with the appropriate inverse radial factors.*

Proof. The Wronskian and $xI_\nu'/I_\nu \ge \nu$, $-xK_\nu'/K_\nu \ge \nu$, imply

$$I_\nu(x)K_\nu(x) \le \frac{1}{2\nu}.$$

Also

$$\frac{K_\nu(xt)}{K_\nu(x)} \le t^{-\nu}.$$

The integral representation $K_\nu(x) = \int_0^\infty e^{-x\cosh v}\cosh(\nu v)\mathrm{d}v$ gives $-K_\nu'/K_\nu \ge 1$, hence the alternative bound $K_\nu(xt)/K_\nu(x) \le e^{-x(t-1)}$. Taking the geometric mean of these two bounds at each endpoint proves . The zero-channel statement follows directly from .

For the positive orders occurring at $n \neq 0$, $\nu > 1$, and

$$-uK_\nu'(u) \le (\nu + u)K_\nu(u).$$

Higher Euler derivatives are controlled using $(u\partial_u)^2 K_\nu = (u^2 + \nu^2)K_\nu$. For $\nu = 0$, the corresponding fixed-order estimates follow from its small-argument expansion and the large-argument expansion. Angular differentiation gives powers of n. This proves the derivative assertion.

The exterior mode representation is a theorem

Let $y = (\tau, z)$ be the two Euclidean longitudinal coordinates. For exterior endpoints $r, r' > \varepsilon$, define

$$\delta G_\varepsilon(X, X') = -\frac{1}{4\pi^2\alpha}\sum_{n\in\mathbb{Z}} e^{in(\varphi-\varphi')}\int_0^\infty p\,\mathrm{d}p\,J_0(p|y-y'|)b_n(p\varepsilon)K_{\nu_n}(pr)K_{\nu_n}(pr').$$

Theorem (Exterior representation and absolute differentiated convergence). *For each smooth profile and $\xi \ge 0$:*

1. *is the difference between the smooth-core spectral Euclidean covariance and the Friedrichs conical covariance at exterior endpoints;*

2. *the sum, integral, and every fixed differentiated sum are absolutely convergent, locally uniformly for $r, r' > \varepsilon$;*

3. *their bounds on sets $r, r' \ge (1+\eta)\varepsilon$, $\eta > 0$, can be chosen independently of the profile and of $\xi \ge 0$, apart from explicit coefficients in an observable's point-splitting operator. In particular,*

$$\delta F_\varepsilon(r) = -\frac{1}{2\pi^2\alpha}\sum_{n=0}^{\infty}{}'\int_0^\infty p\,b_n(p\varepsilon)K_{n/\alpha}(pr)^2\,\mathrm{d}p,$$

and the analogous stress differences are obtained by applying their regular-exterior point-splitting operators term by term. The prime halves the zero term.

Proof. Fourier transform in (τ, z) and expand in normalized angular modes. For each n, p, the radial Green function is fixed by the regular interior solution, the decaying exterior solution $K_{\nu_n}(pr)$, and their Wronskian. Equation therefore gives

$$I_{\nu_n}(pr_<)K_{\nu_n}(pr_>) - b_n(p\varepsilon)K_{\nu_n}(pr)K_{\nu_n}(pr').$$

The first term is the Friedrichs radial resolvent. The angular normalization and two-dimensional longitudinal Fourier inversion give .

For $n \neq 0$, use Lemma with $x = p\varepsilon$, $t = r/\varepsilon$, and $t' = r'/\varepsilon$. After any fixed derivatives, the majorant is a polynomial in $|n|$ and x, multiplied by

$$(tt')^{-|n|/(2\alpha)} e^{-x(t+t'-2)/2}.$$

The angular series and momentum integral both converge. At zero momentum the $p\,\mathrm{d}p$ measure is integrable. For $n = 0$, gives only a logarithm at zero momentum and exponential decay at infinity. Longitudinal derivatives give additional powers of p; the integral representation of J_0 bounds these derivatives without a singularity at $y = y'$.

These majorants justify the inverse transforms and all the stated interchanges. They also show that the correction is smooth across the regular exterior diagonal. The core and cone have the same local subtraction there, so the difference of their renormalized one-point functions is precisely the differentiated correction.

Corollary (Higher angular channels have integrable far tails). *For a unit core and $r \geq 2$, the contribution of $n \neq 0$ satisfies*

$$|\delta F_{\neq 0}(r)| \leq C r^{-2-1/\alpha}, \qquad |\delta T^{\mu}_{\nu,\neq 0}(r)| \leq C_\xi r^{-4-1/\alpha}$$

in a regular exterior orthonormal frame. The same assertion holds after a fixed number of Euler derivatives. The constants are profile-independent for fixed α, ξ.

Proof. In take $t = t' = r$, integrate $x\,\mathrm{d}x$, and sum the geometric series $r^{-|\eta|/\alpha}$ with its polynomial weights. Each stress derivative costs at most the corresponding power of r^{-1}, after the substitution $v = xr$. This gives . The exponent is sufficient here; no claim of an optimal higher-mode decay rate is needed.

Remark (Exactly what happened to hypothesis (M)). The exterior hypothesis (M) of CORE is removed for the smooth profile class: its scalar and stress representations and their differentiated convergence are now established. This theorem is not a uniform estimate at the matching circle for a singular limiting profile. Nor is it a numerical verification of the renormalized interior mode sum. The interior one-point functions are supplied by the ordinary smooth-background construction of Section .

The exterior split agrees with the form used in the finite-core literature [2, 3]. It is a free-field calculation on a prescribed metric, not an acquisition or source equation [10, 12].

Zero-channel matching and a uniform logarithmic expansion

The matching number

Let $d_P = D_0(0)$, and set

$$G = 2\xi(\alpha^{-1} - 1).$$

Lemma (Matching bounds and spectral perturbation). *For* $\xi > 0$,

$$0 < Gs_0^G \le d_P \le G, \qquad 0 \le D_0(x) - d_P \le \frac{x^2}{2}.$$

The zero-energy exterior solution is proportional to

$$1 + d_P \log(r/\varepsilon) = d_P \log(r/L_{\mathrm{eff}}), \qquad L_{\mathrm{eff}} = \varepsilon e^{-1/d_P}.$$

At small x,

$$b_0(x) = \frac{1}{A_P + \log(1/x)} + O\big(x^2(1 + |\log x|)^2\big), \qquad A_P = d_P^{-1} + \log 2 - \gamma_E.$$

Proof. At $x = 0$, put $w = sU'/[PU]$. The integrated equation and positivity show $0 \le w(s) \le G$. Thus $sU'/U = Pw \le G$, and

$$U(s)/U(1) \ge s^G \ge s_0^G$$

on the curvature support. Pairing the integrated equation with $U(1)^{-1}$ gives

$$d_P = \int_0^1 2\xi \frac{P' U(s)}{P^2 U(1)}\, \mathrm{d}s,$$

which proves the first bounds.

Normalize the regular zero-mode solution at $s = 1$. Its Dirichlet-to-Neumann value is the minimum, subject to $u(1) = 1$, of

$$\int_0^1 \left[\frac{s}{P}|u'|^2 + 2\xi \frac{P'}{P^2}|u|^2 + x^2 sP|u|^2 \right] \mathrm{d}s.$$

Comparison with the zero-energy minimizer, which lies between zero and one, gives

$$d_P \le D_0(x) \le d_P + x^2 \int_0^1 sP\, \mathrm{d}s \le d_P + x^2/2.$$

The exterior zero-energy equation gives . Finally insert $D_0(x) = d_P + O(x^2)$ and the Bessel expansions in .

Definition (A nondegenerate family for uniform remainders). Fix

$$0 < \alpha_- \le \alpha \le \alpha_+ < 1, \qquad 0 < \xi_- \le \xi \le \xi_+ < \infty, \qquad s_0 \ge s_- > 0.$$

Profiles may otherwise vary throughout the smooth monotone class. No upper curvature or C^k-profile bound is imposed. Equation then bounds d_P above and away from zero uniformly.

Uniform estimates for local interior values over this class are not asserted. The uniform remainder below concerns exterior tails and the contact expansion after its profile-dependent finite constant has been retained.

Mellin moments

Set

$$k_1 = 1 + \gamma_E - \log 2, \qquad M_P(r) = \log r + 1 + d_P^{-1}.$$

The standard product Mellin transform is

$$f(t) := \int_0^\infty x^{1+t} K_0(x)^2 \, \mathrm{d}x = \frac{\sqrt{\pi}\, \Gamma(1 + t/2)^3}{4\Gamma(3/2 + t/2)}.$$

It follows by inserting two integral representations of K_0 and performing a gamma and beta integral. The product formula is also recorded in [8, Section 10.32]. In particular,

$$f(0) = \frac{1}{2}, \qquad (\log f)'(0) = -k_1, \qquad (\log f)''(0) = 1, \qquad (\log f)'''(0) = \zeta(3) - 2.$$

Define

$$\mathcal{Q}(M) = \log M - \frac{1}{2M^2} + \frac{2 - \zeta(3)}{3M^3}.$$

Theorem (Uniform zero-channel tail through the third centered moment). *For the unit smooth core and $\xi > 0$,*

$$\sigma_\alpha r^2 \delta F_0(r) = -\frac{1}{4\pi}\left[\frac{1}{M_P(r)} + \frac{1}{M_P(r)^3} + \frac{\zeta(3) - 2}{M_P(r)^4}\right] + O(M_P(r)^{-5}).$$

The remainder is uniform on the families , for all sufficiently large r.

Consequently

$$\sigma_\alpha r^2 \delta F(r) + \frac{1}{4\pi}\mathcal{Q}'(M_P(r)) = O(M_P(r)^{-5}) + O(r^{-1/\alpha}).$$

Proof. The zero term of gives

$$\sigma_\alpha r^2 \delta F_0(r) = -\frac{1}{2\pi}\int_0^\infty x\, b_0(x/r) K_0(x)^2 \, \mathrm{d}x.$$

On $x > r^{1/2}$, Lemma and the large-argument Bessel estimates give an exponentially small contribution. On $x \le r^{1/2}$, the error in gives $O(r^{-2}(1 + \log r)^2)$ after integration, which is smaller than every fixed inverse logarithmic power.

Put $L = \log r + A_P$. The remaining denominator is $L - \log x$. Split further at $|\log x| = L/2$. The small-x excluded part

has exponentially small $xK_0(x)^2$-mass, while the large-x excluded part is exponentially suppressed by K_0^2. On the retained set, geometric expansion through cubic order has an integrable remainder bounded by $C|\log x|^4 L^{-5}$.

Normalize $2xK_0(x)^2 \mathrm{d}x$ to a probability measure. For $Z = \log x + k_1$, gives

$$\mathbb{E}Z = 0, \qquad \mathbb{E}Z^2 = 1, \qquad \mathbb{E}Z^3 = \zeta(3) - 2.$$

Since $M_P(r) = L + k_1$, expansion of $\mathbb{E}(M_P(r) - Z)^{-1}$ gives . The bounds on d_P in the nondegenerate class make all the preceding choices uniform. Add Corollary to obtain .

Remark (Uniformity is not asserted as $d_P \operatorname{downarrow} 0$). Allowing $\xi \operatorname{downarrow} 0$, or permitting the core transition to move arbitrarily close to the center, can destroy the lower bound on d_P. A uniform expansion solely in the shifted variable $M = \log r + 1 + d_P^{-1}$ then requires a separate two-parameter estimate. Theorem does not conceal that degeneracy in an unspecified constant.

Full scalar moments, cutoff corrections, and finite constants

All radii in moment formulas are measured in a fixed reference length, set to one. Thus $\log(1/\varepsilon)$ denotes the logarithm of a dimensionless ratio.

The reference finite part and moment normalization

The cone finite part is

$$\operatorname{langle} U_L, f\rangle = \lim_{\eta \operatorname{downarrow} 0} \Big[\int_{r > \eta} r^{-2} f \, \mathrm{d}\Omega_0 + \sigma_\alpha \log(\eta/L) \int f(y, 0) \, \mathrm{d}^2 y \Big].$$

It satisfies

$$(E_x + 2) U_L = \sigma_\alpha \delta_{\mathscr{W}}, \qquad U_{L'} - U_L = -\sigma_\alpha \log(L'/L) \delta_{\mathscr{W}}.$$

These conventions agree with the one-point prescription of I-B-VP-1 and its corrective companions [15, 16, 17].

For one neutral real scalar,

$$s = \frac{\alpha^{-2} - 1}{48\pi^2}, \qquad h = \frac{\alpha^{-4} - 1}{1440\pi^2}.$$

At conformal coupling the exterior cone stress has $T^t_t = h/r^4$, and its derivative-compatible extension is

$\widehat{T}^t_t = (h/4)\Delta U_1$.

Let χ be a smooth compact radial function equal to one near zero. Define, per unit worldsheet volume,

$$c_{P,\chi}(\varepsilon;\mu) = \int F_{\varepsilon,P}\chi \, \mathrm{d}A_{\varepsilon,P} - \mathrm{slangleU_1},\chi\rangle,$$

$$t_{P,\chi}(\varepsilon;\mu) = \int T^t_{\varepsilon,P\,t}\frac{r^2}{4}\chi \, \mathrm{d}A_{\varepsilon,P} - \frac{h}{4}\left\langle \Delta U_1, (r^2/4)\chi \right\rangle, \qquad q = 0.$$

The second definition is used at conformal coupling unless stated otherwise.

The sharp normalization, denoted by sharp, uses $\chi^{\mathrm{sharp}} = 1_{\{r<1\}}$. This is an ordinary core integral, not an unrestricted distributional test. The reference finite parts have a canonical action on it: the function is smooth near the only singular support, and the remaining pairing is with an ordinary function away from zero. Direct finite-part integration gives

$$\mathrm{langleU_1},\chi^{\mathrm{sharp}}\rangle = 0, \qquad \frac{h}{4}\left\langle \Delta U_1, (r^2/4)\chi^{\mathrm{sharp}} \right\rangle = -\frac{\sigma_\alpha h}{4}.$$

The full scalar expansion

Theorem (Complete smooth-core scalar moment). *For every smooth profile and $\xi > 0$, there is a finite constant C_P, independent of ε, μ, such that, with*

$$\ell = \log(1/\varepsilon), \qquad M = \ell + 1 + d_P^{-1},$$

$$\mathrm{boxed}\, c_P^{\mathrm{sharp}}(\varepsilon;\mu) = \sigma_\alpha s\ell - \frac{qK}{8\pi^2}\log(\mu\varepsilon) - \frac{1}{4\pi}\mathscr{Q}(M) + C_P + O(M^{-4}).$$

The remainder, after retaining C_P, is uniform over the nondegenerate families .

For a fixed smooth radial cutoff define

$$m_j(\chi) = \int_0^\infty [\chi(u) - 1_{\{u<1\}}](\log u)^j \frac{du}{u}.$$

Then

$$c_{P,\chi}(\varepsilon;\mu) = \sigma_\alpha s\ell - \frac{qK}{8\pi^2}\log(\mu\varepsilon) + C_P$$
$$-\frac{1}{4\pi}\left[\mathcal{Q}(M) + \frac{m_0(\chi)}{M} - \frac{m_1(\chi)}{M^2} + \frac{m_2(\chi)+m_0(\chi)}{M^3}\right] + O(M^{-4}).$$

Thus the finite constant is cutoff-independent in this reference convention, but the subleading inverse-M coefficients are not.

Proof. Constant rescaling of the smooth heat prescription gives the exact scalar law

$$F_{\varepsilon,P,\mu}(r) = \varepsilon^{-2}F_{1,P,1}(r/\varepsilon) - \frac{qR_{\varepsilon,P}(r)}{8\pi^2}\log(\mu\varepsilon).$$

The fixed finite R-ambiguity obeys the same homogeneous scaling and is included in $F_{1,P,1}$.

Put

$$J_P(\Lambda) = \int_{r<\Lambda} F_{1,P,1}\,dA_{1,P} - \sigma_\alpha s\log\Lambda, \qquad \Lambda > 1.$$

This is finite because the unit-core renormalized scalar is smooth on compact sets. Theorem gives

$$\frac{dJ_P}{d\log\Lambda} = \sigma_\alpha\Lambda^2\delta F_{1,P}(\Lambda).$$

Equation therefore implies

$$J_P(\Lambda) = -\frac{1}{4\pi}\mathcal{Q}(M_P(\Lambda)) + C_P + O(M_P(\Lambda)^{-4}).$$

All exterior remainder bounds are uniform on ; the compact interior enters only the retained constant C_P. Integrate over the sharp ball, use , and set $\Lambda = \varepsilon^{-1}$. This proves .

For sufficiently small ε, the difference between the smooth and sharp moments is entirely exterior:

$$c_{P,\chi} - c_P^{\text{sharp}} = \sigma_\alpha\int_0^\infty r\,\delta F_{\varepsilon,P}(r)[\chi(r) - 1_{\{r<1\}}]\,dr.$$

The bracket is supported away from zero. On that fixed set, Theorem , with $M_P(r/\varepsilon) = M + \log r$, gives

$$\sigma_\alpha r^2 \delta F_{\varepsilon,P}(r) = -\frac{1}{4\pi}\left[\frac{1}{M} - \frac{\log r}{M^2} + \frac{1+(\log r)^2}{M^3}\right] + O(M^{-4}).$$

Integration proves .

Remark (Correction to the claimed absence of $1/M$). For the sharp normalization, centering the Mellin moments indeed removes the $1/M$ term from the integrated expansion. For a general smooth cutoff it reappears with coefficient $-m_0(\chi)/(4\pi)$. Replacing a cutoff by another with merely $o(1)$ difference does not justify an $O(M^{-3})$, or a stronger, cutoff-independent subleading expansion.

A convergent finite-constant formula

Proposition (Finite constant with a controlled exterior truncation). *For $R_0 \geq 2$, write $M_0 = \log R_0 + 1 + d_P^{-1}$. Then*

$$C_P = J_P(R_0) + \frac{1}{4\pi}\mathcal{Q}(M_0) + \int_{\log R_0}^{\infty}\left[\sigma_\alpha e^{2u}\delta F_{1,P}(e^u) + \frac{1}{4\pi}\mathcal{Q}'(u+1+d_P^{-1})\right]\mathrm{d}u.$$

The integral is absolutely convergent, and

$$\left|C_P - \left(J_P(R_0) + \frac{1}{4\pi}\mathcal{Q}(M_0)\right)\right| \leq CM_0^{-4} + CR_0^{-1/\alpha}.$$

The constants are uniform over . No uniform bound for C_P itself is asserted.

Proof. Integrate from $\log R_0$ to infinity. The first remainder integrates to $O(M_0^{-4})$; the higher-mode remainder integrates to $O(R_0^{-1/\alpha})$.

Formula is a convergent representation of the finite part, including the full renormalized unit-core interior integral. It is not a numerical evaluation. The files and numerical tables described in CORE were not supplied as executed artifacts in this calculation and are not relabeled as independent checks.

Proposition (Scalar distributional concentration). *For a fixed smooth profile, fixed μ, and a smooth radial*

$\chi = 1$ *near zero,*

$$\mathscr{F}_{\varepsilon,P} - sU_1 - c_{P,\chi}(\varepsilon;\mu)\delta_{\mathscr{W}} \to 0 \quad \text{strongly in} \mathcal{D}'_{\mathrm{loc}}.$$

The same holds when $\mu = \kappa/\varepsilon$, $\kappa > 0$ *fixed.*

Proof. For a compactly supported test, subtract its apex value times χ, and take the angular average. The resulting transverse test is $O(r^2 p_2)$. On a fixed multiple of the core radius, bounds its pairing by $O(\varepsilon^2(1 + |\log \varepsilon|)p_2)$, with a fixed-profile constant. On the exterior away from that inner region, the reflection bounds give an integrable Cr^{-2} majorant for the correction; its pointwise limit is zero. The additional r^2 Taylor factor permits dominated convergence. The cone finite-part subtraction is treated by its defining angular Taylor subtraction. The estimates are uniform on bounded sets of tests with fixed compact support, proving strong convergence. At $\mu = \kappa/\varepsilon$, the local logarithm in is bounded, so the same proof applies.

Interpretive boundary. Theorem and Proposition concern a specified renormalized free scalar observable. They do not select a canonical contact or derive a source equation from that contact [10, 12].

Stress tails and why projected moments are used

In the flat exterior, point splitting gives

$$\delta T^t{}_t = -\delta C_{\tau\tau} + \left(\xi - \frac{1}{4}\right)\Delta\delta F.$$

For the zero channel, differentiation of yields

$$\delta T^t{}_{t,0}(r) = -\frac{1}{4\pi^2\alpha r^4}\int_0^\infty x^3 b_0(x/r)V_\xi(x)\,\mathrm{d}x, \quad V_\xi = -\frac{1}{2}K_1^2 + \xi\Delta_x(K_0^2)$$

for a unit core. All differentiations are justified by Theorem .

Lemma (The stress logarithmic-channel coefficient). *One has*

$$\int_0^\infty x^3 K_1(x)^2\,\mathrm{d}x = \frac{2}{3}, \qquad \int_0^\infty x^3 \Delta_x(K_0(x)^2)\,\mathrm{d}x = 2, \qquad \int_0^\infty x^3 V_\xi(x)\,\mathrm{d}x = 2q.$$

The zero-channel weighted stress tail is therefore

$$\frac{\sigma_\alpha}{4} r^4 \delta T^{t}_{\ t,0}(r) = -\frac{q}{4\pi M_P(r)} + O(M_P(r)^{-2}).$$

At $q = 0$ *it is* $O(M_P(r)^{-2})$, *integrable in* $\mathrm{d}\log r$.

Proof. The first integral follows from the product Mellin formula at the indicated orders. For the second, integrate the radial Laplacian twice by parts:

$$\int_0^\infty x^3 (f'' + x^{-1} f') \, \mathrm{d}x = 4 \int_0^\infty x f(x) \, \mathrm{d}x.$$

The endpoint terms vanish for $f = K_0^2$, and $\int x K_0^2 = 1/2$. This gives . Use in ; all required logarithmic moments of $x^3 V_\xi$ are absolutely integrable. The same central/endpoint split as in Theorem proves .

For general q, subtracting q times the scalar contact removes the stress $\log\log$ term in the coordinate $\tau = t - qc$. This statement concerns the nonlocal logarithmic-channel contribution. Local curvature running must still be included.

At conformal coupling the unit-core stress constant

$$C_{T,P} = \lim_{\Lambda \to \infty} \left[\int_{r < \Lambda} \frac{r^2}{4} T_{1,P,1\ t}^{\ t} \, \mathrm{d}A_{1,P} - \frac{\sigma_\alpha h}{4} \log \Lambda + \frac{\sigma_\alpha h}{4} \right]$$

exists by Lemma and Corollary . A useful convergent representation is

$$C_{T,P} = \int_{r < R_0} \frac{r^2}{4} T_{1,P,1\ t}^{\ t} \, \mathrm{d}A_{1,P} - \frac{\sigma_\alpha h}{4} \log R_0 + \frac{\sigma_\alpha h}{4}$$

$$+ \frac{\sigma_\alpha}{4} \int_{R_0}^\infty r^3 \delta T_{1,P\ t}^{\ t}(r) \, \mathrm{d}r.$$

The omitted last integral is bounded by

$$C M_0^{-1} + C R_0^{-1/\alpha}.$$

Remark (The stress moment is not the whole tensor). The test $r^2\chi/4$ is blind to a longitudinal tension term

proportional to $\delta_{\mathscr{W}}$. Furthermore the conformal trace anomaly contains curvature-square terms and need not vanish as a distributional core limit. Consequently the pair (c, t) is a declared projection onto two moments. At $q = 0$, write $\tau = t$ when comparing with the contact coordinates of I-B-VP-1, but do not infer that the full smooth-core stress belongs to that paper's traceless class.

Non-sourcing. Neither the stress tail nor its projected moment imposes a gravitational field equation [10, 12].

Independent normalization of local stress running

The local coefficient and a lapse variation

For a real scalar, modulo an integrated total derivative,

$$a_2 = \frac{1}{180}(\mathrm{Riem}^2 - \mathrm{Ric}^2) + \frac{1}{2}q^2R^2 = \frac{1}{120}C^2 - \frac{1}{360}E + \frac{1}{2}q^2R^2.$$

This is the standard smooth Laplace-type heat coefficient [7, Eq. (4.28)] [6]. The conventions of Section fix its effective-action sign and normalization.

Let

$$H_{1,\mu\nu} = \frac{1}{\sqrt{g}}\frac{\delta}{\delta g^{\mu\nu}}\int R^2\sqrt{g}\,\mathrm{d}^4x, \qquad H_{2,\mu\nu} = \frac{1}{\sqrt{g}}\frac{\delta}{\delta g^{\mu\nu}}\int \mathrm{Ric}^2\sqrt{g}\,\mathrm{d}^4x,$$

and $B_{\mu\nu} = H_{2,\mu\nu} - H_{1,\mu\nu}/3$. Thus $\delta\int C^2 = 2\int\sqrt{g}\,B_{\mu\nu}\delta g^{\mu\nu}$.

Theorem (Local running, checked without contact matching). *In the fixed compatible convention,*

$$\frac{\partial F}{\partial\log\mu} = -\frac{qR}{8\pi^2}, \qquad \frac{\partial T^{\mu}{}_{\nu}}{\partial\log\mu} = -\frac{1}{8\pi^2}\left(\frac{1}{60}B^{\mu}{}_{\nu} + \frac{1}{2}q^2H_1{}^{\mu}{}_{\nu}\right).$$

On the product of a flat longitudinal plane and a transverse surface,

$$H_1{}^{t}{}_{t} = 2\Delta R - \frac{1}{2}R^2, \qquad B^{t}{}_{t} = -\frac{1}{6}\Delta R - \frac{1}{12}R^2.$$

The coefficient and sign in are independent of any requirement that scalar and stress contacts fit an improvement identity.

Proof. The scalar statement follows directly from . Varying and gives the stress statement.

Here is an independent component calculation that checks the relevant contraction. Vary only the inverse Euclidean longitudinal metric by $\delta g^{\tau\tau} = \eta(x)$, with compact transverse support. At the unvaried product metric,

$$\delta\sqrt{g} = -\frac{1}{2}\eta\sqrt{g}, \qquad \delta R = \Delta\eta, \qquad \delta(\mathrm{Ric}^2) = \frac{R}{2}\Delta\eta, \qquad \delta(\mathrm{Riem}^2) = 0.$$

Since $\mathrm{Ric}^2 = R^2/2$ and $\mathrm{Riem}^2 = R^2$, integration by parts gives

$$\frac{1}{\sqrt{g}}\frac{\delta}{\delta g^{\tau\tau}}\int a_2\sqrt{g} = -\frac{\Delta R}{360} - \frac{R^2}{720} + q^2\left(\Delta R - \frac{R^2}{4}\right).$$

This equals $B^{\tau}{}_{\tau}/60 + q^2 H_1{}^{\tau}{}_{\tau}/2$ with . It also verifies the longitudinal component directly, without importing a contact normalization.

An explicit spectral normalization check

Proposition (Constant-curvature spectral check). *On* $\mathbb{R}^2 \times S^2_a$, *at* $\xi = 1/6$, *the local heat coefficient is* $a_2 = R^2/360$. *The mixed longitudinal scale derivative is*

$$\frac{\partial T^t{}_t}{\partial \log \mu} = \frac{R^2}{5760\pi^2},$$

in agreement with .

Proof. The transverse spectral heat kernel on the diagonal is

$$\frac{1}{4\pi a^2}\sum_{\ell=0}^{\infty}(2\ell+1)e^{-u[\ell(\ell+1)+2\xi]/a^2}.$$

Euler–Maclaurin summation gives

$$\sum_{\ell\geq 0}(2\ell+1)e^{-v\ell(\ell+1)} = v^{-1} + \frac{1}{3} + \frac{v}{15} + O(v^2).$$

Multiplication by $e^{-2\xi v}$ at $\xi = 1/6$ leaves the v^2 coefficient inside the normalized heat expansion equal to 1/90. Since $R = 2/a^2$, this is $R^2/360$. A constant longitudinal lapse variation changes only the volume factor in the local

integrated coefficient. Equations and the factor 2 in the stress definition then give $a_2/(16\pi^2)$, as claimed.

Remark (The requested mode-sum check and its exact limitation). Proposition is an explicit spectral check on a homogeneous smooth product, and Theorem gives a local lapse-variation check on every smooth core. An independent numerical renormalized stress mode sum for one flat-center core was not executed.

More importantly, differentiating only the exterior reflection correction with respect to μ gives zero: the reflection coefficients and exterior local geometry have no such scale dependence. That operation cannot test the interior-supported running. A genuine interior mode-sum check must retain the local subtractions before the μ -derivative. This residual obligation is not called a numerical discrepancy.

Shape moments and three exact smooth examples

Put

$$Q_2[P] = \int_0^1 sP'(s)^2 P(s)^{-5}\,\mathrm{d}s, \qquad \mathscr{A}_\alpha = \alpha^{-2} - \alpha.$$

Lemma (Geometric moments). *For every smooth profile,*

$$\int \frac{r^2}{4} R^2\,\mathrm{d}A = 2\pi\alpha Q_2[P],$$

$$\int R\Delta(r^2/4)\,\mathrm{d}A = \frac{4\pi}{3}\mathscr{A}_\alpha - 2\pi\alpha Q_2[P],$$

$$M_B[P] := \int \frac{r^2}{4} B^t{}_t\,\mathrm{d}A = -\frac{2\pi}{9}\mathscr{A}_\alpha + \frac{\pi\alpha}{6} Q_2[P],$$

$$M_1[P] := \int \frac{r^2}{4} H_1{}^t{}_t\,\mathrm{d}A = \frac{8\pi}{3}\mathscr{A}_\alpha - 5\pi\alpha Q_2[P].$$

All these moments are independent of ε.

Proof. Inside the core,

$$\Delta(r^2/4) = P^{-2} - \frac{sP'}{2P^3}.$$

Insert . The first moment is immediate, while the second becomes

$$4\pi\alpha\int_0^1 P'P^{-4}\,\mathrm{d}s - 2\pi\alpha Q_2[P].$$

The first integral is $\frac{4\pi}{3}(\alpha^{-2}-\alpha)$. Since R is smooth and compactly supported, integration by parts transfers Δ from R to $r^2/4$. Now use .

Proposition (An exact smooth family realizing every positive Q_2). *Fix a smooth increasing step $S:\mathbb{R}\to[0,1]$, with $S=0$ on $(-\infty,0]$, $S=1$ on $[1,\infty)$, and all endpoint derivatives zero. For $T>0$, define*

$$P_T(s)^{-3/2} = \alpha^{-3/2} - (\alpha^{-3/2}-1)S\Big(\frac{\log s + T}{T}\Big), \qquad s>0.$$

Extend it by $P_T(0)=\alpha$. Then P_T is an admissible smooth profile and

$$Q_2[P_T] = \frac{4(\alpha^{-3/2}-1)^2}{9T}\int_0^1 S'(v)^2\,\mathrm{d}v.$$

Hence Q_2 ranges over $(0,\infty)$.

Proof. The profile is constant for $s\le e^{-T}$ and for $s\ge 1$, is smooth at both endpoints, and is monotone. With $u=\log s$ and $Y=P^{-3/2}$,

$$Q_2 = \frac{4}{9}\int_{-\infty}^0 |Y'(u)|^2\,\mathrm{d}u.$$

The substitution $v=(u+T)/T$ proves .

Define

$$B_\alpha = \frac{2\pi}{9}\mathscr{A}_\alpha, \qquad Q_* = \frac{4}{3\alpha}\mathscr{A}_\alpha.$$

Choose three members of Proposition with $Q_2 = Q_*/2, Q_*, 2Q_*$. Their moments are exactly

$$M_B = -B_\alpha/2, \qquad 0, \qquad B_\alpha.$$

This gives three genuinely smooth examples with positive, zero, and negative running coefficients in the invariant defined below, without an unexecuted numerical profile fit.

The nonsmooth benchmarks are not silently promoted

The Gott–Hiscock cap has the useful geometric value

$$Q_2^{\mathrm{GH}} = \frac{(1-\alpha)^2(2+\alpha)}{3\alpha^3}.$$

Its standard piecewise-smooth matching and the finite-thickness literature are described in [4, 2]. The sharp flower-pot limit has $Q_2 = \infty$. Neither is a member of the smooth flat-center class used in the full one-point theorems.

In particular, a sharp flower-pot stress has surface singularities. It is not a third finite stress-contact datum to be compared numerically with two smooth profiles. One must specify a smooth approximation and control its renormalized surface limit. This is why the three rigorous examples above are smooth.

Projected stress running and the dilation-invariant verdict

For the neutral one-point contact class, the dilation direction is

$$(c,\tau)\mathrm{longmapsto}\,(c,\tau) + \sigma_\alpha \log\lambda\,(s, h/4).$$

The linear invariant is

$$I = hc - 4s\tau.$$

For a smooth core at $q = 0$, define the projected diagnostic

$$I_{P,\chi} = h\,c_{P,\chi} - 4s\,t_{P,\chi}.$$

If the full one-point data belonged to the declared two-contact class, this would be its invariant. Here it is used only as a necessary moment test for that interpretation.

Proposition (Running of the projected coordinates). *With $\chi = 1$ on the core, the scalar running is*

$$r_F := \frac{\partial c}{\partial \log \mu} = -\frac{qK}{8\pi^2}.$$

For the weighted longitudinal moment,

$$\frac{\partial t}{\partial \log \mu} = -\frac{1}{8\pi^2}\left(\frac{M_B}{60} + \frac{q^2 M_1}{2}\right).$$

If $\tau = t - qc$*, then*

$$\frac{\partial \tau}{\partial \log \mu} = -\frac{1}{8\pi^2}\left[\frac{M_B}{60} + q^2\left(\frac{M_1}{2} - K\right)\right].$$

At $q = 0$*, put*

$$r_{T,P} = -\frac{M_B[P]}{480\pi^2}, \qquad \kappa_P = 4s\, r_{T,P} = -\frac{sM_B[P]}{120\pi^2}.$$

Then $\partial I/\partial \log \mu = -\kappa_P$.

Proof. Pair Theorem with the two test weights and use Lemma .

Theorem (Fixed-scale non-universality of the projected invariant). *Let* $\xi = 1/6$*, and fix the finite renormalization constants. For each smooth profile,*

$$t_P^{\mathrm{sharp}}(\varepsilon;\mu) = \frac{\sigma_\alpha h}{4}\ell + r_{T,P}\log(\mu\varepsilon) + C_{T,P} + O(M^{-1}),$$

$$I_{P,\chi}(\varepsilon;\mu) = -\frac{h}{4\pi}\log \ell + \kappa_P \log\frac{1}{\mu\varepsilon} + I_P^{\infty} + o(1),$$

$$I_P^{\infty} = hC_P - 4sC_{T,P}.$$

The finite constant is independent of the chosen smooth $\chi = 1$ *near zero in the reference convention.*

At fixed μ*, no profile has a finite limiting projected invariant. If* $Q_2[P] \neq Q_2[P']$*, then*

$$I_{P,\chi} - I_{P',\chi} = -\frac{s\alpha}{720\pi}(Q_2[P] - Q_2[P'])\log\frac{1}{\mu\varepsilon} + O(1).$$

No shape-independent subtraction makes all these projected invariants converge at fixed μ*.*

Proof. The stress scaling law in the fixed compatible scheme is the tensor counterpart of . At $q = 0$, pairing it with $r^2\chi^{\mathrm{sharp}}/4$, using , and then , gives the first line. The conformal exterior tail is integrable in $d\log r$, so its remainder is

$O(M^{-1})$. Smooth cutoff changes in that stress moment are $O(\ell^{-2})$; scalar cutoff changes are $O(\ell^{-1})$.

Insert Theorem into $I = hc - 4st$. The $\sigma_{\alpha} sh\ell$ terms cancel, leaving . Since $h > 0$, the invariant diverges even when $\kappa_P = 0$; otherwise its linear logarithm dominates. Subtracting the two expansions and using proves . The exact smooth family of Proposition supplies different values of κ_P, excluding a common subtraction.

Remark (What the verdict proves). Theorem is an obstruction to convergence into a common dilation orbit as tested by these two moments. It does not construct a full stress-distribution extension. The latter also has to account for tension and anomaly contacts. A diverging necessary moment is sufficient to rule out the proposed finite contact-pair interpretation; it is not a substitute for a positive full-tensor theorem.

For the three smooth profiles in Section , the coefficients are exactly

$$\kappa_1 = \frac{sB_{\alpha}}{240\pi^2}, \qquad \kappa_2 = 0, \qquad \kappa_3 = -\frac{sB_{\alpha}}{120\pi^2}.$$

Their fixed-scale separation is therefore a proved analytical comparison, not a fit to truncated modes.

Non-sourcing. The invariant is a normalization diagnostic. Neither its divergence nor its shape dependence supplies Einstein dynamics or a physical string equation [10, 12].

Core-adapted scales, finite parts, and nominal-radius rescaling

What a core-adapted scale does not remove

Set $\mu = \kappa/\varepsilon$, with $\kappa > 0$ fixed. Equation becomes

$$I_{P,\chi}(\varepsilon; \kappa/\varepsilon) = -\frac{h}{4\pi}\log \ell + I_P^{\infty} - \kappa_P \log \kappa + o(1).$$

Thus the raw invariant still has no finite limit. Define the separately subtracted finite remainder

$$\Im_P(\kappa) := \lim_{\varepsilon \text{downarrow} 0} \left[I_{P,\chi}(\varepsilon; \kappa/\varepsilon) + \frac{h}{4\pi}\log \ell \right] = I_P^{\infty} - \kappa_P \log \kappa.$$

This extra common subtraction is not a dilation: I is invariant under the declared dilation direction.

Equations and provide convergent representations for I_P^{∞}, with explicit exterior truncation errors. Its complete numerical value still requires the renormalized unit-core interior contribution.

For two profiles with different κ_P, their finite remainders can coincide at at most one κ. For three prescribed profiles, simultaneous coincidence requires the corresponding three affine functions of $\log \kappa$ to meet. Different slopes alone do not prove that such a meeting never occurs at one specified value.

An exact rescaling obstruction on the stated class

Theorem (Profile rescaling and adapted finite parts). *Let* $0 < a < 1$ *and define* $P_a(s) = P(s/a)$. *This remains an admissible profile with nominal radius one, but its actual curvature support is contained in* $s \le a$. *Then*

$$\frac{1}{d_{P_a}} = \frac{1}{d_P} + \log(1/a), \qquad L_{\text{eff}}(P_a, \varepsilon) = L_{\text{eff}}(P, a\varepsilon),$$

and

$$Q_2[P_a] = Q_2[P], \qquad M_B[P_a] = M_B[P], \qquad \kappa_{P_a} = \kappa_P.$$

At conformal coupling,

$$I_{P_a}^{\infty} = I_P^{\infty} - \kappa_P \log a, \qquad \Im_{P_a}(\kappa) - \Im_P(\kappa) = -\kappa_P \log a.$$

Consequently, if $\kappa_P \neq 0$, *no single core-adapted* κ *makes the finite remainder shape-independent on the entire stated nominal-radius class.*

Proof. Between $s = a$ and $s = 1$, the zero-energy solution is conical and proportional to $1 + d_P \log(s/a)$. Its logarithmic derivative at $s = 1$ is

$$d_{P_a} = \frac{d_P}{1 + d_P \log(1/a)},$$

proving . A substitution in Q_2 proves its invariance.

The geometries satisfy the exact identity $g_{\varepsilon, P_a} = g_{a\varepsilon, P}$. Their moments at the same μ are therefore equal. Write $\ell' = \log(1/(a\varepsilon)) = \ell - \log a$ in . Since $\log \ell' - \log \ell \to 0$, comparison of finite constants gives the first equation of .

The slopes are equal, so the second follows from .

Remark (Nominal radius versus minimal physical radius). This theorem identifies a genuine obstruction on the profile class as it was stated in CORE. It also exposes a normalization issue: an unused conical collar changes the nominal radius without changing the underlying physical family after reparametrization.

If one instead requires that the minimal radius outside which $P = 1$ be exactly the declared radius, the profiles P_a are not equal-radius comparison examples. For that narrower problem, the finite constants for three independently normalized profiles have not been evaluated here. Their common-κ coincidence question remains open. The fixed-μ non-universality in Theorem does not need this rescaling argument and remains valid.

Stopping condition (Exact finite-part ending). The finite constants are constructed by convergent representations and have controlled exterior truncations. They are not supplied as certified numerical values. The adapted raw invariant diverges. The additionally subtracted finite remainder is non-universal on the full nominal-radius class by Theorem . No stronger assertion about three fixed equal-minimal-radius finite constants is made.

Twisted fields: the missing flux input and a computable local contribution

A smooth core cannot carry unspecified nontrivial flat holonomy

A nontrivial meridian holonomy cannot extend as a smooth flat connection across a disk. To pose a smooth charged-core problem one must supply the interior connection. For definiteness, let

$$D = \nabla + \mathrm{i}A, \qquad A = \beta a(r/\varepsilon)\,\mathrm{d}\varphi, \qquad a(s) = O(s^2)\text{smoothly at}0, \qquad a(s) = 1\text{for}s \geq 1.$$

Thus the smooth magnetic curvature is $\mathscr{F} = \mathrm{d}A$, and $\int \mathscr{F} = 2\pi\beta$. Different a's are additional physical background data, not determined by the exterior holonomy.

The radial equation has

$$\frac{n^2 P}{\alpha^2 s} \quad \text{replaced by} \quad \frac{(n + \beta a(s))^2 P}{\alpha^2 s}.$$

Its coefficients remain nonnegative for $\xi \geq 0$; the regular logarithmic derivative is therefore nonnegative. Outside the core the orders are

$$\nu_n = \frac{|n+\beta|}{\alpha}.$$

Proposition (Exterior twisted convergence and absence of the logarithmic channel). *For a fixed smooth flux profile and* $0 < \beta < 1$*, the exterior representation and differentiated absolute convergence of Theorem remain valid with these orders. There is no zero-order logarithmic channel.*

Writing $\nu_* = \min(\beta, 1-\beta)/\alpha > 0$*, the same estimates give, for a unit core,*

$$|\delta F(r)| \leq C r^{-2-\nu_*}, \qquad |\delta T(r)| \leq C r^{-4-\nu_*}, \qquad |\delta J(r)| \leq C r^{-3-\nu_*}, \qquad r \geq 2,$$

in regular exterior frames. Consequently the corresponding weighted exterior moment tails are integrable in $\mathrm{dlog}\, r$.

Proof. Apply and to every order $\nu_n \geq \nu_*$. There are finitely many orders below one, treated with fixed-order Bessel estimates; all other orders use Lemma . The angular series remains geometric. The far-tail proof of Corollary now starts at ν_*. Current point splitting costs one derivative and an angular factor, giving the third estimate.

This proves absence of the nonlocal log log mechanism, not absence of local geometric or magnetic running.

A declared charged normalization

For one complex scalar, use

$$F = \langle \Phi^\dagger \Phi \rangle, \qquad J^\mu = \mathrm{i}\big(\Phi^\dagger D^\mu \Phi - (D^\mu \Phi)^\dagger \Phi\big).$$

The scalar F is one covariance component; its stress multiplicity is $n = 2$. Thus the sentence ``the complex field doubles every number'' is not correct for F in this convention.

For $D = \nabla + \mathrm{i}A$, the Euclidean effective action satisfies $J^\mu = -(\sqrt{g})^{-1}\delta W/\delta A_\mu$. The connection part of the scalar heat coefficient is $-\mathscr{F}_{\mu\nu}\mathscr{F}^{\mu\nu}/12$. The complex determinant has twice the real-scalar effective-action normalization.

Proposition (Local magnetic running of a first-jet moment). *In these conventions,*

$$\left.\frac{\partial J^{\mu}}{\partial \log \mu}\right|_{\mathrm{mag}} = \frac{1}{24\pi^{2}} \nabla_{\nu} \mathscr{F}^{\nu\mu}.$$

Let $R^{i} = \varepsilon^{i}_{\ j} x^{j}$, *with* $\varepsilon = \begin{pmatrix} 0 & -1 \\ 1 & 0 \end{pmatrix}$, *and extract a supported circulating first jet by the moment* $-\frac{1}{2}\int R_{i} J^{i} \chi \, \mathrm{d}A$, *using the auxiliary Cartesian lowering and* $\chi = 1$ *on the flux support. Its magnetic running contribution is*

$$\left.\frac{\partial w}{\partial \log \mu}\right|_{\mathrm{mag}} = \frac{\beta}{12\pi\alpha} \int_{0}^{1} \frac{a'(s)}{P(s)} \, \mathrm{d}s.$$

The magnetic contribution to the weighted longitudinal stress moment is

$$\left.\frac{\partial t}{\partial \log \mu}\right|_{\mathrm{mag}} = -\frac{\beta^{2}}{96\pi\alpha} \int_{0}^{1} \frac{s\, a'(s)^{2}}{P(s)} \, \mathrm{d}s.$$

These are additional flux-profile-dependent local terms.

Proof. The magnetic part of $\partial W/\partial \log \mu$ is

$$\frac{1}{96\pi^{2}} \int \mathscr{F}_{\mu\nu} \mathscr{F}^{\mu\nu} \sqrt{g} \, \mathrm{d}^{4}x.$$

Gauge variation gives $\delta \int \mathscr{F}^{2} = -4 \int \nabla_{\nu} \mathscr{F}^{\nu\mu} \delta A_{\mu} \sqrt{g}$, proving .

Integration by parts against R_{i} gives

$$-\frac{1}{2} \int R_{i} \nabla_{\nu} \mathscr{F}^{\nu i} \, \mathrm{d}A = \int \mathscr{F}^{12} \, \mathrm{d}A.$$

For , the latter equals

$$\frac{2\pi\beta}{\alpha} \int_{0}^{1} \frac{a'}{P} \, \mathrm{d}s,$$

which proves . Metric variation of $\int \mathscr{F}^{2}$ has longitudinal mixed component $-\mathscr{F}^{2}/2$. With the factor 2 in the stress definition, its scale derivative is $-\mathscr{F}^{2}/(96\pi^{2})$. Finally,

$$\int \frac{r^{2}}{4} \mathscr{F}^{2} \, \mathrm{d}A = \frac{\pi\beta^{2}}{\alpha} \int_{0}^{1} \frac{s\, a'^{\wedge}2}{P} \, \mathrm{d}s,$$

giving .

The first-jet extraction is normalized because

$$-\frac{1}{2}\mathrm{langlew}\varepsilon^{i}{}_{k}\partial_{k}\delta_{\mathscr{H}},R_{i}\chi\rangle = w.$$

It still does not determine the finite part of w.

For generic $s \neq 0$, the exterior contact dilation direction has the two linear invariants

$$I_1 = hc - 4s\tau, \qquad I_2 = jc + 2\alpha^2 s\, w,$$

where j is the regular-current coefficient in the same charged-current convention [22]. At degenerate $s = 0$, a different independent basis of covectors annihilating the dilation direction is required.

Stopping condition (Twisted ending). The exterior differentiated convergence and integrable power tails are proved for a supplied smooth flux profile. Equations – evaluate local magnetic contributions in an explicitly declared convention.

The complete finite w, a full three-contact stress/current completion, and a universality classification for both invariants are not evaluated. They require the interior charged one-point prescription and normalization matching to the chosen regular-current record. Exterior holonomy alone does not supply those data. No Maxwell or Einstein response is derived [22, 10, 12].

Operator selection, trace contacts, and the remaining boundaries

Operator selection remains a different result

I-B-VP-5 reports Friedrichs strong-resolvent selection in its stated monotone core class and identifies L_{eff} in the matching problem [14]. The present proof of the exterior moment law neither depends on nor contradicts that operator conclusion.

Strong resolvent convergence does not control an inverse covariance weight without additional threshold

estimates. For example, on $\ell^2(\mathbb{N})$, let $B_n = I$ except on its n-th basis vector, where the eigenvalue is 2^{-8n}. Then $B_n \to I$ strongly in the resolvent sense. For the fixed vector $v_j = 2^{-j}$,

$$\text{langlev}, B_n^{-1/2} v\rangle \geq 2^{2n} \to \infty.$$

This is a logical counterexample, not a core model.

Thus the following statements must remain separate:

operator limit, fixed exterior observable limit, renormalized apex moment limit.

Trace anomaly and the contact class

At conformal coupling the smooth-core trace anomaly has the standard local curvature-square contribution, with a scheme-dependent Box R term [6, 7]. Its moments are not automatically zero. The weighted scalar I was therefore described as a projected invariant throughout.

The full stress problem must still specify:

1. subtraction of the possible power-divergent tension;

2. the trace-contact prescription;

3. the complete transverse tensor contact module;

4. the chosen Ward representative on the auxiliary completion;

5. convergence on arbitrary tensor tests. Two moment limits, even if they existed, would not prove all five.

Negative coupling

For a compact curvature core with $\int R\mathrm{d}A = K > 0$, the logarithmic variational test equal to one on the curvature support has kinetic energy $2\pi\alpha/\log(b/R)$. For $\xi < 0$, its curvature energy is the fixed negative number ξK. For large b, the form is negative. This reproduces the scoped instability argument of Ex1 [16]. It obstructs the nonnegative spectral-ground-state construction, not every nonstationary Hadamard state.

Standing programme stops

This manuscript updates no unrelated programme status. The supplied boundaries are retained as version-qualified scope records:

• Problems 1, 2, and 8 remain unfinished; Problems 3–7 and 9–11 are not reached, and Problem 12 remains unresolved.

• XVI-O's standing entry remains nolinkurl $O.1 = NOT_C LOSED$ in its prescription. A separately reported scoped two-point closure in another prescription is not revoked or imported.

• XVI-N and Route B are not discharged.

• The downstream quantum-admission exit associated with XV-B is not opened.

• The historical H2 ending in XIV-H is distinguished from XIV-I's constructed I2 internal amplitude and the still unresolved full maps at I3 and J2. The supplied characteristic manuscripts concern different distributional products [25, 26]. No mode-sum estimate here constructs their full joint map, joint section, transverse estimates, contact compatibility, output wavefront theorem, or quantum admission.

No spacetime emergence, backreaction, Page curve, de Sitter state, cosmological bounce, interacting apex algebra, or ultraviolet completion is constructed. OpenAI catalogue entries remain external records, not analytical premises.

Reliability ledger and exact release

Evidence and residual retention

The integrity distinction of XVI-O1-ST is retained [23]:

\texttt{ANALYTIC} A result proved by the displayed argument with identified smooth-background analytical inputs.

\texttt{SUPPLIED} A version-qualified source statement or historical computational report not independently

executed here.

\texttt{UNCERTIFIED} A numerical approximation lacking the artifacts and total error bounds needed for certification. An unproved analytical estimate is marked open, not converted into an ``uncertified term" in a theorem.

toprule Target Ending Exact scope or residual

midrule endfirsthead toprule Target Ending Exact scope or residual

midrule endhead bottomrule endfoot

Exterior hypothesis (M) ANALYTIC Removed for smooth monotone cores and $\xi \geq 0$; all fixed differentiated sums converge on compact exterior sets.

Uniform higher-mode estimates ANALYTIC Profile-independent majorant and integrable far tails; not optimal decay exponents.

Full scalar contact moment ANALYTIC Includes the smooth renormalized interior through scale covariance; formula .

Uniform logarithmic remainder ANALYTIC $O(M^{-4})$ on the nondegenerate class ; no assertion as $d_P$downarrow 0.

Cutoff-independent inverse-M cancellation Corrected False without normalization; gives the missing cutoff terms.

Finite constants Constructed, not numerically evaluated Convergent formulas , and explicit exterior truncation errors.

Stress scale normalization ANALYTIC Heat-coefficient/lapse-variation calculation and constant-curvature spectral check.

Independent numerical smooth-core stress check OPEN No interior renormalized mode-sum computation was executed. Exterior differentiation in μ is identically zero.

Fixed-scale projected orbit universality Excluded Shape-dependent slopes, including three exact smooth examples.

Core-adapted raw invariant Divergent The universal $-h\log \ell/(4\pi)$ term remains.

Adapted finite remainder on nominal-radius class Non-universal Exact profile-rescaling identity, Theorem .

Three equal-minimal-radius finite constants OPEN No complete numerical or closed-form comparison; different slopes alone do not decide one crossing point.

Full stress-distribution limit OPEN Tension, trace contacts, complete tensor normalization, and arbitrary-test convergence remain separate.

Twisted exterior sums and local running Scoped calculation A smooth flux profile is declared; finite *w* and full invariant classification remain open.

CORE numerical tables SUPPLIED No scripts or raw logs were independently executed; their numbers are not used as proof.

Finite coverage

Proposition (Coverage is an independent hypothesis). *Let Z be an independently proved finite-dimensional space containing every residual admissible for a claim. A finite linear evaluation map $E : Z \to \mathbb{R}^m$ detects every residual if and only if E is injective. Without membership and injectivity, finite vanishing does not imply a distributional identity.*

Proof. The exact assertion is $\operatorname{Ker} E = \{0\}$. Neither the choice of finitely many tests nor their vanishing proves that all admissible residuals belong to Z.

This is the finite-test distinction documented in Oai282-A [24]. The exterior representation above is proved on arbitrary compact exterior tests and all angular modes. It is not inferred from finitely many samples or a truncated numerical fit.

Release theorem

Theorem (I-B-VP-5-Ex1 mathematical release). *For the smooth core class, field prescription, density*

identification, and renormalization convention of this paper:

1. *the exterior Green correction and all fixed differentiated mode sums are constructed with profile-independent summable majorants;*

2. *the complete scalar moment and scalar distributional concentration satisfy Theorem and Proposition ;*

3. *the sharp logarithmic expansion has the evaluated third centered Mellin coefficient and uniform* $O(M^{-4})$ *remainder on the stated nondegenerate families;*

4. *smooth-cutoff subleading corrections are ;*

5. *the finite scalar and conformal stress constants have the convergent representations and tail bounds –;*

6. *the local scalar and stress running is fixed independently of contact matching by Theorem ;*

7. *fixed-scale universality of the projected dilation invariant fails, with three exact smooth profile examples;*

8. *a core-adapted scale leaves a universal divergent term, and its separately subtracted finite remainder obeys the exact nominal-radius rescaling obstruction;*

9. *the twisted exterior statement and local magnetic contributions are established only with the explicit smooth flux input of Section . No complete stress-contact distribution, equal-minimal-radius finite-part universality decision, independent numerical smooth-core stress computation, or interacting or gravitational completion is released.*

Proof. Items (i)–(iv) are Theorems , , , and Proposition . Item (v) is Proposition and –. Item (vi) is Theorem and Proposition . Items (vii)–(viii) are Theorems and , with Proposition . Item (ix) is Propositions and . The last sentence records constructions and comparisons not supplied by these proofs.

appendix

Corrections to the interpretation of I-B-VP-CORE

1. Exterior modes. Hypothesis (M) is no longer needed for the smooth class and compact exterior differentiated

sums. It is not thereby proved uniformly at a singular flower-pot ring.

2. Full scalar divergence. For the complete moment,

$$c^{\mathrm{sharp}} = \sigma_{\alpha} s\ell - \frac{qK}{8\pi^{2}}\log(\mu\varepsilon) - \frac{1}{4\pi}\log M + O(1).$$

The double logarithm is the nonlocal logarithmic-channel part, not the only divergence of the full raw moment.

3. Subleading cutoff dependence. An $o(1)$ change of cutoff may be $O(M^{-1})$. It cannot be ignored in a claimed $O(M^{-3})$ expansion. The sharp and smooth formulas are distinguished.

4. Additional evaluated term. The third centered moment is $\zeta(3) - 2$, producing $(2 - \zeta(3))/(3M^{3})$ in the integrated primitive. No numerical quadrature is required for this coefficient.

5. Stress normalization. The local running is fixed by a compatible effective-action convention and checked by a lapse variation. It is not justified by requiring agreement with a preselected contact improvement.

6. Projected versus complete stress. The stress moment does not see a tension contact. Its use as a coordinate does not prove a traceless full stress limit. The trace anomaly and complete tensor module remain visible.

7. Adapted scales. At $\mu = \kappa/\varepsilon$, the universal $\log\log$ term remains. Only the additionally subtracted finite remainder is affine in $\log\kappa$.

8. Rescaled profiles. At a fixed nominal boundary,

$$d_{P_a}^{-1} = d_{P}^{-1} + \log(1/a).$$

Thus profile rescaling does not leave d unchanged. It leaves Q_2 unchanged and rescales L_{eff}.

9. Nonsmooth examples. The sharp flower-pot is not a finite conformal stress contact example. The Gott–Hiscock benchmark is not silently made a smooth flat-center profile. Their smooth approximation problems need separate estimates.

10. Complex multiplicity. With $F = \langle \Phi^{\dagger}\Phi \rangle$, the complex stress multiplicity is two, but this scalar one-component

Wick square is not twice its own covariance normalization.

11. State selection. Nonnegativity of the spatial operator and smoothness do not prove uniqueness of every algebraic ground state. The zero-mean spectral state is explicitly selected.

12. Numerical evidence. The named historical scripts and tables are attributed reports. They were not executed here and do not become independent or certified evidence through restatement.

Verification and submission declarations

Analytical dependencies. The exterior matching, convergence bounds, logarithmic expansion, shape moments, and rescaling identities are proved in the text. The ordinary local heat/Hadamard construction and the standard smooth Laplace-type heat coefficients are identified external analytical inputs. No theorem of an unavailable companion is reconstructed as a premise.

Online source scope. The philosophy page and the En09-A and XVI-O1-ST records were consulted for the documentary framework. Primary literature records, the indicated primary articles, and DLMF were consulted for attribution and normalization. The I-B, I-B-VP-5, and I-B-G-S browse endpoints were not available through the consulted interface; references to their supplied material are therefore explicitly source-qualified. No exhaustive corpus search or priority audit is asserted.

Execution. No successful pdfLaTeX run, numerical integration, symbolic-program execution, repository rebuild, proof-assistant verification, independent referee review, or journal acceptance is asserted. No new numerical profile values or error enclosures are reported.

Source format and authorship. This is one pdfLaTeX source with an internal bibliography, standard packages, and no external figure, data, code, or bibliography-file dependency. Two runs are intended to resolve cross-references and the contents. The author name follows the supplied manuscript series. Submission requires the author's approval of the mathematics, attribution, and scope.

Final distinction. A controlled exterior sum is not a complete interior normalization. A convergent finite-part

representation is not a numerical evaluation. A projected stress invariant is not the entire tensor. A field observable on a prescribed cone is not a derived source of that cone.

References

[1] B. Allen, B. S. Kay, and A. C. Ottewill, Long range effects of cosmic string structure, *Physical Review D* **53** (1996), 6829–6846. https://arxiv.org/abs/gr-qc/9510058. Long-range core-sensitive scalar effects. No full apex-contact theorem is imported.

[2] E. R. Bezerra de Mello, V. B. Bezerra, A. A. Saharian, and A. S. Tarloyan, Vacuum polarization induced by a cylindrical boundary in the cosmic string spacetime, *Physical Review D* **74** (2006), 025017. https://arxiv.org/abs/hep-th/0605231. Exterior core representation and asymptotic comparison, especially Sections 5–6. The convergence theorem in this paper is proved separately.

[3] E. R. Bezerra de Mello, V. B. Bezerra, A. A. Saharian, and H. H. Harutyunyan, Vacuum currents induced by a magnetic flux around a cosmic string with finite core, *Physical Review D* **91** (2015), 064034. https://arxiv.org/abs/1411.1258. Finite-core flux-profile and current context.

[4] N. R. Khusnutdinov and M. Bordag, Ground state energy of massive scalar field in the background of finite thickness cosmic string, *Physical Review D* **59** (1999), 064017. https://arxiv.org/abs/gr-qc/9810066. Finite-thickness and spherical-cap comparison. Its different dimensional and perturbative calculation is not identified with the present four-dimensional moments.

[5] S. Hollands and R. M. Wald, Local Wick polynomials and time ordered products of quantum fields in curved spacetime, *Communications in Mathematical Physics* **223** (2001), 289–326. https://arxiv.org/abs/gr-qc/0103074. Smooth-background normalization and scaling framework; no interacting singular-core limit is imported.

[6] Y. Décanini and A. Folacci, Hadamard renormalization of the stress-energy tensor for a quantized scalar field in a general spacetime of arbitrary dimension, *Physical Review D* **78** (2008), 044025. https://arxiv.org/abs/gr-qc/0512118. Smooth-background local ambiguities, integrated heat coefficients, and

effective-action normalization. Metric-variation conventions are explicitly fixed in this paper.

[7] D. V. Vassilevich, Heat kernel expansion: user's manual, *Physics Reports* **388** (2003), 279–360. https://arxiv.org/abs/hep-th/0306138. Especially the smooth Laplace-type coefficient in Eq. (4.28), including the connection-curvature term. Our a_2 denotes the coefficient of u^2 inside the normalized heat expansion.

[8] F. W. J. Olver et al., editors, *NIST Digital Library of Mathematical Functions*, Chapter 10. https://dlmf.nist.gov/10. Modified-Bessel recurrences, positivity, integral representations, endpoint expansions, and the product Mellin representation in Section 10.32.

Cheung source and companion records. These entries distinguish complete supplied sources, supplied excerpts, and documentary comparisons. Citation does not assert an independently executed computation or verification of an entire corpus.

[9] N. Cheung, *The Philosophy of Enter: Destructive Readout, Quantum Recovery, and the Refusal of Identification*, Cheung Ngo Medical Limited (2026). Framing essay; not a theorem premise. Programme note: https://enter.grok.me/enter.

[10] N. Cheung, *I-B-v1: A Finite Enter Spacetime: Calibrated Destructive Time Readout on a Prescribed Conical Product*, version 1, supplied source descriptions and excerpts, Enter Calculus Companion. https://enter.grok.me/browse/I-B. Prescribed host, thm:instrument, and prop:not-source. Companion record; not a Scholar-indexed journal article.

[11] N. Cheung, *En09-A*, version 1, Enter Calculus Companion. https://enter.grok.me/browse/En09-A. Complete destructive clock record–residual interface and non-extraction boundary. Companion record; not a Scholar-indexed journal article.

[12] N. Cheung, *X-A-ST*, version 1, supplied excerpts, Enter Calculus Companion. prop:noeinstein and the distinction between finite reporting and supplied continuum evaluations. https://enter.grok.me/browse/X-A-ST. Companion record; not a Scholar-indexed journal article.

[13] N. Cheung, *I-B-VP-CORE: Curvature cores of a conical string: The* log log *law for the apex Wick-square contact, and non-universality of the conformal stress contact up to dilation*, 18 October 2026. Complete supplied source I-B-VP-CORE.tex, Enter Calculus Companion. Immediate source improved and corrected here. Its historical numerical artifacts were not independently executed. Companion record; not a Scholar-indexed journal article.

[14] N. Cheung, *I-B-VP-5*, version 1, supplied version-qualified excerpts and the descriptions retained in CORE, Enter Calculus Companion. Matching length, Friedrichs operator selection, and contact-separation context. No unavailable full proof is represented as independently audited. https://enter.grok.me/browse/I-B-VP-5. Companion record; not a Scholar-indexed journal article.

[15] N. Cheung, *I-B-VP-1: Prescribed apex finite parts on a conical Enter spacetime: Wick square, stress tensor, and charged current: strong extension, Ward identities, and classified contact freedom*, version 3, October 2026. Complete supplied source I-B-VP-1-v4.tex, Enter Calculus Companion. Cone finite-part conventions and the class-relative contact/dilation comparison. https://enter.grok.me/browse/I-B-VP-1. Companion record; not a Scholar-indexed journal article.

[16] N. Cheung, *I-B-VP-1-Ex1: Worldsheet domains, core diagnostics, and first-order apex dependence*, version 1, 18 October 2026. Complete supplied source I-B-VP-1-Ex1-v2.tex, Enter Calculus Companion. Exterior diagnostic, transfer obligation, negative-coupling variational proof, and stress-scale caution. https://enter.grok.me/browse/I-B-VP-1-Ex1. Companion record; not a Scholar-indexed journal article.

[17] N. Cheung, *I-B-VP-1-Ex2: Spectral tails, exterior contact limits, and admissibility on a prescribed conical spacetime*, version 1, 18 October 2026. Complete supplied source I-B-VP-1-Ex2-v2.tex, Enter Calculus Companion. Exterior concentration and moment-controlled transfer comparison. No unrelated boundary-field theorem is needed here. https://enter.grok.me/browse/I-B-VP-1-Ex2. Companion record; not a Scholar-indexed journal article.

[18] N. Cheung, *I-B-G: Intrinsic and holonomy-twisted Green operators on a prescribed conical spacetime*,

version 2, supplied excerpts, Enter Calculus Companion. Bessel and extension conventions, positivity, and transfer-domain distinctions. The radial estimates used here are proved explicitly. https://enter.grok.me/browse/I-B-G. Companion record; not a Scholar-indexed journal article.

[19] N. Cheung, *I-B-G-S*, companion identifier named in the supplied plan, Enter Calculus Companion. No complete version-qualified analytical passage was supplied or accessed for use as a premise. No Bernoulli-jet theorem is inferred from the identifier. https://enter.grok.me/browse/I-B-G-S. Companion record; not a Scholar-indexed journal article.

[20] N. Cheung, *I-B-VP: Vacuum polarization of the Friedrichs ground state on a prescribed conical spacetime*, version-qualified excerpts supplied with the problem, Enter Calculus Companion. Spectral prescription, infrared estimates, and regular-host one-point conventions. https://enter.grok.me/browse/I-B-VP. Companion record; not a Scholar-indexed journal article.

[21] N. Cheung, *I-B-VP-AB*, version 2, supplied companion description, Enter Calculus Companion. Regular coefficient normalization comparison; no unprovided proof is imported. Companion record; not a Scholar-indexed journal article.

[22] N. Cheung, *I-B-VP-0: The holonomy-odd vacuum current of the charged Friedrichs ground state*, version 1, supplied version-qualified descriptions and excerpts, Enter Calculus Companion. Charged complex-first-jet convention and regular-current comparison only. The smooth flux profile and its local running are separately specified here. https://enter.grok.me/browse/I-B-VP-0. Companion record; not a Scholar-indexed journal article.

[23] N. Cheung, *XVI-O1-ST*, version 5, supplied integrity descriptions and consulted documentary record, Enter Calculus Companion. https://enter.grok.me/browse/XVI-O1-ST. Evidence classes, residual retention, complete reporting, and execution-scope discipline. Companion record; not a Scholar-indexed journal article.

[24] N. Cheung, *Oai282-A*, version 1, supplied excerpts prop:coverage and prop:invisible, Enter Calculus Companion. Class-relative finite-test coverage only; no external OpenAI catalogue theorem is a premise. https://enter.grok.me/browse/Oai282-A. Companion record; not a Scholar-indexed journal article.

[25] N. Cheung, *XIV-H*, *XIV-I*, *XIV-J*, and *XV-B*, version-qualified excerpts supplied with the problem, Enter Calculus Companion. Historical H2 ending, constructed I2 internal amplitude, unresolved I3/J2 full map, and the distinct unopened downstream quantum-admission exit. No characteristic coefficient is constructed by this paper. https://enter.grok.me/browse/XIV-H. https://enter.grok.me/browse/XIV-I. https://enter.grok.me/browse/XIV-J. https://enter.grok.me/browse/XV-B. Companion record; not a Scholar-indexed journal article.

[26] N. Cheung, *XVI-N*, version r12; *XVI-O*, version o70-sync; and the separately scoped *XVI-Q* record, supplied version-qualified scope descriptions, Enter Calculus Companion. Documentary programme boundaries only. No unrelated status is independently verified or changed. https://enter.grok.me/browse/XVI-N. https://enter.grok.me/browse/XVI-O. https://enter.grok.me/browse/XVI-Q. Companion record; not a Scholar-indexed journal article.

[27] N. Cheung, *Black-Hole Information as an Access Problem: Enter, Quantum Recovery, and Complete Residuals*, figshare (2026). DOI: 10.6084/m9.figshare.34335024.v1. https://doi.org/10.6084/m9.figshare.34335024.v1.

[28] N. Cheung, *Operational Wormhole Certificates—Existence, Dimension-Dependent Scaling, Operational Functions, and Composition*, figshare (2026). DOI: 10.6084/m9.figshare.34549575.v1. https://doi.org/10.6084/m9.figshare.34549575.v1.

[29] N. Cheung, *Enter, Quantum Magic, and Quantum Gravity: Readout, Recovery, and the Limits of Geometric Response*, figshare (2026). DOI: 10.6084/m9.figshare.34335582.v1. https://doi.org/10.6084/m9.figshare.34335582.v1.

[30] N. Cheung, *Operational Consistency Does Not Select Lorentzian Geometry*, figshare (2026). DOI: 10.6084/m9.figshare.34441758.v1. https://doi.org/10.6084/m9.figshare.34441758.v1.

[31] N. Cheung, *Realization of Relational Time by Destructive Enter on a Clock Register*, figshare (2026). DOI: 10.6084/m9.figshare.34442907.v1. https://doi.org/10.6084/m9.figshare.34442907.v1.