I-B-VP-5-Ex2-B Audited smooth flux-core selection on a prescribed cone

Exhausted-annulus Mosco convergence, Dirac sign conventions, angular truncation, and threshold square domains

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Abstract

We give a self-contained robustness audit and extension of I-B-VP-5-Ex2 for free fields on prescribed smooth curvature-and-flux cores.

Scalar package. For every admissible family and fixed nonnegative curvature coupling, the transported magnetic forms Mosco-converge to the twisted Friedrichs form. We identify each weak derivative on exhausted regular annuli, retain the exact transport commutator, and prove a quantitative axis-removal estimate. A correction to the proposed proof is essential: bounded-energy sequences can retain nonzero energy in the shrinking core. We construct such a sequence. Core-energy disappearance is instead proved for the resolvent minimizers themselves. The signed matching coefficient remains profile dependent, while the established localized upper estimate is uniform over profiles for fixed channel and spectral data.

Dirac package. A direct calculation fixes the spin connection, radial transport, Green form, selected boundary line, and Pauli-square signs in one consistent convention. An explicit angular-tail estimate completes the passage from radial channels to the full transverse strong-resolvent limit. In the critical window the smooth centre selects $A = 0$, and the square is unitarily equivalent to $h_{\nu_+,\mathrm{F}} \oplus h_{\nu_-,\mathrm{E}_0}$. At the equal-weight point this is Dirichlet–Neumann on the flattened half-line. At both limit-point thresholds the order-zero logarithmic alternative is excluded by the

first-order graph condition. We also prove strong-resolvent continuity to these thresholds, including simultaneous shrinking cores with varying flux approaching a threshold.

The contribution is a proof-level consolidation, with explicit counterexamples, domain identifications, and parameter-endpoint control; it is not a priority claim for Aharonov–Bohm regularization or extension theory. Its potential utility is to prevent concentration defects, radial sign choices, or nonuniform boundary traces from being mistaken for different physical core limits. Limiting-domain independence does not mean profile-independent finite-radius scattering. No one-point contact normalization, renormalized vacuum limit, interacting product, or gravitational source law is obtained.

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not claimed to be Google Scholar-indexed. The published Enter Calculus article is cited only for finite-instrument / operational interface discussions, never as a PDE premise.

Book expansion: change log and concentration pair

Change log vs Ex2. Corrections to Ex2 are listed in the body; this book note flags the pair: (i) arbitrary bounded-energy concentration example (blocks an overclaim); (ii) resolvent-minimizer (narrower positive result). Threshold square-domain analysis stays distinct from the subcritical boundary-trace argument. **Boxed obstruction.** One cannot impose componentwise Friedrichs conditions and assume they are the square of a Dirac extension. Scalar Mosco convergence, Dirac first-order selection, and the squared-domain consequence remain separate sections.

Framework, source scope, and the corrected task

Enter, interpretation, and prescribed spacetime

The operational Enter calculus and the philosophical account of Enter distinguish destructive readout from quantum return, and a classical record from an identification with a continuum physical object [9, 10]. The philosophy essay is interpretive background, not a premise of the operator proofs.

For a finite calibrated clock C, surviving register A, and orthonormal clock basis $\{|\tau_j\rangle\}$, put

$$D_j = \langle \tau_j|_C \otimes I_A, \qquad \mathscr{M}(X) = \sum_j \text{lvertj}, w_j\rangle\text{langlej}, w_j|_R \otimes D_j X D_j^\dagger.$$

Its Kraus operators satisfy

$$\sum_j D_j^\dagger D_j = I_{CA}, \qquad \mathrm{Tr}_R \circ \mathscr{M}(X) = \mathrm{Tr}_C X.$$

Both identities follow by clock matrix-unit expansion and hold with every untouched finite reference. The output retains the actual conditional residual on a positive-probability branch; a zero branch has no assigned normalized state. The measured quantum names are not returned. This is the I-B/En09-A interface [11, thm:instrument] [19].

I-B places this finite interface on a separately prescribed conical spacetime [11]. It does not acquire an unknown operator domain or flux profile, consume the geometric time coordinate, or derive the background metric.

Scope boundary (Non-extraction and non-sourcing). The metric, actual flux lift, charge convention, spin structure, couplings, Hilbert-space identifications, and smooth-core realizations are declared inputs. An operator limit or a conserved tensor does not impose an Einstein or Maxwell equation. This is the separation of I-B prop:not-source and X-A-ST prop:noeinstein [11, 20]. It applies to every domain, square, and contact statement below.

What is being extended

The complete supplied source I-B-VP-5-Ex2-v2.tex is the immediate predecessor [12]. The complete supplied I-B-VP-5-Ex1-v2.tex is a distinct scalar-observable companion [13]. The latter's exterior sums and apex moments are not premises for the operator convergence proved here.

I-B-VP-5 supplies the untwisted comparison [14]. I-B-G and I-B-VP supply exterior-extension and holonomy context [15, 17]. I-B-G-S concerns different field and source-test compatibility conditions [16]. The results required for this paper are stated and proved again, rather than inferred from those documentary relationships.

The extension-theoretic setting is established literature: conical boundary conditions, inverse-square realizations, and regularized Dirac/Pauli flux tubes have been studied independently of Enter [2, 3, 4, 5, 6, 7]. The present contribution is the explicit audit of this specified core class, not a new general extension theory.

Three necessary corrections to the proposed argument

1. Discarding positive energy is not proving that it vanishes. The lower Mosco inequality uses nonnegativity of the core contribution. It does not require that contribution to tend to zero for every bounded-energy sequence. Such a conclusion is false; Section gives a counterexample.

2. The transport commutator need not be globally small. No bound on p_ε' is assumed. The commutator is retained exactly, and it is exactly zero on a fixed exterior annulus once ε is small. There is no need to estimate it

there by a nonzero error.

3. Equal weights and logarithmic thresholds are different. At $\gamma = 0$, the square has order 1/2, not order zero: there is no logarithmic Frobenius solution. At $\gamma = \pm 1/2$, one square component has order zero and admits a logarithmic alternative at the level of its maximal second-order domain. The first-order graph condition excludes that alternative.

A further clarification concerns uniformity: the limiting domain is independent of the admissible profiles; finite-radius matching coefficients generally are not. Nevertheless, several upper bounds in this paper are uniform over all those profiles. Profile dependence of a coefficient does not invalidate a uniform upper bound.

Core class and fixed Hilbert-space identifications

Fix

$$0 < \alpha < 1, \qquad 0 < \beta < 1, \qquad \xi \geq 0.$$

Here $2\pi(1 - \alpha)$ is the conical deficit and $2\pi\beta$ is the *actual* flux. It is not silently replaced by an arbitrary integer translate: a punctured-plane gauge transformation need not extend smoothly through the core centre.

For each $\varepsilon > 0$, let

$$g_\varepsilon = -\mathrm{d}t^2 + p_\varepsilon(r)^2\mathrm{d}r^2 + \alpha^2 r^2 \mathrm{d}\varphi^2 + \mathrm{d}z^2, \qquad D_\varepsilon = \mathrm{d} + \mathrm{i}a_\varepsilon(r)\mathrm{d}\varphi.$$

Assumption (Admissible cores). The profiles are smooth and satisfy:

1. p_ε is nondecreasing,

$$\alpha \leq p_\varepsilon \leq 1, \qquad p_\varepsilon = 1 \quad (r \geq \varepsilon),$$

and $p_\varepsilon = \alpha$ in a neighbourhood of zero.

2. a_ε is nondecreasing,

$$0 \leq a_\varepsilon \leq \beta, \qquad a_\varepsilon = \beta \quad (r \geq \varepsilon),$$

and near zero $a_\varepsilon(r) = r^2 b_\varepsilon(r^2)$ for a smooth b_ε.

3. The exterior gluing is smooth. Profiles may depend arbitrarily on ε; no uniform derivative bounds are imposed.

4. The scalar has no additional short-range interaction. The Dirac operator has minimal magnetic coupling and the spin structure extending over the smooth disk. No electric potential or anomalous magnetic moment is added.

The transverse measure, curvature, and magnetic field are

$$\mathrm{d}A_\varepsilon = \alpha p_\varepsilon r \,\mathrm{d}r\,\mathrm{d}\varphi, \qquad R_\varepsilon = \frac{2p_\varepsilon'}{r p_\varepsilon^3}, \qquad \mathscr{B}_\varepsilon = \frac{a_\varepsilon'}{\alpha p_\varepsilon r}.$$

In particular

$$R_\varepsilon, \mathscr{B}_\varepsilon \geq 0, \qquad \int R_\varepsilon \mathrm{d}A_\varepsilon = 4\pi(1-\alpha), \qquad \int \mathscr{B}_\varepsilon \mathrm{d}A_\varepsilon = 2\pi\beta.$$

These formulas follow by using geodesic distance $\mathrm{d}s = p_\varepsilon \mathrm{d}r$ and circumference function $f(s) = \alpha r(s)$, for which $R = -2f''/f$. The flat centre is smooth in Cartesian coordinates. The connection is smooth because

$$r^2 \mathrm{d}\varphi = x^1 \mathrm{d}x^2 - x^2 \mathrm{d}x^1.$$

The class is nonempty: choose smooth monotone steps $P = \alpha$ near zero, $P = 1$ near and beyond one, and $F = 0$ near zero, $F = 1$ near and beyond one, and set

$$p_\varepsilon(r) = P(r/\varepsilon), \qquad a_\varepsilon(r) = \beta F(r/\varepsilon).$$

For scalars put

$$N = \mathbb{R}_+ \times S^1 \times \mathbb{R}, \qquad \mathrm{d}V_0 = \alpha r \,\mathrm{d}r\,\mathrm{d}\varphi\,\mathrm{d}z, \qquad \mathscr{H}_0 = L^2(N, \mathrm{d}V_0),$$

$$\mathscr{H}_\varepsilon = L^2(N, p_\varepsilon \mathrm{d}V_0), \qquad U_\varepsilon u = p_\varepsilon^{1/2} u.$$

All scalar resolvent limits use U_ε. Spinor radial identifications are specified independently in Section .

Scalar form domain, transport, and axis removal

The magnetic form domain

Define on $C_c^\infty(N)$

$$q_{\mathrm{F},\beta}[u] = \int_N e_\beta[u]\,\mathrm{d}V_0, \qquad e_\beta[u] = |\partial_r u|^2 + \frac{|(\partial_\varphi + \mathrm{i}\beta)u|^2}{\alpha^2 r^2} + |\partial_z u|^2.$$

Its closure is the exterior Friedrichs form.

Lemma (Domain and quantitative removal of the axis). *Let $d_\beta = \min(\beta, 1-\beta) > 0$. The maximal finite-energy space for is its closed form domain, and*

$$\operatorname{Dom} q_{\mathrm{F},\beta} = H^1(\mathbb{R}^3) \cap L^2(\mathbb{R}^3, r^{-2}\mathrm{d}V_0).$$

For every radial measurable set E,

$$\int_E \frac{|u|^2}{r^2}\mathrm{d}V_0 \le \frac{\alpha^2}{d_\beta^2}\int_E \frac{|(\partial_\varphi + \mathrm{i}\beta)u|^2}{\alpha^2 r^2}\mathrm{d}V_0.$$

Choose $0 \le \eta_\delta \le 1$, zero for $r \le \delta$, one for $r \ge 2\delta$, with $|\eta_\delta'| \le C/r$. Then

$$\|(1-\eta_\delta)u\|^2 \le 4\delta^2 \int_{r<2\delta} \frac{|u|^2}{r^2}\mathrm{d}V_0,$$

$$q_{\mathrm{F},\beta}[(1-\eta_\delta)u] \le C_{\alpha,\beta,\eta} \int_{r<2\delta} e_\beta[u]\mathrm{d}V_0.$$

Both right sides tend to zero. Consequently $C_c^\infty(N)$, with supports avoiding the axis, is form dense.

Proof. Fourier expansion of periodic scalar components gives

$$\int_0^{2\pi} |(\partial_\varphi + \mathrm{i}\beta)u|^2 \mathrm{d}\varphi \ge d_\beta^2 \int_0^{2\pi} |u|^2 \mathrm{d}\varphi.$$

Integration over E proves . The angular ordinary derivative is controlled by

$$|\partial_\varphi u| \le |(\partial_\varphi + \mathrm{i}\beta)u| + \beta|u|.$$

Thus finite magnetic energy controls the ordinary Cartesian H^1 energy away from the axis, and conversely H^1 together with $u/r \in L^2$ controls the magnetic energy.

The first cutoff estimate follows from $r < 2\delta$. For the second, the only extra derivative is $-\eta_\delta' u$. Its squared norm is bounded by $C\int_{\delta < r < 2\delta} |u|^2/r^2$. Apply on that annulus; all other terms are bounded by the energy in $r < 2\delta$.

For a maximal finite-energy function on the punctured space, these cutoffs form a Cauchy sequence in the ordinary H^1 norm as well. Their L^2 limit is the original function, proving its global H^1 extension and . Cutoffs at spatial infinity and smoothing on regular annuli complete the density proof.

par *Interpretive boundary.* This domain or operator statement concerns prescribed continuum data. It supplies neither a finite Enter acquisition map nor a Maxwell or Einstein source equation [11, 19, 20]. par

The exact transported form

The smooth-core scalar form is

$$q_\varepsilon[u] = \int_N \left(p_\varepsilon^{-2} |\partial_r u|^2 + \frac{|(\partial_\varphi + \mathrm{i} a_\varepsilon) u|^2}{\alpha^2 r^2} + |\partial_z u|^2 + \xi R_\varepsilon |u|^2 \right) p_\varepsilon \mathrm{d}V_0.$$

For fixed ε, its domain is ordinary Cartesian $H^1(\mathbb{R}^3)$, with equivalent form norm. The connection and potential are smooth and bounded for that fixed core. Let B_ε be its associated nonnegative operator.

On the fixed space define $Q_\varepsilon[v] = q_\varepsilon[p_\varepsilon^{-1/2} v]$. Direct differentiation gives

$$\text{boxed}\, Q_\varepsilon[v] = \int_N \left[p_\varepsilon^{-2} \left| \partial_r v - \frac{p_\varepsilon'}{2p_\varepsilon} v \right|^2 + \frac{|(\partial_\varphi + \mathrm{i} a_\varepsilon) v|^2}{\alpha^2 r^2} + |\partial_z v|^2 + \xi R_\varepsilon |v|^2 \right] \mathrm{d}V_0.$$

Its domain is $U_\varepsilon H^1(\mathbb{R}^3)$. For fixed ε, multiplication by $p_\varepsilon^{\pm 1/2}$ preserves H^1, so this set also equals H^1; the multiplier bounds on derivatives need not be uniform in ε.

The commutators are exactly

$$= \pm \frac{1}{2} p_\varepsilon^{\pm 1/2 - 1} p_\varepsilon', \qquad [\partial_\varphi, p_\varepsilon^{\pm 1/2}] = [\partial_z, p_\varepsilon^{\pm 1/2}] = 0.$$

They are not discarded globally.

For every fixed $h \in \mathscr{H}_0$,

$$\left\|(p_\varepsilon^{\pm 1/2}-1)h\right\| \le C_\alpha \left\|1_{\{r<\varepsilon\}}h\right\| \to 0.$$

On every region $r > a > \varepsilon$, the multipliers are exactly one and all commutators in are exactly zero. This is the required control of transport in the lower-bound argument; no unsupported smallness of $p_\varepsilon^{'}$ is needed.

Expanded Mosco proof and the concentration correction

Theorem (Scalar selection with explicit annular lower bounds). *For every family in Assumption , Q_ε Mosco-converges to $q_{\mathrm{F},\beta}$. For every $s > 0$ and $f \in \mathscr{H}_0$,*

$$(U_\varepsilon B_\varepsilon U_\varepsilon^{-1}+s)^{-1}f \to (B_{\mathrm{F},\beta}+s)^{-1}f \quad \text{strongly in} \mathscr{H}_0.$$

The limiting form has no additional axis term.

Proof.

Step 1: the weakly convergent sequence and transport. It suffices to consider a sequence $\varepsilon_\ell \mathrm{downarrow}\, 0$ realizing a finite lower limit and satisfying

$$v_\ell \mathrm{rightharpoonup}\, v, \qquad \sup_\ell \left(Q_{\varepsilon_\ell}[v_\ell] + \mathrm{lVertv_}\ell\|^2\right) < \infty.$$

Write $p_\ell = p_{\varepsilon_\ell}$ and $u_\ell = p_\ell^{-1/2} v_\ell$. For any $h \in \mathscr{H}_0$,

$$|\mathrm{ip}\, h u_\ell - v_\ell| \le C_\alpha \mathrm{lVertv_}\ell\| \left\|1_{\{r<\varepsilon_\ell\}}h\right\| \to 0.$$

Hence $u_\ell \mathrm{rightharpoonup}\, v$. No differentiation of a strongly convergent multiplier has been inferred from .

Step 2: identify every derivative on a fixed annulus. For $m \ge 2$, let

$$\Omega_m = \{m^{-1} < r < m,\ |z| < m\} \times S^1.$$

For sufficiently large ℓ,

$$p_\ell = 1, \quad a_{\varepsilon_\ell} = \beta, \quad R_{\varepsilon_\ell} = 0, \quad u_\ell = v_\ell \quad \mathrm{on} \Omega_m.$$

Nonnegativity in therefore gives

$$\int_{\Omega_m} \left(|\partial_r v_\ell|^2 + \left| \frac{(\partial_\varphi + \mathrm{i}\beta) v_\ell}{\alpha r} \right|^2 + |\partial_z v_\ell|^2 \right) \mathrm{d}V_0 \le Q_{\varepsilon_\ell}[v_\ell].$$

Together with the L^2 bound, this controls ordinary $H^1(\Omega_m)$, since r is bounded above and below and the magnetic angular derivative controls ∂_φ up to an L^2 term.

The weak limits of the three derivatives are identified distributionally. For a compactly supported smooth h inside Ω_m, for example,

$$\int \overline{h}\, \partial_r v_\ell \, \mathrm{d}V_0 = -\int \left(\partial_r \overline{h} + \frac{\overline{h}}{r} \right) v_\ell \, \mathrm{d}V_0.$$

The right side converges by weak L^2 convergence. The longitudinal derivative is identified by the usual integration by parts, and the angular derivative by periodic integration by parts with the fixed smooth coefficient $1/(\alpha r)$. Thus, along a subsequence if necessary,

$$\partial_r v_\ell \rightharpoonup \partial_r v, \qquad \frac{(\partial_\varphi + \mathrm{i}\beta) v_\ell}{\alpha r} \rightharpoonup \frac{(\partial_\varphi + \mathrm{i}\beta) v}{\alpha r}, \qquad \partial_z v_\ell \rightharpoonup \partial_z v$$

in the corresponding $L^2(\Omega_m)$ spaces.

Step 3: lower semicontinuity and exhaustion. Apply weak lower semicontinuity to the norm of the three-component derivative vector in :

$$\liminf_{\ell \to \infty} Q_{\varepsilon_\ell}[v_\ell] \ge \int_{\Omega_m} e_\beta[v] \mathrm{d}V_0.$$

The sets Ω_m increase to N. Monotone convergence of the nonnegative energy density gives

$$\liminf_{\ell \to \infty} Q_{\varepsilon_\ell}[v_\ell] \ge \int_{N} e_\beta[v] \mathrm{d}V_0.$$

Lemma identifies this maximal finite-energy space with $\operatorname{Dom} q_{\mathrm{F},\beta}$, proving the lower Mosco condition.

There was no integration by parts across the axis and no limiting boundary pairing. The core was omitted only because every term of its unexpanded transported energy is nonnegative.

Step 4: recovery and absence of an extra axis energy. For $v \in \operatorname{Dom} q_{\mathrm{F},\beta}$, use Lemma and cutoffs at infinity to

find $\varphi_j \in C_c^\infty(N)$ converging in form norm. For fixed j, all sufficiently small cores miss $\operatorname{supp}\varphi_j$, so exactly

$$U_\varepsilon \varphi_j = \varphi_j, \qquad Q_\varepsilon[\varphi_j] = q_{\mathrm{F},\beta}[\varphi_j].$$

A diagonal sequence is a strong recovery sequence. Its core energy is identically zero. The lower bound and this exact recovery exclude an additional positive axis contribution to the limiting form. These are the Mosco conditions [1].

Step 5: strong resolvents. Let v_ε minimize

$$J_\varepsilon[w] = Q_\varepsilon[w] + s\|w\|^2 - 2\mathrm{Re}\,\mathrm{ip}\,fw,$$

and let v minimize $J_{\mathrm{F},\beta}$. The lower bound, weak compactness, and recovery show that v_ε rightharpoonup v and the minimum values converge. For a recovery sequence w_ε of v, the variational equation gives

$$J_\varepsilon[w_\varepsilon] - J_\varepsilon[v_\varepsilon] = Q_\varepsilon[w_\varepsilon - v_\varepsilon] + s\mathrm{lVertw_}\varepsilon - v_\varepsilon\|^2.$$

The left side tends to zero; hence $v_\varepsilon \to v$ strongly. The minimizers are exactly the resolvents in .

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Why arbitrary bounded-energy sequences can retain core energy

Proposition (A shrinking-core energy defect). *There are admissible self-similar profiles and $v_\varepsilon \to 0$ strongly in $\mathscr{H}_0$ such that*

$$\sup_\varepsilon Q_\varepsilon[v_\varepsilon] < \infty, \qquad \lim_{\varepsilon\mathrm{downarrow}\,0} Q_\varepsilon[v_\varepsilon;\, r < \varepsilon] > 0.$$

Thus vanishing of discarded core energy for every bounded-energy sequence is not a valid step of the Mosco proof.

Proof. Choose admissible P, F and a nonzero $w \in C_c^\infty((c,d))$, $0 < c < d < 1$. Let $g \in C_c^\infty(\mathbb{R})$ be nonzero, and use the normalized zero angular mode:

$$u_\varepsilon(r,\varphi,z) = \frac{w(r/\varepsilon)g(z)}{\sqrt{2\pi\alpha}}, \qquad v_\varepsilon = \sqrt{P(r/\varepsilon)}\,u_\varepsilon.$$

Its squared norm is

$$\mathrm{lVertv_}\varepsilon\|^2 = \varepsilon^2\|g\|^2\int_0^1 sP(s)|w(s)|^2\mathrm{d}s \;\to\; 0.$$

Its entire support lies inside the core. A change of variable gives

$$Q_\varepsilon[v_\varepsilon] = E_w\|g\|^2 + \varepsilon^2\mathrm{lVertg}'\|^2\int_0^1 sP(s)|w(s)|^2\mathrm{d}s,$$

where

$$E_w = \int_0^1 \left[\frac{s}{P}|w'|^2 + \frac{P\beta^2F^2}{\alpha^2 s}|w|^2 + \frac{2\xi P'}{P^2}|w|^2\right]\mathrm{d}s > 0.$$

The derivative term is strictly positive for a nonzero compactly supported w. This proves the assertion.

The same scaling can make the separated transport commutator have nonvanishing norm when w meets the metric transition. The square in , not its separately expanded pieces, is the controlled energy.

Proposition (Core energy does vanish for resolvent minimizers). *For the resolvent vectors v_ε, v in the proof of Theorem ,*

$$Q_\varepsilon[v_\varepsilon] \to q_{\mathrm{F},\beta}[v], \qquad Q_\varepsilon[v_\varepsilon;\, r<\varepsilon] \to 0.$$

No rate in ε is claimed here.

Proof. Convergence of minimum values, the linear term, and the strongly convergent norms implies convergence of total form values.

For fixed $a > 0$ and $\varepsilon < a$,

$$Q_\varepsilon[v_\varepsilon;\, r<\varepsilon] \le Q_\varepsilon[v_\varepsilon] - q_{\mathrm{F},\beta}[v_\varepsilon;\, r>a].$$

The same annular lower semicontinuity, exhausted also in r and z, yields

$$\operatorname*{limsup}_{\varepsilon\,\mathrm{downarrow}\,0} Q_\varepsilon[v_\varepsilon;\, r<\varepsilon] \le q_{\mathrm{F},\beta}[v;\, r\le a].$$

Let *a*downarrow 0. The last integral tends to zero because the limiting energy density is integrable.

This is the precise valid replacement for the proposed universal core-energy disappearance argument.

Scalar channels and the meaning of a profile-uniform bound

In the normalized angular basis $e^{in\varphi}/\sqrt{2\pi\alpha}$, the exterior transverse expression is

$$h_\nu = -\partial_r^2 - r^{-1}\partial_r + \nu^2 r^{-2}, \qquad \nu = \frac{|n+\beta|}{\alpha} > 0,$$

on $L^2(\mathbb{R}_+, r\,dr)$. It is deficient exactly when $0 < \nu < 1$. There are at most two deficient scalar channels: $n = 0$ if $\beta < \alpha$, and $n = -1$ if $1 - \beta < \alpha$. In such a channel write

$$f(r) = Ar^\nu + Br^{-\nu} + o(r^\nu).$$

Friedrichs means $B = 0$, and E_0 means $A = 0$. These are the Bessel conventions used here [2, 3, 15]. Theorem selects Friedrichs in each deficient channel.

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For completeness we retain the matching estimate needed to distinguish profile-dependent coefficients from profile-uniform upper bounds.

Proposition (Signed matching and localized resolvent control). *Let v be the positive regular zero-energy core solution, normalized by* $v(\varepsilon) = 1$, *and set*

$$\Lambda_{\varepsilon,n} = \varepsilon v'(\varepsilon).$$

Then $\Lambda_{\varepsilon,n} \geq 0$, *and its exterior continuation obeys*

$$\frac{B_\varepsilon}{A_\varepsilon} = \varepsilon^{2\nu}\frac{\nu - \Lambda_{\varepsilon,n}}{\nu + \Lambda_{\varepsilon,n}}, \qquad \left|\frac{B_\varepsilon}{A_\varepsilon}\right| \leq \varepsilon^{2\nu}.$$

The regular coefficient A_ε *is nonzero.*

At spectral parameter $-k^2$, $k>0$, let $\Lambda_{\varepsilon,n}(k)$ be the corresponding regular logarithmic derivative. Then

$$0 \le \Lambda_{\varepsilon,n}(k) - \Lambda_{\varepsilon,n}(0) \le k^2\varepsilon^2/2.$$

The exterior radial resolvent correction is

$$c_{\varepsilon,n}(k)K_\nu(kr)K_\nu(kr'),$$

where, for $t = k\varepsilon$,

$$c_{\varepsilon,n}(k) = \frac{tI_\nu'(t) - \Lambda_{\varepsilon,n}(k)I_\nu(t)}{\Lambda_{\varepsilon,n}(k)K_\nu(t) - tK_\nu'(t)}.$$

For fixed $\nu, k, a > 0$, with $P_a = 1_{\{r \ge a\}}$,

$$\begin{aligned}\left\| P_a\left[(U_\varepsilon h_{\varepsilon,n} U_\varepsilon^{-1} + k^2)^{-1} - (h_{\nu,\mathrm{F}} + k^2)^{-1}\right]P_a\right\| \\ = |c_{\varepsilon,n}(k)| \int_a^\infty K_\nu(kr)^2 r\,\mathrm{d}r \le C_{\nu,k,a}\varepsilon^{2\nu}.\end{aligned}$$

The upper-bound constant can be chosen independently of the admissible profiles. For $0 < \nu < 1$,

$$c_{\varepsilon,n}(k) = \frac{2^{1-2\nu}(k\varepsilon)^{2\nu}\ \nu - \Lambda_{\varepsilon,n}(0)}{\Gamma(\nu)\Gamma(1+\nu)\,\nu + \Lambda_{\varepsilon,n}(0)} + O((k\varepsilon)^{4\nu}),$$

again with a profile-uniform remainder for fixed ν, k.

Proof. The regular equation is

$$\left(\frac{r}{p_\varepsilon}v'\right)' = \left[\frac{p_\varepsilon(n+a_\varepsilon)^2}{\alpha^2 r} + \xi p_\varepsilon r R_\varepsilon\right]v.$$

Its regular positive leading coefficient and the nonnegative right side give $v > 0$, $v' \ge 0$. The normalized solution is at most one inside the core.

Its logarithmic derivative is the minimum, subject to $w(\varepsilon) = 1$, of

$$\int_0^\varepsilon \left[\frac{r}{p_\varepsilon}|w'|^2 + \left(\frac{p_\varepsilon(n+a_\varepsilon)^2}{\alpha^2 r} + \xi p_\varepsilon r R_\varepsilon\right)|w|^2\right]\mathrm{d}r.$$

Integration by parts identifies the minimum with $\varepsilon v'(\varepsilon)$. Adding $k^2 p_\varepsilon r|w|^2$ gives the negative-energy minimum.

Evaluating it on the zero-energy minimizer proves .

Matching $Ar^{\nu} + Br^{-\nu}$ and its logarithmic derivative proves . A pure singular solution would have logarithmic derivative $-\nu$, which is impossible.

The regular exterior negative-energy solution is $I_{\nu} + cK_{\nu}$; matching gives . The denominator is positive, and

$$|c_{\varepsilon,n}(k)| \le \max\left\{\frac{I_{\nu}(t)}{K_{\nu}(t)}, \frac{I_{\nu}'(t)}{-K_{\nu}'(t)}\right\} = O(t^{2\nu}).$$

The radial Wronskian gives the stated rank-one correction. Its norm on $r \ge a$ is exactly .

For $0 < \nu < 1$, insert

$$I_{\nu}(t) = \frac{2^{-\nu}}{\Gamma(1+\nu)} t^{\nu}(1 + O(t^2)),$$

$$K_{\nu}(t) = 2^{\nu-1}\Gamma(\nu)t^{-\nu} + 2^{-\nu-1}\Gamma(-\nu)t^{\nu} + O(t^{2-\nu})$$

and their differentiated expansions. After factoring $(\nu + \Lambda)t^{-\nu}$ in the denominator, all ratios involving $\Lambda \ge 0$ are uniformly bounded. Equation then replaces $\Lambda(k)$ by $\Lambda(0)$. The error is $O(t^{4\nu} + t^{2\nu+2}) = O(t^{4\nu})$. The Bessel identities are standard [8, Sections 10.28–10.30].

The coefficient in can vary with the profiles, can vanish, and need not converge when the profiles vary with ε. For self-similar profiles its impedance is the fixed number

$$\min_{w(1)=1} \int_0^1 \left[\frac{x}{P}|w'|^2 + \frac{P(n+\beta F)^2}{\alpha^2 x}|w|^2 + \frac{2\xi P'}{P^2}|w|^2\right] \mathrm{d}x.$$

This explicitly records profile dependence.

The signed effective data are

$$\sigma_{\varepsilon,n} = \operatorname{sign}(B_{\varepsilon}/A_{\varepsilon}), \qquad L_{\mathrm{eff},\varepsilon,n} = |B_{\varepsilon}/A_{\varepsilon}|^{1/(2\nu)} \le \varepsilon.$$

They are not a universally retained length. Equation is a localized, fixed-channel upper bound, not a full operator-norm convergence claim.

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Dirac sign audit in a fixed spin and charge convention

We use the standard Pauli matrices

$$\sigma_1 = \begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix}, \qquad \sigma_2 = \begin{pmatrix} 0 & -\mathrm{i} \\ \mathrm{i} & 0 \end{pmatrix}, \qquad \sigma_3 = \begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix},$$

and the inner product $\mathrm{ip}\, uv = \int u^{\dagger} v$, linear in its second entry.

The bounding spin structure is antiperiodic in the polar orthonormal frame:

$$\psi(\varphi + 2\pi) = -\psi(\varphi), \qquad j \in \mathbb{Z} + \frac{1}{2}.$$

It is the spin structure extending over the disk. The magnetic connection is the separate $\mathrm{i} a_{\varepsilon} \mathrm{d}\varphi$.

Proposition (Audited radial and transported signs). *In the coframe $e^1 = p_{\varepsilon}\mathrm{d}r$, $e^2 = \alpha r \mathrm{d}\varphi$, one has*

$$\omega^1{}_2 = -\frac{\alpha}{p_{\varepsilon}}\mathrm{d}\varphi, \qquad \Gamma_{\varphi} = -\frac{\mathrm{i}\alpha}{2p_{\varepsilon}}\sigma_3.$$

The transverse Dirac expression is

$$\mathscr{D}_{\varepsilon} = -\mathrm{i}\sigma_1 p_{\varepsilon}^{-1}\left(\partial_r + \frac{1}{2r}\right) - \frac{\mathrm{i}\sigma_2}{\alpha r}(\partial_{\varphi} + \mathrm{i}a_{\varepsilon}).$$

With angular normalization $e^{\mathrm{i}j\varphi}/\sqrt{2\pi\alpha}$, the radial unitary to $L^2(\mathbb{R}_+, \mathrm{d}r; \mathbb{C}^2)$ is

$$\mathscr{U}_{\varepsilon} f = \sqrt{p_{\varepsilon} r}\, f.$$

It gives

$$d_{\varepsilon, j} = -\mathrm{i}\sigma_1 p_{\varepsilon}^{-1}\left(\partial_r - \frac{p_{\varepsilon}'}{2p_{\varepsilon}}\right) + \sigma_2 \frac{j + a_{\varepsilon}}{\alpha r}.$$

Writing

$$T_{\varepsilon} = p_{\varepsilon}^{-1}\left(\partial_r - \frac{p_{\varepsilon}'}{2p_{\varepsilon}}\right), \qquad w_{\varepsilon,j} = \frac{j + a_{\varepsilon}}{\alpha r},$$

the matrix is

$$d_{\varepsilon,j} = -\mathrm{i}\begin{pmatrix} 0 & T_{\varepsilon} + w_{\varepsilon,j} \\ T_{\varepsilon} - w_{\varepsilon,j} & 0 \end{pmatrix}.$$

Outside the core,

$$d_{\gamma_j} = -\mathrm{i}\sigma_1\partial_r + \sigma_2\frac{\gamma_j}{r}, \qquad \gamma_j = \frac{j + \beta}{\alpha}.$$

For compactly supported maximal-domain vectors on an interval (a, b),

$$\mathrm{i}\mathrm{p}\, u d_{\varepsilon,j} v - \mathrm{i}\mathrm{p}\, d_{\varepsilon,j} u v = -\mathrm{i}\left[p_{\varepsilon}^{-1} u^{\dagger}\sigma_1 v\right]_a^b.$$

Proof. Cartan's equation for the displayed coframe gives the connection one-form. The spin representation gives $\Gamma_{\varphi} = \frac{1}{2}\omega_{12,\varphi}\sigma_1\sigma_2$, hence the displayed sign. Its contribution to $-\mathrm{i}\sigma_2(\alpha r)^{-1}\Gamma_{\varphi}$ is $-\mathrm{i}\sigma_1/(2p_{\varepsilon}r)$, proving .

For $f = (p_{\varepsilon}r)^{-1/2}u$,

$$\left(\partial_r + \frac{1}{2r}\right)f = (p_{\varepsilon}r)^{-1/2}\left(\partial_r u - \frac{p_{\varepsilon}'}{2p_{\varepsilon}}u\right).$$

This proves and its matrix form. Finally,

$$T_{\varepsilon} = p_{\varepsilon}^{-1}\partial_r + \frac{1}{2}(p_{\varepsilon}^{-1})'$$

is formally skew-adjoint in $L^2(\mathrm{d}r)$. Integration by parts proves .

As an independent flat-centre check, the change from polar to Cartesian spin components is

$$\psi_{\mathrm{Cart}} = e^{-\mathrm{i}\varphi\sigma_3/2}\psi_{\mathrm{polar}}.$$

The lower component in $j = -1/2$, with constant physical radial leading term, is therefore Cartesian smooth. This checks the chirality used in the core continuation below.

The signs in Proposition agree with the displayed convention of the supplied Ex2. This is an analytic calculation, not a symbolic-program run or a verification of signs in every cited source. Other sources can use the opposite charge, angular, or spin-frame convention [4, 5, 7, 12].

First-order boundary traces and smooth-centre selection

Critical traces and their quantitative remainders

For each fixed smooth core the radial realization is the unique self-adjoint smooth-centre realization. Near zero $p_\varepsilon = \alpha$ and $a_\varepsilon = O(r^2)$; the flattened leading powers are $r^{\pm j}$. Since $|j| \geq 1/2$, the centre is limit point. Infinity is also limit point.

For the exterior expression d_γ, zero is limit circle exactly when $|\gamma| < 1/2$. This follows from the two leading zero-energy solutions

$$(r^\gamma, 0)^T, \qquad (0, r^{-\gamma})^T;$$

the nonreal-energy regular-singular system has the same integrability classification. Infinity is limit point. We use the standard first-order Weyl alternative here, as in the Dirac extension literature [4, 5, 7].

For $0 < \beta < 1$, only $j_* = -1/2$ can be critical. Its parameter and window are

$$\gamma = \frac{\beta - \frac{1}{2}}{\alpha}, \qquad |\gamma| < \frac{1}{2} \quad \Leftrightarrow \quad \frac{1-\alpha}{2} < \beta < \frac{1+\alpha}{2}.$$

Lemma (Boundary traces, Green form, and a graph core). *Suppose $|\gamma| < 1/2$. For u in the maximal domain and $f = d_\gamma u$, the traces*

$$A(u) = \lim_{r \downarrow 0} r^{-\gamma} u_+(r), \qquad B(u) = \lim_{r \downarrow 0} r^{\gamma} u_-(r)$$

exist and are graph-norm continuous. More precisely,

$$|u_+(r) - A(u) r^\gamma| \leq \frac{r^{1/2}}{\sqrt{1-2\gamma}} \| f - \|_{L^2(0,r)},$$

$$|u_{-}(r) - B(u) r^{-\gamma}| \le \frac{r^{1/2}}{\sqrt{1+2\gamma}} \mathrm{lVertf}_{-} + \|_{L^2(0,r)}.$$

The endpoint Green form is

$$\mathrm{ip}\, u d_{\gamma} v - \mathrm{ip}\, d_{\gamma} u v = \mathrm{i}\big(\overline{A(u)} B(v) + \overline{B(u)} A(v)\big).$$

The minimal closed domain has $A = B = 0$. All self-adjoint boundary lines are

$$A = \mathrm{i}tB, \quad t \in \mathbb{R}, \qquad \text{or} \qquad B = 0.$$

In particular,

$$\mathrm{Dom}\, d_{\gamma}^{\mathrm{sel}} = \{u \in \mathrm{Dom}\, d_{\gamma,\max} : A(u) = 0\}$$

is self-adjoint and has the operator core

$$C_c^{\infty}(\mathbb{R}_{+}; \mathbb{C}^2) + \mathrm{span}\,\{(0, \chi r^{-\gamma})^T\},$$

where χ is compactly supported and equals one near zero.

Proof. The matrix in gives

$$u_{+}(r) = r^{\gamma}\left(A + \mathrm{i}\int_0^r s^{-\gamma} f_{-}(s)\mathrm{d}s \right),$$

$$u_{-}(r) = r^{-\gamma}\left(B + \mathrm{i}\int_0^r s^{\gamma} f_{+}(s)\mathrm{d}s \right).$$

Cauchy–Schwarz proves . Graph continuity follows by estimating a trace at a fixed regular interval, where the equation controls ordinary H^1, and then using these integral formulas.

Equation with $p = 1$, together with the remainders in , gives ; the infinity term vanishes on the maximal domain by the limit-point property.

For completeness, zero traces really give the minimal closure. If $A = B = 0$, the weighted Hardy estimate for the integral formulas gives, locally,

$$\mathrm{lVertu}_{+}/r\|_{L^2(0,R)} \le \frac{2}{1-2\gamma}\mathrm{lVertf}_{-}\|_{L^2(0,R)}, \qquad \mathrm{lVertu}_{-}/r\|_{L^2(0,R)} \le \frac{2}{1+2\gamma}\mathrm{lVertf}_{+}\|_{L^2(0,R)}.$$

One obtains this estimate by integration by parts in $\int_0^R r^{2\gamma-2}|\int_0^r s^{-\gamma}f(s)\mathrm{d}s|^2\mathrm{d}r$; the endpoint at R has the favourable sign. Ordinary radial cutoffs therefore remove the origin in graph norm. Cutoffs at infinity and smoothing finish the minimal-domain approximation. Conversely, the continuous traces vanish on its closure.

Cutoff zero solutions realize arbitrary pairs (A, B). The maximal isotropic lines of the nondegenerate boundary form are precisely the listed lines. Subtracting $B(u)(0, \chi r^{-\gamma})^T$ from a vector with $A = 0$ leaves zero traces, proving .

par *Interpretive boundary*. This domain or operator statement concerns prescribed continuum data. It supplies neither a finite Enter acquisition map nor a Maxwell or Einstein source equation [11, 19, 20]. par

A smooth-core continuation of the boundary generator

In the critical channel define

$$b_\varepsilon(r) = \frac{p_\varepsilon(r)(-\frac{1}{2} + a_\varepsilon(r))}{\alpha}, \qquad \gamma_+ = \max(\gamma, 0),$$

and

$$z_\varepsilon(r) = \begin{pmatrix} 0 \\ \sqrt{p_\varepsilon(r)}\,\varepsilon^{-\gamma}\exp\left(-\int_\varepsilon^r b_\varepsilon(s)\frac{\mathrm{d}s}{s}\right) \end{pmatrix}.$$

The off-diagonal matrix shows directly that $d_{\varepsilon,j_*} z_\varepsilon = 0$. For $r \ge \varepsilon$, $z_\varepsilon = (0, r^{-\gamma})^T$. Near zero its lower entry is a constant times $r^{1/2}$ times a smooth function of r^2, so the physical spinor is smooth in the Cartesian spin frame.

The profile restrictions imply $b_\varepsilon \le \gamma_+$: if $\beta \le 1/2$ it is nonpositive; if $\beta > 1/2$ its positive part is at most $(\beta - 1/2)/\alpha$. Consequently

$$\int_0^\varepsilon |z_\varepsilon|^2\mathrm{d}r \le \frac{\varepsilon^{1-2\gamma}}{1-2\gamma_+}.$$

Lemma (Graph convergence implies strong resolvent convergence). *Let T_ε, T be self-adjoint on one Hilbert space. Suppose a core $\mathscr{C}$ of T admits $u_\varepsilon \in \mathrm{Dom}\,T_\varepsilon$ with*

$$u_\varepsilon \to u, \qquad T_\varepsilon u_\varepsilon \to Tu \quad (u \in \mathscr{C}).$$

Then $T_\varepsilon \to T$ strongly in the resolvent sense.

Proof. For $\zeta \notin \mathbb{R}$ and $f = (T - \zeta)u$,

$$(T_\varepsilon - \zeta)^{-1} f - u_\varepsilon = (T_\varepsilon - \zeta)^{-1}((T - \zeta)u - (T_\varepsilon - \zeta)u_\varepsilon).$$

This tends to zero. The set $(T - \zeta)\mathscr{C}$ is dense, and every resolvent is bounded by $1/|\mathrm{Im}\,\zeta|$. Density completes the proof.

Theorem (Channel selection and its uniform boundary-generator estimate). *For every admissible profile family:*

1. *in each noncritical channel, the smooth-core radial operators converge strongly in the resolvent sense to the unique exterior self-adjoint realization;*

2. *in the critical channel, they converge to d_γ^{sel} with $A = 0$. For the additional core generator $u = (0, \chi r^{-\gamma})^T$, set $u_\varepsilon = \chi z_\varepsilon$. For sufficiently small ε,*

$$\mathrm{lVertu_}\varepsilon - u\| \le \left(\frac{1}{\sqrt{1 - 2\gamma_+}} + \frac{1}{\sqrt{1 - 2\gamma}}\right)\varepsilon^{1/2 - \gamma}, \qquad d_{\varepsilon, j_*} u_\varepsilon = d_\gamma u.$$

The estimate is uniform over profiles for fixed γ. It is a graph-generator estimate, not a Dirac operator-norm resolvent rate.

Proof. On compactly supported functions avoiding zero, the expressions agree for all sufficiently small cores. Such functions form a core in a noncritical channel.

In the critical channel use . The approximant u_ε is a smooth-core domain vector. Equation and

$$\int_0^\varepsilon r^{-2\gamma} \mathrm{d}r = \frac{\varepsilon^{1 - 2\gamma}}{1 - 2\gamma}$$

prove its norm estimate. Where $\chi = 1$, both differential images vanish. On the support of χ', the expressions and zero solutions agree exactly when ε is small. The graph images are therefore equal. Apply Lemma .

par *Interpretive boundary*. This domain or operator statement concerns prescribed continuum data. It supplies neither a finite Enter acquisition map nor a Maxwell or Einstein source equation [11, 19, 20]. par

The selected line is not inferred from semibounded scalar forms: the first-order Dirac operator has no Friedrichs extension. The calculation instead uses its actual graph domain and boundary Green form.

Angular direct sums with an explicit tail estimate

Let

$$\mathcal{H} = \operatorname{bigoplus}_{j \in \mathbb{Z}+1/2} L^2(\mathbb{R}_+, \mathrm{d}r; \mathbb{C}^2).$$

Define

$$\mathbf{d}_\varepsilon = \operatorname{bigoplus}_j d_{\varepsilon,j}, \qquad \mathbf{d}_{\mathrm{sel}} = \operatorname{bigoplus}_j d_j^{\mathrm{sel}},$$

where d_j^{sel} is the unique realization in a noncritical channel and in the critical channel. Their direct-sum domains are

$$\operatorname{Dom} \mathbf{d}_\bullet = \left\{ (u_j) : u_j \in \operatorname{Dom} d_{\bullet,j}, ?\sum_j (\mathrm{lVertu_}j\|^2 + \mathrm{lVertd_}\bullet, ju_j\|^2) < \infty \right\}.$$

The direct sums are self-adjoint: their nonreal resolvents are the diagonal sums of the channel resolvents, uniformly bounded by $1/|\operatorname{Im}\zeta|$.

Theorem (Uniform angular truncation and full convergence). *Let Π_J project onto $|j| \le J$, with $J \ge 1/2$. For $f = (f_j) \in \mathcal{H}$, $\zeta \notin \mathbb{R}$, and $R_\varepsilon = (\mathbf{d}_\varepsilon - \zeta)^{-1}$, $R = (\mathbf{d}_{\mathrm{sel}} - \zeta)^{-1}$,*

$$\text{boxed}\, \|(R_\varepsilon - R)f\|^2 \le \sum_{|j| \le J} \left\| ((d_{\varepsilon,j} - \zeta)^{-1} - (d_j^{\mathrm{sel}} - \zeta)^{-1}) f_j \right\|^2 + \frac{4}{|\operatorname{Im}\zeta|^2} \|(1 - \Pi_J) f\|^2.$$

The tail estimate is uniform in ε, j, and the admissible profiles. Hence

$$\mathbf{d}_\varepsilon \to \mathbf{d}_{\mathrm{sel}} \quad \text{in the strong-resolvent sense.}$$

If L is angular multiplication by j and $f \in \operatorname{Dom}(1 + L^2)^{s/2}$, $s > 0$, the tail term is at most

$$\frac{4}{|\operatorname{Im}\zeta|^2}(1+J^2)^{-s}\|(1+L^2)^{s/2}f\|^2.$$

Proof. The angular channels are mutually orthogonal and reduce both resolvents. Therefore the squared norm of the difference is the sum of its squared channel norms. For each omitted channel,

$$\left\|((d_{\varepsilon,j}-\zeta)^{-1}-(d_j^{\mathrm{sel}}-\zeta)^{-1})f_j\right\| \le \frac{2}{|\operatorname{Im}\zeta|}\mathrm{lVertf_}j\|.$$

Summation proves . First choose J so the tail is small; then use Theorem on the finitely many retained channels. The weighted estimate follows by

$$\sum_{|j|>J}\mathrm{lVertf_}j\|^2 \le (1+J^2)^{-s}\sum_j (1+j^2)^s\mathrm{lVertf_}j\|^2.$$

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No uniform-in-j channel convergence rate was assumed. The needed uniformity is the universal nonreal resolvent bound. In particular, the proof does not need to import a finite-propagation theorem from I-B-G into a different spinor domain.

The tail bound controls a fixed input, or an input class with a proved angular norm bound. It is not an all-input operator-norm estimate tending to zero as $J \to \infty$.

Exact square domains, equal weights, and thresholds

Radial unitary notation

Let

$$\mathscr{W}: L^2(\mathbb{R}_+, r\,\mathrm{d}r) \to L^2(\mathbb{R}_+, \mathrm{d}r), \qquad \mathscr{W}f = \sqrt{r}\,f.$$

Then

$$\mathscr{W}h_\nu\mathscr{W}^{-1} = \ell_\nu := -\partial_r^2 + \frac{\nu^2-\frac{1}{4}}{r^2}.$$

The symbols $h_{v,\mathrm{F}}$ and h_{v,E_0}, $0 < v < 1$, refer to the radial $r\,\mathrm{d}r$ space and respectively to the regular and pure singular boundary coefficient. The order-zero Friedrichs realization $h_{0,\mathrm{F}}$ has no logarithmic coefficient. For $v \geq 1$, $h_{v,\mathrm{F}}$ denotes the unique limit-point realization. These conventions agree with the inverse-square extension framework [3].

The complete selected square

Theorem (Square domains in all exterior channels). *For every exterior parameter γ,*

$$d_\gamma^2 = \begin{pmatrix} -\partial_r^2 + \gamma(\gamma-1)r^{-2} & 0 \\ 0 & -\partial_r^2 + \gamma(\gamma+1)r^{-2} \end{pmatrix}.$$

Put $\mathscr{W}_2 = \mathscr{W} \oplus \mathscr{W}$. For the selected realization:

1. *If $|\gamma| < 1/2$, let $v_+ = 1/2 - \gamma$, $v_- = 1/2 + \gamma$. Then the exact operator identity is*

$$(d_\gamma^{\mathrm{sel}})^2 = \mathscr{W}_2 (h_{v_+,\mathrm{F}} \oplus h_{v_-,\mathrm{E}_0}) \mathscr{W}_2^{-1}.$$

2. *If $|\gamma| \geq 1/2$, then*

$$d_\gamma^2 = \mathscr{W}_2 (h_{|\gamma-\frac{1}{2}|,\mathrm{F}} \oplus h_{|\gamma+\frac{1}{2}|,\mathrm{F}}) \mathscr{W}_2^{-1}.$$

At a zero order the condition is no logarithm. These are identities of self-adjoint operators with their domains, not just formal expressions. The Bessel operators without $\mathscr{W}_2$ are unitarily equivalent operators on a different Hilbert space.

Proof. Matrix multiplication in gives

$$d_\gamma^2 = -\partial_r^2 + \gamma^2 r^{-2} - \gamma \sigma_3 r^{-2},$$

proving .

In the critical channel, maximal second-order vectors have the boundary expansions

$$u_+ = a_+ r^{\gamma} + b_+ r^{1-\gamma} + o(r^{3/2}), \qquad u_- = a_- r^{-\gamma} + b_- r^{1+\gamma} + o(r^{3/2}),$$

with differentiated remainders $o(r^{1/2})$. These follow from variation of constants for $\ell_{\nu_\pm} u \in L^2$; the coefficients are second-order boundary coefficients, not pointwise assumptions on arbitrary first-order domain vectors.

Membership in $\operatorname{Dom} d_\gamma^{\mathrm{sel}}$ requires $a_+ = 0$. Membership in its square additionally requires $d_\gamma u \in \operatorname{Dom} d_\gamma^{\mathrm{sel}}$. The upper component of $d_\gamma u$ is

$$-\mathrm{i}(\partial_r + \gamma/r)u_- = -\mathrm{i}(1+2\gamma)b_- r^\gamma + o(r^{1/2}).$$

Thus its A-trace is zero exactly when $b_- = 0$. After division by $\sqrt{r}$, these are the upper Friedrichs and lower E_0 conditions.

For $\gamma > 1/2$, the only potentially deficient upper square order is $\gamma - 1/2 < 1$. Its alternative power is $r^{1-\gamma}$, and

$$(\partial_r - \gamma/r)r^{1-\gamma} = (1-2\gamma)r^{-\gamma} \notin L^2(0,1).$$

It is therefore excluded by $d_\gamma u \in L^2$. The lower square is limit point. For $\gamma < -1/2$, interchange the components: the alternative lower power $r^{1+\gamma}$ gives $(1+2\gamma)r^\gamma \notin L^2$. The threshold logarithms are checked explicitly below.

The chirality matrix preserves each selected first-order domain and anticommutes with the operator. Hence its square reduces into the two components. The resulting square is self-adjoint and satisfies the displayed separated Bessel conditions. The Bessel operator with those conditions is also self-adjoint, so the inclusion is equality.

par *Interpretive boundary.* This domain or operator statement concerns prescribed continuum data. It supplies neither a finite Enter acquisition map nor a Maxwell or Einstein source equation [11, 19, 20]. par

The equal-weight point γ = 0

Proposition (Exact traces and Dirichlet–Neumann square at $\gamma = 0$). *At $\gamma = 0$,*

$$d_0 = -\mathrm{i}\sigma_1\partial_r, \qquad \operatorname{Dom} d_0^{\mathrm{sel}} = \{u \in H^1(\mathbb{R}_+;\mathbb{C}^2) : u_+(0) = 0\}.$$

The first-order traces are

$$A(u) = u_+(0), \qquad B(u) = u_-(0).$$

Its square domain is exactly

$$\mathrm{Dom}\,(d_0^{\mathrm{sel}})^2 = \{u \in H^2(\mathbb{R}_+;\mathbb{C}^2) : u_+(0) = 0,\ u_-'(0) = 0\}.$$

Thus

$$(d_0^{\mathrm{sel}})^2 = (-\partial_r^2)_{\mathrm{D}} \oplus (-\partial_r^2)_{\mathrm{N}}.$$

After $\mathscr{W}_2^{-1}$, this is $h_{1/2,\mathrm{F}} \oplus h_{1/2,\mathrm{E}_0}$. There is no logarithmic alternative at this point.

Proof. The first-order maximal domain is H^1. The selected trace is $u_+(0) = 0$. For a square-domain vector, $d_0 u \in H^1$ gives $u \in H^2$, and

$$(d_0 u)_+(0) = -\mathrm{i}u_-'(0) = 0.$$

These conditions are also sufficient. The homogeneous second-order flattened solutions are $1, r$, not a logarithmic pair.

par *Interpretive boundary*. This domain or operator statement concerns prescribed continuum data. It supplies neither a finite Enter acquisition map nor a Maxwell or Einstein source equation [11, 19, 20]. par

At $\gamma = 0$, every first-order self-adjoint boundary line is dilation invariant, because the two traces have the same dilation weight. The smooth-centre theorem still selects the particular line $A = 0$; scale invariance alone would not select it.

The exact thresholds γ = ±1/2

Proposition (Threshold logarithms and the failure of unchanged critical traces). *At $\gamma = 1/2$, the unique first-order realization has square*

$$d_{1/2}^2 = \mathscr{W}_2(h_{0,\mathrm{F}} \oplus h_{1,\mathrm{F}})\mathscr{W}_2^{-1}.$$

At $\gamma = -1/2$,

$$d_{-1/2}^2 = \mathscr{W}_2(h_{1,\mathrm{F}} \oplus h_{0,\mathrm{F}})\mathscr{W}_2^{-1}.$$

In the order-zero component write the maximal second-order expansion as

$$u(r) = r^{1/2}(a + b\log r) + \text{remainder}.$$

The square domain has $b = 0$.

The two critical first-order traces do not extend as finite traces on the entire maximal first-order threshold domain. No new boundary parameter is to be imposed there.

Proof. At $\gamma = 1/2$, the order-zero component is upper. Its logarithmic power satisfies

$$(\partial_r - \tfrac{1}{2r})(r^{1/2}\log r) = r^{-1/2},$$

which is not square integrable. The same first-order derivative annihilates $r^{1/2}$. Thus $d_{1/2}u \in L^2$ excludes precisely the logarithmic coefficient. The lower component has order one and is limit point.

At $\gamma = -1/2$, the order-zero component is lower, and its derivative in the upper Dirac image is again $\partial_r - 1/(2r)$. The same calculation applies. This also supplies the threshold step in Theorem .

To see why the critical trace formulas cannot simply be reused, choose a cutoff supported in $0 < r < e^{-2}$ and equal to one near zero. At $\gamma = 1/2$, the vector

$$u(r) = \begin{pmatrix} \chi(r)r^{1/2}\log\log(1/r) \\ 0 \end{pmatrix}$$

belongs to the maximal first-order domain: its nontrivial derivative near zero is a constant multiple of $r^{-1/2}/\log(1/r)$, which is in $L^2(0, e^{-2})$. But $r^{-1/2}u_+(r)$ has no finite limit. The lower-component counterpart works at $\gamma = -1/2$. The first-order operator is limit point at the threshold, so its maximal self-adjoint domain is unique, not a two-trace extension family.

par *Interpretive boundary.* This domain or operator statement concerns prescribed continuum data. It supplies neither a finite Enter acquisition map nor a Maxwell or Einstein source equation [11, 19, 20]. par

The example is not in the square domain. It distinguishes a maximal first-order vector from a maximal

second-order boundary expansion.

Approach to a threshold and simultaneous core limits

Theorem (Strong-resolvent continuity to the thresholds). *Keep α fixed and set*

$$\beta_{\pm} = \frac{1 \pm \alpha}{2}.$$

If $\beta_\ell \to \beta_{\pm}$, the selected exterior transverse operators converge strongly in the resolvent sense to the unique operator at $\beta_{\pm}$. Their squares converge as well.

More generally, if $\varepsilon_\ell$downarrow0, $\beta_\ell \to \beta_{\pm}$, and for each ℓ the smooth profiles satisfy Assumption with actual flux β_ℓ, then the transported smooth-core transverse Dirac operators converge strongly in the resolvent sense to that same threshold operator. No joint quantitative rate is asserted.

Proof. At either threshold every angular channel is limit point. Finite angular sums of smooth functions compactly supported away from zero form an operator core of the threshold direct sum.

For such a fixed test in channel j,

$$(d_{\gamma_j(\beta_\ell)} - d_{\gamma_j(\beta_{\pm})})u = \sigma_2 \frac{\beta_\ell - \beta_{\pm}}{\alpha r} u \to 0.$$

It belongs to every exterior realization because its support avoids zero. Lemma gives the exterior result, using the finite-angular core.

For smooth cores the test is exterior once ℓ is large, so exactly the same difference formula holds. The same lemma proves simultaneous core convergence. One may equivalently use after proving the channel limits.

For a self-adjoint T and $s > 0$,

$$(T^2 + s)^{-1} = \frac{(T - \mathrm{i}\sqrt{s})^{-1} - (T + \mathrm{i}\sqrt{s})^{-1}}{2\mathrm{i}\sqrt{s}}.$$

Convergence of the two nonreal resolvents proves the square statement.

par *Interpretive boundary.* This domain or operator statement concerns prescribed continuum data. It supplies

neither a finite Enter acquisition map nor a Maxwell or Einstein source equation [11, 19, 20]. par

The critical trace estimates become nonuniform at the endpoints, and is not a uniform positive-power estimate as *y*uparrow 1/2. Theorem does not extrapolate those estimates. It uses the different, limit-point core of the threshold target.

Global square domains and longitudinal products

The exact angular square domain is

$$\operatorname{Dom} \mathbf{d}_{\mathrm{sel}}^{2} = \left\{ (u_j) : u_j \in \operatorname{Dom} (d_j^{\mathrm{sel}})^2, ?\sum_j \left(\mathrm{lVertu_}j\|^2 + \left\| (d_j^{\mathrm{sel}})^2 u_j \right\|^2 \right) < \infty \right\}.$$

Indeed,

$$\mathrm{lVertd_}j^{\mathrm{sel}} u_j \|^2 \leq \frac{1}{2} \left(\mathrm{lVertu_}j\|^2 + \left\| (d_j^{\mathrm{sel}})^2 u_j \right\|^2 \right)$$

by the spectral inequality $x^2 \leq (1 + x^4)/2$. Thus is equivalent to $u \in \operatorname{Dom} \mathbf{d}_{\mathrm{sel}}$ and $\mathbf{d}_{\mathrm{sel}} u \in \operatorname{Dom} \mathbf{d}_{\mathrm{sel}}$. The same holds for each smooth core. Consequently

$$\mathbf{d}_{\varepsilon}^{2} \to \mathbf{d}_{\mathrm{sel}}^{2}$$

strongly in the resolvent sense by the identity used in Theorem . The exact Bessel domain is the angular direct sum of Theorem after the declared radial unitaries.

par *Interpretive boundary.* This domain or operator statement concerns prescribed continuum data. It supplies neither a finite Enter acquisition map nor a Maxwell or Einstein source equation [11, 19, 20]. par

For a four-component product realization, let τ_i be another Pauli triple, $M \geq 0$, and Fourier transform in z. Define

$$H_{\varepsilon}(k) = \tau_1 \otimes \mathbf{d}_{\varepsilon} + k\tau_2 \otimes I + M\tau_3 \otimes I,$$

and define $H_{\mathrm{sel}}(k)$ similarly. For fixed k, the last two terms are bounded. Tensoring the graph approximants with $\mathbb{C}^2$ proves fiberwise strong-resolvent convergence. The bound $1/|\operatorname{Im} \zeta|$, independent of k, permits dominated convergence of the direct-integral resolvents. The Clifford relations give

$$H_{\mathrm{sel}}(k)^2 = I \otimes \mathbf{d}_{\mathrm{sel}}^2 + (k^2 + M^2)I.$$

No longitudinal-momentum-dependent apex relation is introduced.

par *Interpretive boundary.* This domain or operator statement concerns prescribed continuum data. It supplies neither a finite Enter acquisition map nor a Maxwell or Einstein source equation [11, 19, 20]. par

Remark (Why the scalar and Dirac-square conclusions differ). In the charge convention used here, the smooth Dirac square contains the spinor connection Laplacian, $R_{\varepsilon}/4$, and $+\sigma_3\mathscr{B}_{\varepsilon}$. The lower spin component therefore has a magnetic term of the attractive sign, although the full square is nonnegative. The scalar form does not contain this Pauli term. The selected E_0 square component is not a counterexample to scalar Friedrichs selection. This distinction is consistent with the Dirac/Pauli extension literature [6, 7].

Profile independence, contact separation, and companion scope

The exact meaning of profile independence

For fixed α, β, ξ, the scalar limiting form and domain are the same for every admissible family. For fixed α, β, bounding spin structure, and minimal Dirac coupling, the limiting first-order domain and its square are likewise the same for every such family.

This does not assert equality of:

• the core solutions or Dirichlet-to-Neumann values;

• the finite-ε matching coefficient;

• the actual leading coefficient in a resolvent asymptotic expansion;

• a sharp convergence rate when the leading coefficient cancels;

• a renormalized one-point observable or its apex contact.

In contrast, , , and are genuinely profile-uniform upper bounds in their stated fixed-parameter regimes. The plan's unqualified suggestion that every resolvent rate must remain profile dependent would incorrectly weaken these

established estimates. No uniformity in β approaching an integer, in v approaching zero, or in critical trace constants at $|\gamma| = 1/2$ is asserted.

Strong resolvent convergence does not normalize contacts

The contacts c, τ, w in the supplied I-B-VP-1 one-point completion are c-number extension data for different observables [18]. Changing them changes none of the spatial operators constructed here. The operator theorems therefore do not normalize them.

Proposition (An inverse-weight counterexample). *There are strictly positive self-adjoint T_n converging strongly in the resolvent sense to I, and a fixed v, such that*

$$\mathrm{ip}\, v T_n^{-1/2} v \to +\infty.$$

Proof. On $\ell^2(\mathbb{N})$, let $T_n = I$ except on the n-th basis vector, where its eigenvalue is 2^{-8n}. For each $s > 0$ and $f \in \ell^2$, the resolvent difference has norm at most $C_s|f_n| \to 0$.

Choose $v_j = 2^{-j}$. The n-th inverse-weight contribution is

$$|v_n|^2 (2^{-8n})^{-1/2} = 2^{-2n} 2^{4n} = 2^{2n}.$$

Each individual quadratic form is finite, but the sequence diverges.

This counterexample concerns the logical strength of the topology, not the existence of this mechanism in the specified flux-core models. It shows why threshold estimates and a prescribed state construction cannot be omitted when passing to an inverse covariance weight. Coincidence subtraction and contact normalization require further control beyond that weight as well.

The scalar-observable companion I-B-VP-5-Ex1 gives separate exterior mode and moment calculations [13]. Those results do not turn the present Dirac graph estimate into a renormalized spinor one-point limit. The disclosed Spinelly–Bezerra de Mello coefficient discrepancy in I-B-VP-1 remains a separate comparison [18]. This paper neither evaluates that coefficient nor identifies a branch by an unverified naming convention.

par *Interpretive boundary*. This domain or operator statement concerns prescribed continuum data. It supplies neither a finite Enter acquisition map nor a Maxwell or Einstein source equation [11, 19, 20]. par

No transfer to gauge-field or characteristic-product problems

The channel-diagonal source-test criteria of I-B-G-S refer to vector, tensor, and ghost domains [16]. The present spinor theorem does not certify its curvature-regular tensor branch as a smooth-core limit.

The characteristic-product programme has different objects, test spaces, and admission gates. The supplied XIV-H ending is unresolved at H2 in that historical release. XIV-I subsequently constructs its named internal I2 amplitude but leaves the full I3 joint map unresolved. XIV-J leaves its required full map unresolved at J2 [23]. We preserve those version-qualified distinctions: the I2 advance is not withdrawn, and it is not promoted to a complete characteristic map.

The technical manuscript XV-B exists and proves its stated interfaces; the distinct downstream quantum-admission exit is not opened by those interfaces or by the present free-field result [23]. No full characteristic composition, joint section, transverse estimate, contact gluing, output-wavefront theorem, or quantum coefficient is supplied here.

Reliability ledger, corrections, and release theorem

Evidence discipline

We retain the supplied XVI-O1-ST distinction [21]:

\texttt{ANALYTIC} A displayed mathematical proof or an explicitly named analytical input on its stated domain.

\texttt{SUPPLIED} A source-qualified manuscript statement or historical report not independently executed here.

\texttt{UNCERTIFIED} A numerical approximation without complete artifacts and justified total error bounds.

\texttt{OPEN} A construction or comparison not established by this release.

No numerical integration, symbolic program, compilation, or machine proof was executed for this manuscript. The sign audit is the displayed algebra. The record of a proof is not itself an execution certificate.

toprule Audit target Ending Exact result or retained limit

midrule endfirsthead toprule Audit target Ending Exact result or retained limit

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Scalar transported derivatives ANALYTIC Exact commutator retained; all transport terms vanish identically on fixed exterior annuli for small cores. No global derivative-smallness claim.

Exhausted-annulus lower bound ANALYTIC Each weak derivative is identified distributionally; vector L^2 lower semicontinuity and monotone exhaustion give the full magnetic energy bound.

Hidden axis energy Excluded from limiting form Quantitative Hardy cutoff and exact exterior recovery leave no additional axis term.

Core energy of arbitrary bounded-energy sequences Proposed claim corrected Can remain positive; Proposition constructs a concentration defect.

Core energy of resolvent minimizers ANALYTIC Vanishing follows from total-energy convergence and localized lower semicontinuity, not from the false universal claim.

Scalar coefficient and localized rate ANALYTIC Signed coefficient can depend on profiles; fixed-channel upper bound is profile uniform.

Dirac radial signs ANALYTIC Spin connection, transport, off-diagonal matrix, Green form, and square signs are consistent in the declared convention.

Cross-source sign dictionary OPEN No identification of every convention or named branch in the vacuum-polarization literature is asserted.

Critical Dirac line ANALYTIC Smooth-centre continuation selects $A = 0$; its boundary-generator graph bound is profile uniform for fixed γ.

Angular direct sum ANALYTIC Explicit uniform tail estimate ; no uniform-in-channel convergence rate assumed.

Equal-weight point ANALYTIC Ordinary traces; flattened square is Dirichlet–Neumann. No logarithmic mode occurs.

Exact thresholds ANALYTIC Unique first-order domain; order-zero square has no log. Critical trace formulas are not reused at the endpoints.

Approaching thresholds ANALYTIC Strong-resolvent continuity, also for simultaneous shrinking cores and varying flux. No joint rate.

Square-domain identification ANALYTIC Exact after the displayed radial unitaries; global square summability is explicit.

One-point contacts and Dirac coefficient discrepancy OPEN here Not normalized or resolved by operator convergence.

Compilation and formal verification Not asserted No compiler, symbolic-program, or proof-assistant run.

Finite coverage is an additional theorem

A finite probe family cannot replace the quantified annular, recovery, or graph arguments. More precisely, if all admissible residuals are independently proved to belong to a finite-dimensional space Z, a linear probe map $E : Z \to \mathbb{R}^m$ detects every residual exactly when $\ker E = \{0\}$. This is immediate linear algebra. Membership in Z and injectivity are separate hypotheses; finite vanishing does not prove either. This is the coverage discipline attributed to the supplied Oai282-A material [22]. No external catalogue theorem is used as a premise.

Theorem (I-B-VP-5-Ex2-B release). *For Assumption , fixed* $0 < \alpha < 1$, *actual flux* $0 < \beta < 1$, *and the declared identifications:*

1. Scalar selection. *For fixed $\xi \geq 0$, the transported smooth-core forms Mosco-converge to the twisted Friedrichs form. The derivative transport, annular lower semicontinuity, axis density, and recovery steps are given explicitly.*

2. Concentration correction. *Arbitrary bounded-energy sequences need not lose their core energy. Resolvent minimizers do lose it, by the separate saturation argument .*

3. Scalar quantitative scope. *The signed matching identity and localized fixed-channel resolvent estimates are –. Finite-radius coefficients may depend on the profiles; the stated upper bounds are uniform over profiles for fixed channel and spectral data.*

4. Dirac selection. *The audited first-order convention selects $A = 0$ in the critical channel and the unique exterior realization elsewhere. The singular boundary generator has the explicit graph approximation .*

5. Full angular limit. *The tail estimate completes strong-resolvent convergence on the entire transverse spinor Hilbert space.*

6. Squares and endpoints. *The square domains are exactly – under the radial unitaries. At $\gamma = 0$ the flattened conditions are Dirichlet–Neumann. At $\gamma = \pm 1/2$, the order-zero component is Friedrichs with no logarithm. Strong-resolvent continuity to both thresholds, including the stated simultaneous core limits, holds.*

7. Product scope. *The direct-sum square domain is , and the declared four-component longitudinal product has the corresponding strong-resolvent limit.*

8. Separation. *None of these conclusions normalizes one-point contacts, proves renormalized one-point convergence, resolves the I-B-VP-1 Dirac coefficient discrepancy, or constructs an interacting or gravitational completion. The limiting domains are independent of the admissible profiles within this class. No classification is made for additional electric potentials, anomalous moments, negative scalar curvature coupling, other actual flux lifts, non-product boundary laws, or interacting products.*

Proof. Item (i) is Lemma and Theorem . Item (ii) is Propositions and . Item (iii) is Proposition . Item (iv) is Proposition , Lemma , and Theorem . Item (v) is Theorem . Item (vi) is Theorem , Propositions –, and Theorem .

Item (vii) is Section . Item (viii) is the distinct-object argument and Proposition in Section . All constructions and estimates use only the declared class; the final exclusions list hypotheses not supplied.

par *Interpretive boundary.* This domain or operator statement concerns prescribed continuum data. It supplies neither a finite Enter acquisition map nor a Maxwell or Einstein source equation [11, 19, 20]. par

Unchanged programme boundaries

For the present cone reading,

$$\text{nolinkurl}\, O.1 = NOT_{C}LOSED, \qquad \text{texttt}\, bulk_symmetry_restored = \text{texttt}\, false.$$

XVI-N and its spatially compact Route B are not discharged. The downstream XV-B quantum-admission exit is not opened. Problems 1, 2, and 8 remain unfinished. Other problem numbers are not advanced by this release.

The historical H2 ending remains as stated in XIV-H; this paper does not discharge it. The later constructed I2 internal amplitude is retained, while the full I3 and J2 maps remain unresolved [23, 24]. No separately scoped closure elsewhere is verified, revoked, or transferred.

No Maxwell or Einstein backreaction, spacetime emergence, Page curve, de Sitter state, cosmological bounce, ultraviolet completion, or interacting quantum Ward identity is constructed. OpenAI catalogue entries remain external records, not analytical premises.

Conclusion. The scalar and spinor selections survive the requested audit, but not every proposed proof step does. The discarded scalar core energy need not vanish on arbitrary bounded-energy sequences; the transport commutator need not be globally small; and critical first-order traces cannot be continued unchanged to a limit-point threshold. Replacing those shortcuts by the arguments above gives the same fixed-parameter limiting domains, an explicit full-angular convergence proof, and controlled threshold limits.

These results identify realizations arising from the declared smooth cores. They do not identify an operator domain with a vacuum contact, a finite record, or a source of the prescribed spacetime.

appendix

Compact matrix audit

The following table fixes the convention without relying on an external branch name.

toprule Quantity Audited formula

midrule endfirsthead toprule Quantity Audited formula

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Charge convention $D = \mathrm{d} + \mathrm{i}a_{\varepsilon}\mathrm{d}\varphi$.

Polar spin connection $\Gamma_{\varphi} = -\mathrm{i}\alpha\sigma_{3}/(2p_{\varepsilon})$.

Physical radial connection term $-\mathrm{i}\sigma_{1}p_{\varepsilon}^{-1}(\partial_{r} + 1/(2r))$.

Radial unitary $f \mapsto \sqrt{p_{\varepsilon} r}\, f$.

Transported radial term $-\mathrm{i}\sigma_{1}p_{\varepsilon}^{-1}(\partial_{r} - p_{\varepsilon}^{'}/(2p_{\varepsilon}))$.

Exterior matrix $d_{\gamma} = -\mathrm{i}\begin{pmatrix} 0 & \partial_{r} + \gamma/r \\ \partial_{r} - \gamma/r & 0 \end{pmatrix}$.

Exterior zero modes $(r^{\gamma}, 0)^{T}$, $(0, r^{-\gamma})^{T}$.

Green form at zero $\mathrm{i}(\overline{A(u)}B(v) + \overline{B(u)}A(v))$ for an inner product linear in its second entry.

Smooth positive-flux critical line $A = 0$, retaining the lower zero-mode boundary generator.

Upper square potential $\gamma(\gamma - 1)/r^{2}$.

Lower square potential $\gamma(\gamma + 1)/r^{2}$.

Magnetic Pauli sign $+\sigma_{3}\mathscr{B}_{\varepsilon}$.

Critical square Upper Friedrichs, lower E_{0}, after the specified $\mathscr{W}_{2}$ transformation.

For a direct flat-chart magnetic check, put $\pi_{i} = -\mathrm{i}\partial_{i} + A_{i}$. Then

$$[\pi_{1}, \pi_{2}] = -\mathrm{i}\mathscr{B}, \qquad (\sigma_{1}\pi_{1} + \sigma_{2}\pi_{2})^{2} = \pi_{1}^{2} + \pi_{2}^{2} + \sigma_{3}\mathscr{B}.$$

For the inverse-square sign,

$$(-\mathrm{i}\sigma_1\partial_r + \sigma_2 w)^2 = -\partial_r^2 + w^2 + \sigma_3 w', \qquad w = \gamma/r.$$

Both reproduce the table.

The uniform angular estimate is not a sign-dependent Bessel approximation:

$$\|(1 - \Pi_J)(R_\varepsilon - R)f\| \le \frac{2}{|\operatorname{Im}\zeta|}\|(1 - \Pi_J)f\|.$$

It depends only on self-adjointness, angular reduction, and the common nonreal spectral parameter.

Source, execution, and submission declarations

Complete supplied core sources. The complete sources supplied for this development are I-B-VP-5-Ex1-v2 and I-B-VP-5-Ex2-v2. Other internal records are used at the version-qualified excerpt or bibliographic-description level indicated in the bibliography. No complete audit of unavailable companion sources is claimed.

Online consultation. The Enter philosophy and I-B browse pages were consulted for their stated interpretive and finite-interface roles. Primary Dirac/Pauli and inverse-square records, DLMF, and the publisher record for the Enter calculus article were consulted for attribution. A source page or publisher record is not a verification of the complete internal series. No exhaustive priority or corpus audit is asserted.

Analytical inputs. Standard closed-form representation, the first-order limit-point/limit-circle alternative, and ordinary Bessel maximal-domain boundary expansions are used in their stated settings. The transport identities, lower-bound argument, recovery, concentration counterexample, graph construction, angular-tail estimate, endpoint exclusions, and threshold-continuity arguments are supplied explicitly.

Novelty. The claimed contribution is the audited implementation and the corrective concentration and endpoint statements for this specified smooth-core class. General Mosco convergence, self-adjoint extension theory, and regularized Aharonov–Bohm Dirac/Pauli selection are not claimed as new.

Verification. No successful pdfLaTeX execution, numerical or symbolic-program execution, proof-assistant

verification, independent rebuild, referee acceptance, or journal acceptance of this manuscript is asserted. An analytically consistent convention is not a machine-checked convention.

Submission. This is a single pdfLaTeX source with an internal bibliography and no external figure, data, code, or bibliography-file dependency. Two runs are intended to resolve references and contents. The author field follows the supplied series; submission requires the author's approval of the mathematics, attribution, and scope.

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