I-B-VP-5-Ex2 Smooth flux-core selection on a prescribed cone

Scalar Friedrichs universality, Dirac-window selection, and quantitative exterior matching

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Abstract

We determine the zero-radius operator limits of a declared class of smooth rotational curvature and magnetic-flux cores on the prescribed conical product of spacetime Enter. The scalar and Dirac answers differ.

For the minimally coupled charged scalar with nonnegative curvature coupling, every admissible core family selects the twisted Friedrichs operator in the strong-resolvent sense, including families whose profiles vary with the core radius. The proof uses the correct magnetic form domain and does not transfer the untwisted global energy comparison. For each deficient scalar channel we obtain an exact signed zero-energy matching parameter,

$$\frac{B_\varepsilon}{A_\varepsilon} = \varepsilon^{2\nu}\frac{\nu - \Lambda_\varepsilon}{\nu + \Lambda_\varepsilon}, \qquad \Lambda_\varepsilon \geq 0,$$

and an exterior localized resolvent estimate of order $\varepsilon^{2\nu}$. Thus no singular scalar E_0 realization is selected within this positive-core class.

For the minimally coupled Dirac operator, with the spin structure that extends across the smooth disk, a separate first-order argument gives a profile-independent limit. In the window

$$\left|\beta - \frac{1}{2}\right| < \frac{\alpha}{2},$$

the unique critical radial channel selects one explicitly stated chiral boundary condition. Its square has Friedrichs in one component and the singular nonnegative E_0 Bessel realization in the other. The proof constructs a

smooth-core zero-mode continuation of the singular boundary generator, with a quantitative graph approximation; it does not infer a Dirac domain from independent scalar form limits.

The contribution is this profile-uniform cone implementation, the scalar–spinor contrast, and the normalized matching estimates. Flux regularization and Dirac/Pauli extension selection are established subjects, not priority claims of the Enter programme. The results identify which idealized domains arise from the specified cores, but do not normalize vacuum-polarization contacts, prove convergence of renormalized one-point functions, or construct an interacting state. The metric and flux remain inputs: no finite Enter record acquires a continuum operator, and no Einstein dynamics or spacetime emergence is inferred.

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not claimed to be Google Scholar-indexed. The published Enter Calculus article is cited only for finite-instrument / operational interface discussions, never as a PDE premise.

Book expansion: scalar forms vs Dirac graph

Use separate subsection tracks for scalar forms and Dirac graph convergence. The scalar magnetic form domain is *not* ordinary H^1; highlight the Hardy estimate. Before boundary conditions, make explicit: actual-flux lift, spin structure, charge convention, radial unitary.

Purpose, interpretation, and corrections to the plan

The question and the supplied baseline

The supplied I-B-VP-5 selection theorem concerns an untwisted scalar on monotone flat-centre curvature cores, with $\xi \geq 0$, after a specified unitary identification [11, thm:selection]. The present paper treats a smooth magnetic flux distributed inside a shrinking core. It proves the twisted scalar limit and, separately, a minimally coupled Dirac limit.

The operator problem is classical extension theory. Boundary conditions for conical scalar and fermionic systems, including their independence from the exterior differential expression, are established subjects [1, 3, 2]. Regularized flux tubes have also been used to select Dirac and Pauli realizations [4, 5, 6]. We do not claim that non-Friedrichs spinor limits originate in the Enter framework.

The principal contribution here is a self-contained selection proof for the declared conical core class, uniform over its radius-dependent profiles, together with explicit scalar matching and a direct Dirac graph approximation.

Enter and spacetime Enter

The *Philosophy of Enter* distinguishes destructive readout, quantum return, an installed description, and a physical identification [9]. Those distinctions fix interpretation, not the hypotheses of the operator proofs.

For a finite calibrated clock C, surviving register A, and orthonormal clock basis $\{|\tau_j\rangle\}$, put

$$D_j = \langle \tau_j|_C \otimes I_A, \qquad \mathscr{M}(X) = \sum_j \mathrm{lvertj}, w_j\rangle\mathrm{langlej}, w_j|_R \otimes D_j X D_j^\dagger.$$

Then

$$\sum_j D_j^\dagger D_j = I_{CA}, \qquad \mathrm{Tr}_R \mathscr{M}(X) = \mathrm{Tr}_C X,$$

including every untouched-reference extension. Both statements follow by expanding X in clock matrix units. The measured quantum names are not returned. This is the I-B/En09-A record–residual interface [10, thm:instrument] [20].

The channel does not acquire an unknown continuum domain, determine a magnetic profile, or choose a core radius. Spacetime Enter places that finite interface on a separately prescribed conical host [10]. The geometric time coordinate is not consumed.

Scope boundary (Non-sourcing). The background, flux, couplings, spin structure, and operator realizations are prescribed data. An operator limit, a conserved current, or a contact classification is not thereby a source law for the cone. The distinctions of I-B prop:not-source and X-A-ST prop:noeinstein remain in force [10, 21].

Corrections required before selection can be proved

1. Scalar and Dirac domains must not be conflated. A Dirac operator is not semibounded and has no Friedrichs extension. A scalar Mosco argument cannot select its components independently.

2. The twisted form domain changes. For nonintegral flux the exterior magnetic energy controls $r^{-2}|u|^2$. Its form domain is strictly smaller than ordinary Cartesian H^1.

3. The spin structure is additional data. A smooth disk selects the bounding spin structure. In the polar orthonormal frame its spinors are antiperiodic. The other spin structure of the punctured cone does not extend across the disk.

4. Power-law matching is not logarithmic matching. For $0 < \nu < 1$, the invariant zero-energy datum is a signed quantity with dimension length$^{2\nu}$. A nonnegative length alone loses its sign.

5. A core-class theorem is not an unrestricted physical classification. No electric potential, anomalous magnetic moment, independently tuned short-range interaction, or extra apex boundary condition is included.

6. No measure on profile space is supplied. The conclusions quantify over every core in the declared class. We make no ``positive-measure set of cores" assertion.

The scalar and Dirac proofs below are independent of any unavailable full proof in a companion manuscript.

A nonempty smooth curvature and flux-core class

Geometry and actual flux

Fix

$$0 < \alpha < 1, \qquad 0 < \beta < 1.$$

The parameter α is the opening ratio. The exterior deficit is $2\pi(1 - \alpha)$.

We fix the actual flux lift $2\pi\beta$, not merely its class modulo 2π. An integer-flux gauge transformation on the punctured plane need not extend smoothly through the centre. Different actual flux lifts are therefore not silently identified as smooth-core problems.

For each $\varepsilon > 0$, consider

$$g_\varepsilon = -\mathrm{d}t^2 + p_\varepsilon(r)^2\mathrm{d}r^2 + \alpha^2 r^2 \mathrm{d}\varphi^2 + \mathrm{d}z^2.$$

Assumption (Monotone flat-centre cores with distributed flux). The functions $p_\varepsilon, a_\varepsilon$ satisfy:

1. p_ε is smooth, nondecreasing,

$$\alpha \le p_\varepsilon \le 1, \qquad p_\varepsilon = 1 \quad (r \ge \varepsilon).$$

For each fixed ε, it equals α in some neighbourhood of $r = 0$.

2. a_ε is smooth, nondecreasing, and

$$0 \le a_\varepsilon \le \beta, \qquad a_\varepsilon = \beta \quad (r \ge \varepsilon).$$

Near zero,

$$a_\varepsilon(r) = r^2 b_\varepsilon(r^2)$$

for a smooth function b_ε.

3. All gluing at the exterior boundary is smooth. No uniform bounds on derivatives of the profiles are required as $\varepsilon \to 0$.

4. The scalar covariant derivative in the global periodic gauge is

$$D = \mathrm{d} + \mathrm{i} a_\varepsilon(r)\mathrm{d}\varphi.$$

The same charge convention is used for the spinor. No electric or additional short-range potential is present.

Profiles may depend arbitrarily on ε, subject to these conditions.

Near the centre, $s = \alpha r$ gives the ordinary flat metric $\mathrm{d}s^2 + s^2\mathrm{d}\varphi^2$. The one-form $a_\varepsilon \mathrm{d}\varphi$ is smooth there because

$$r^2\mathrm{d}\varphi = x^1\mathrm{d}x^2 - x^2\mathrm{d}x^1.$$

The magnetic field and scalar curvature are

$$\mathscr{B}_\varepsilon = \frac{a_\varepsilon'}{\alpha p_\varepsilon r}, \qquad R_\varepsilon = \frac{2p_\varepsilon'}{r p_\varepsilon^3}.$$

They are smooth and bounded for each fixed core, though their bounds may diverge as the core shrinks.

Proposition (Geometry, flux, and nonemptiness). *The completed core geometry is smooth and complete. Its transverse curvature and flux obey*

$$\int R_\varepsilon \,\mathrm{d}A_\varepsilon = 4\pi(1-\alpha), \qquad \int \mathscr{B}_\varepsilon \,\mathrm{d}A_\varepsilon = 2\pi\beta,$$

where

$$\mathrm{d}A_\varepsilon = \alpha p_\varepsilon r \,\mathrm{d}r\,\mathrm{d}\varphi.$$

Both densities are nonnegative and supported in $r \le \varepsilon$.

The class is nonempty. For example, choose a smooth nondecreasing P equal to α near zero and to 1 *near* 1*, and a smooth nondecreasing F equal to zero near zero and to one near* 1*. Then*

$$p_\varepsilon(r) = P(r/\varepsilon), \qquad a_\varepsilon(r) = \beta F(r/\varepsilon)$$

is admissible.

Proof. In geodesic distance $\mathrm{d}s = p_\varepsilon \mathrm{d}r$, the circumference function is $f(s) = \alpha r(s)$. Thus

$$f' = \frac{\alpha}{p_\varepsilon}, \qquad f'' = -\frac{\alpha p_\varepsilon'}{p_\varepsilon^3},$$

and $R = -2f''/f$, giving . Integration gives

$$4\pi\alpha \int_0^\infty \frac{p_\varepsilon'}{p_\varepsilon^2}\mathrm{d}r = 4\pi(1-\alpha).$$

The flux integral is $2\pi \int a_\varepsilon' \mathrm{d}r$. Smoothness and completeness follow from the flat centre, smooth gluing, and $p_\varepsilon \geq \alpha > 0$. The displayed scaled profiles establish nonemptiness.

Exterior gauge convention

For $r > \varepsilon$, periodic scalar components u have angular covariant derivative

$$D_\varphi u = (\partial_\varphi + \mathrm{i}\beta)u.$$

Writing

$$\Phi_{\mathrm{par}} = e^{\mathrm{i}\beta\varphi} u$$

in a lifted parallel frame gives

$$\Phi_{\mathrm{par}}(\varphi + 2\pi) = e^{2\pi\mathrm{i}\beta}\Phi_{\mathrm{par}}(\varphi).$$

This fixes the sign of the holonomy-sector convention. Parallel transport of the connection has the corresponding inverse phase on components. There is no assertion that a nontrivial flat connection extends flatly through the centre: its curvature is precisely the smooth core flux.

Scalar exterior channels and the magnetic form domain

Deficiency and boundary coefficients

In the angular basis

$$e_n(\varphi) = \frac{e^{in\varphi}}{\sqrt{2\pi\alpha}}, \qquad n \in \mathbb{Z},$$

the exterior transverse radial expression on $L^2((0,\infty), r\,\mathrm{d}r)$ is

$$h_{\nu_n} = -\partial_r^2 - r^{-1}\partial_r + \nu_n^2 r^{-2}, \qquad \nu_n = \frac{|n+\beta|}{\alpha}.$$

Lemma (Scalar deficient channels). *Infinity is limit point. At zero a channel is limit circle exactly when* $0 < \nu_n < 1$, *equivalently*

$$|n+\beta| < \alpha.$$

There are at most two such channels: $n = 0$ *when* $\beta < \alpha$, *and* $n = -1$ *when* $1-\beta < \alpha$. *Each has deficiency indices* $(1,1)$.

Use boundary coefficients

$$u(r) = Ar^{\nu} + Br^{-\nu} + o(r^{\nu}).$$

The Friedrichs condition is $B = 0$*; the scale-invariant singular condition* E_0 *is* $A = 0$. *The boundary form is, up to the orientation of the endpoint convention,*

$$2\nu(\overline{B}_u A_\nu - \overline{A}_u B_\nu).$$

Proof. The two zero-energy solutions are $r^{\nu}, r^{-\nu}$. The second belongs to $L^2(r\,\mathrm{d}r)$ near zero exactly when $\nu < 1$. The boundary form follows by inserting their expansions in $r(\overline{u}v' - \overline{u'}v)$. At infinity the nonreal-energy decaying solution is unique, giving limit point. This is the usual Bessel extension analysis [1, 2]; the coefficient convention is fixed here.

The symbols A, B are deliberately tied to the displayed powers. They need not coincide with the letter conventions of an untwisted logarithmic companion.

The limiting form

Let

$$N = (0,\infty)_r \times S^1_\varphi \times \mathbb{R}_z, \qquad \mathrm{d}V_0 = \alpha r\,\mathrm{d}r\,\mathrm{d}\varphi\,\mathrm{d}z, \qquad \mathscr{H}_0 = L^2(N, \mathrm{d}V_0).$$

Define

$$q_{\mathrm{F},\beta}[u] = \int_N \left(|\partial_r u|^2 + \frac{|(\partial_\varphi + \mathrm{i}\beta)u|^2}{\alpha^2 r^2} + |\partial_z u|^2 \right) \mathrm{d}V_0,$$

initially on $C_c^\infty(N)$, and then by closure.

Lemma (Hardy control and exact form domain). *Put* $d_\beta = \min(\beta, 1-\beta) > 0$. *Then*

$$q_{\mathrm{F},\beta}[u] \geq \frac{d_\beta^2}{\alpha^2} \int_N \frac{|u|^2}{r^2} \mathrm{d}V_0.$$

Under the Cartesian identification $x = (r\cos\varphi, r\sin\varphi)$,

$$\mathrm{Dom}\, q_{\mathrm{F},\beta} = \left\{ u \in H^1(\mathbb{R}^3) : r^{-1}u \in L^2(\mathbb{R}^3) \right\}.$$

Equivalently, the domain is the maximal finite-energy space for . Compactly supported smooth functions avoiding the axis are form dense.

Proof. Fourier expansion in φ gives

$$\int_0^{2\pi} |(\partial_\varphi + \mathrm{i}\beta)u|^2 \mathrm{d}\varphi \geq d_\beta^2 \int_0^{2\pi} |u|^2 \mathrm{d}\varphi,$$

which proves . Furthermore,

$$|\partial_\varphi u| \leq |(\partial_\varphi + \mathrm{i}\beta)u| + \beta|u|,$$

so finite magnetic energy and imply ordinary H^1 regularity. The converse follows from the same inequality with

the roles reversed.

To prove density, choose a radial cutoff η_δ equal to zero for $r < \delta$, equal to one for $r > 2\delta$, and satisfying

$$|\eta_\delta'| \le C/r \quad \text{on its transition annulus.}$$

For a finite-energy u, the extra derivative energy is bounded by

$$C \int_{r<2\delta} \frac{|u|^2}{r^2} dV_0 \to 0.$$

Every other omitted form term tends to zero by absolute continuity of its integral. Cutoffs at infinity and ordinary smooth approximation on regular annuli finish the proof.

Remark (The untwisted proof cannot be copied literally). A smooth function nonzero near the axis belongs to ordinary H^1, but not to . Also, because $a_\varepsilon \neq \beta$ inside the core, there is no general global comparison $q_\varepsilon \ge \alpha q_{F,\beta}$ of the untwisted type. The selection proof will use exterior localization instead.

Scalar strong-resolvent selection

Fix $\xi \ge 0$. On

$$\mathscr{H}_\varepsilon = L^2(N, p_\varepsilon dV_0)$$

take the ordinary smooth-core finite-energy form

$$q_\varepsilon[u] = \int_N (p_\varepsilon^{-2}|\partial_r u|^2 + \frac{|(\partial_\varphi + i a_\varepsilon)u|^2}{\alpha^2 r^2} + |\partial_z u|^2 + \xi R_\varepsilon |u|^2) p_\varepsilon dV_0.$$

For each fixed core this is a closed nonnegative form on ordinary Cartesian $H^1(\mathbb{R}^3)$. Indeed its connection is smooth and bounded in an orthonormal frame for fixed ε, and its curvature potential is smooth and bounded. Denote its operator by B_ε.

The unitary identification is

$$U_\varepsilon : \mathscr{H}_\varepsilon \to \mathscr{H}_0, \qquad U_\varepsilon u = p_\varepsilon^{1/2} u.$$

Let

$$Q_\varepsilon[v] = q_\varepsilon[p_\varepsilon^{-1/2} v].$$

Theorem (Twisted scalar Friedrichs universality). *For every family satisfying Assumption and every fixed* $\xi \geq 0$*, the forms* Q_ε *Mosco-converge to* $q_{\mathrm{F},\beta}$*. Consequently, for every* $s > 0$ *and* $f \in \mathscr{H}_0$*,*

$$\left(U_\varepsilon B_\varepsilon U_\varepsilon^{-1} + s\right)^{-1} f \to (B_{\mathrm{F},\beta} + s)^{-1} f \quad \text{strongly in} \mathscr{H}_0.$$

The selected boundary condition is $B = 0$ *in every deficient scalar channel of Lemma . No scalar* E_0 *or finite non-Friedrichs extension is selected in this class.*

Proof.

Weak lower bound. Suppose v_ε rightharpoonup v in $\mathscr{H}_0$ and the form values and norms are bounded. Put $u_\varepsilon = p_\varepsilon^{-1/2} v_\varepsilon$. The multipliers $p_\varepsilon^{\pm 1/2}$ are uniformly bounded and converge strongly to one, since they differ from one only in $r \leq \varepsilon$. Thus u_ε rightharpoonup v.

On every fixed regular annulus $r > a > 0$, the coefficients are exactly those of once $\varepsilon < a$. The local form bound gives local weak H^1 control. Weak lower semicontinuity, nonnegativity of the discarded core terms, and exhaustion of the regular region give

$$\liminf_{\varepsilon \,\mathrm{downarrow}\, 0} Q_\varepsilon[v_\varepsilon] \geq q_{\mathrm{F},\beta}[v].$$

Lemma identifies the maximal finite-energy limit with the closed Friedrichs form. No global untwisted comparison was used.

Strong recovery. For $v \in \mathrm{Dom}\, q_{\mathrm{F},\beta}$, choose $\varphi_j \in C_c^\infty(N)$ converging in its form norm, using Lemma . For each fixed j, sufficiently small cores miss $\mathrm{supp}\, \varphi_j$, and therefore

$$U_\varepsilon \varphi_j = \varphi_j, \qquad Q_\varepsilon[\varphi_j] = q_{\mathrm{F},\beta}[\varphi_j].$$

A diagonal choice $j = j(\varepsilon) \to \infty$ is a strong recovery sequence. Outside the limiting form domain, the recovery

upper bound with value +∞ imposes no restriction. These are the two Mosco conditions [7].

Resolvents. For completeness, minimize

$$J_\varepsilon[w] = Q_\varepsilon[w] + s\|w\|^2 - 2\mathrm{Re}\,\mathrm{ip}\,fw.$$

Its minimizer v_ε is the transported resolvent applied to f. The lower bound and recovery sequence imply that every weak cluster point is the unique minimizer v of the limiting functional, and that the minimum values converge. For a recovery sequence w_ε of v,

$$J_\varepsilon[w_\varepsilon] - J_\varepsilon[v_\varepsilon] = Q_\varepsilon[w_\varepsilon - v_\varepsilon] + s\mathrm{lVertw_}\varepsilon - v_\varepsilon\|^2.$$

Hence $v_\varepsilon \to v$ strongly.

Finally, the singular power $r^{-\nu}$ has infinite energy in ; its coefficient must vanish. Thus the limit has $B = 0$ in each deficient channel.

Scope boundary (Class-relative exclusion and non-sourcing). The theorem excludes non-Friedrichs scalar limits only for the stated minimally coupled, nonnegative scalar forms. It does not exclude limits produced by additional attractive interactions or magnetic-moment terms. It is an operator theorem, not an Einstein source law [10, 21].

Scalar matching: the signed effective scale

Zero-energy impedance

Fix an angular mode n, and put $\nu = |n + \beta|/\alpha > 0$. The core radial expression is

$$h_{\varepsilon,n} = -\frac{1}{p_\varepsilon r}\partial_r\left(\frac{r}{p_\varepsilon}\partial_r\right) + \frac{(n + a_\varepsilon)^2}{\alpha^2 r^2} + \xi R_\varepsilon.$$

At zero energy its regular positive solution satisfies

$$\left(\frac{r}{p_\varepsilon}v'\right)' = V_{\varepsilon,n}v, \qquad V_{\varepsilon,n} = \frac{p_\varepsilon(n + a_\varepsilon)^2}{\alpha^2 r} + \xi p_\varepsilon r R_\varepsilon.$$

Regularity means $v \sim r^{|\eta|}$ for $n \neq 0$ and $v(0) > 0$ for $n = 0$.

Define

$$\Lambda_{\varepsilon,n} = \varepsilon \frac{v'(\varepsilon)}{v(\varepsilon)}.$$

Lemma (Positive impedance and its variational characterization). *The regular solution is positive and nondecreasing. In particular,*

$$\Lambda_{\varepsilon,n} \geq 0.$$

Normalized by $v(\varepsilon) = 1$, it minimizes

$$\Lambda_{\varepsilon,n} = \min_{w(\varepsilon)=1} \int_0^{\varepsilon} \left(\frac{r}{p_\varepsilon} |w'|^2 + V_{\varepsilon,n} |w|^2 \right) \mathrm{d}r,$$

over the smooth-core radial finite-energy domain. Moreover $0 < v \leq 1$ on the core.

Proof. Starting with its positive regular leading coefficient, implies that rv'/p_ε is nondecreasing as long as $v > 0$. Its initial limit is nonnegative. The solution cannot develop a first zero, and its derivative stays nonnegative.

Integration by parts gives the energy in equal to $\varepsilon v'(\varepsilon)$; the inner boundary term vanishes by regularity. For any $w = v + \eta$ with $\eta(\varepsilon) = 0$, the cross term vanishes by the equation. The remaining energy is nonnegative, proving minimality and uniqueness. Monotonicity gives $0 < v \leq 1$.

Theorem (Exact signed matching and exclusion of scalar resonance). *The exterior zero-energy continuation is*

$$v(r) = A_\varepsilon r^{\nu} + B_\varepsilon r^{-\nu}.$$

Its regular coefficient is nonzero, and

$$\eta_{\varepsilon,n} := \frac{B_\varepsilon}{A_\varepsilon} = \varepsilon^{2\nu} \frac{\nu - \Lambda_{\varepsilon,n}}{\nu + \Lambda_{\varepsilon,n}}.$$

Consequently

$$|\eta_{\varepsilon,n}| \leq \varepsilon^{2\nu}.$$

Define the signed effective matching data by

$$\sigma_{\varepsilon,n} = \operatorname{sign} \eta_{\varepsilon,n}, \qquad L_{\mathrm{eff},\varepsilon,n} = |\eta_{\varepsilon,n}|^{1/(2\nu)}.$$

Then $0 \le L_{\mathrm{eff},\varepsilon,n} \le \varepsilon$. *The sign is part of the data and cannot be discarded.*

A pure singular E_0 *exterior solution would require* $\Lambda_{\varepsilon,n} = -\nu$, *which is impossible in this class. More generally, a nonzero finite limit* $\eta_{\varepsilon,n} \to \eta \neq 0$ *would require*

$$\Lambda_{\varepsilon,n} = -\nu + \frac{2\nu}{\eta}\varepsilon^{2\nu} + o(\varepsilon^{2\nu}),$$

also impossible when $\Lambda_{\varepsilon,n} \ge 0$.

Proof. Evaluate v and rv' at $r = \varepsilon$ and solve the two linear equations for $B_\varepsilon/A_\varepsilon$. If $A_\varepsilon = 0$, the logarithmic derivative is $-\nu$, contradicting Lemma . The bound follows from

$$\left|\frac{\nu - \Lambda}{\nu + \Lambda}\right| \le 1 \quad (\Lambda \ge 0).$$

Inverting gives the final expansion.

Profile dependence without domain dependence

For self-similar profiles $p_\varepsilon = P(r/\varepsilon)$, $a_\varepsilon = \beta F(r/\varepsilon)$, the impedance is independent of ε. Its dimensionless formula is

$$\Lambda_n[P,F,\xi] = \min_{w(1)=1} \int_0^1 \Big[\tfrac{x}{P}|w'|^2 + \frac{P(n+\beta F)^2}{\alpha^2 x}|w|^2 + \frac{2\xi P'}{P^2}|w|^2 \Big] dx.$$

Thus the dimensionless factor multiplying $\varepsilon^{2\nu}$ is profile dependent even though the selected limit is not.

Proposition (A strict profile comparison). *Fix* P, ξ, *take* $n = 0$, *and let* $F_2 \ge F_1 \ge 0$ *be two admissible flux profiles. Then*

$$\Lambda_0[P,F_2,\xi] \ge \Lambda_0[P,F_1,\xi].$$

The inequality is strict if $F_2^2 > F_1^2$ *on an open subinterval. Accordingly the signed matching factor* $(\nu - \Lambda)/(\nu + \Lambda)$ *strictly decreases.*

Proof. The energy difference in is a nonnegative potential. Evaluate it on the positive minimizer for the second profile and use the minimum principle for the first. Strict positivity on an open interval gives strictness. The derivative of the matching factor is $-2\nu/(\nu+\Lambda)^2 < 0$.

This supplies a genuine profile-sensitive exterior datum, not a universal retained length at the zero-radius limit.

Finite-energy matching and an exterior resolvent rate

At spectral parameter $-k^2$, $k > 0$, let $\Lambda_{\varepsilon,n}(k)$ be the regular core logarithmic derivative. Its variational formula adds $k^2 p_\varepsilon r|w|^2$ to .

Lemma (Uniform energy dependence of the impedance). *For every admissible profile,*

$$0 \le \Lambda_{\varepsilon,n}(k) - \Lambda_{\varepsilon,n}(0) \le \frac{k^2\varepsilon^2}{2}.$$

Proof. The lower bound follows by comparing the two variational minima. For the upper bound use the normalized zero-energy minimizer v_0, for which $0 < v_0 \le 1$:

$$\Lambda(k) \le \Lambda(0) + k^2\int_0^\varepsilon p_\varepsilon r v_0^2 \mathrm{d}r \le \Lambda(0) + k^2\varepsilon^2/2.$$

Theorem (Exterior Bessel resolvent coefficient). *For $r, r' > \varepsilon$, the radial resolvent kernel, relative to $r'\mathrm{d}r'$, is*

$$(h_{\varepsilon,n} + k^2)^{-1}(r,r') = I_\nu(kr_<)K_\nu(kr_>) + c_{\varepsilon,n}(k)K_\nu(kr)K_\nu(kr'),$$

where, with $t = k\varepsilon$ and $\Lambda = \Lambda_{\varepsilon,n}(k)$,

$$c_{\varepsilon,n}(k) = \frac{tI_\nu'(t) - \Lambda I_\nu(t)}{\Lambda K_\nu(t) - tK_\nu'(t)}.$$

The denominator is positive.

For fixed $k, \nu > 0$,

$$|c_{\varepsilon,n}(k)| \le C_\nu(k\varepsilon)^{2\nu}$$

for sufficiently small kε, uniformly in the core profiles. For $0 < \nu < 1$*, more precisely,*

$$c_{\varepsilon,n}(k) = \frac{2^{1-2\nu}}{\Gamma(\nu)\Gamma(1+\nu)}(k\varepsilon)^{2\nu}\frac{\nu - \Lambda_{\varepsilon,n}(0)}{\nu + \Lambda_{\varepsilon,n}(0)} + O((k\varepsilon)^{4\nu}).$$

The remainder bound is uniform over the admissible profiles.

Proof. The regular exterior solution is $I_\nu + cK_\nu$. Imposing its logarithmic derivative at the interface gives . Its Wronskian with the decaying solution K_ν is unchanged by adding cK_ν, proving .

For $t > 0$, $K_\nu > 0$, $K_\nu' < 0$, and $\Lambda \geq 0$, so the denominator is positive. Also

$$|c_{\varepsilon,n}(k)| \leq \max\left\{\frac{I_\nu(t)}{K_\nu(t)}, \frac{I_\nu'(t)}{-K_\nu'(t)}\right\}.$$

The small-argument expansions give the stated bound.

For $0 < \nu < 1$, write

$$I_\nu(t) = \frac{2^{-\nu}}{\Gamma(1+\nu)}t^\nu(1 + O(t^2)),$$

$$K_\nu(t) = 2^{\nu-1}\Gamma(\nu)t^{-\nu} + 2^{-\nu-1}\Gamma(-\nu)t^\nu + O(t^{2-\nu}),$$

and differentiate these expansions. In the denominator factor out $(\nu + \Lambda)2^{\nu-1}\Gamma(\nu)t^{-\nu}$. All remaining ratios involving Λ are bounded uniformly for $\Lambda \geq 0$. This gives the displayed leading coefficient with $\Lambda(k)$, and an error $O(t^{4\nu} + t^{2\nu+2})$. Equation replaces $\Lambda(k)$ by $\Lambda(0)$ with error $O(t^{2\nu+2})$. Since $\nu < 1$, that is bounded by $O(t^{4\nu})$. The Bessel identities used here are classical [8, Sections 10.25 and 10.30].

Corollary (Localized norm-resolvent estimate in one channel). *Let* P_a *multiply by* $1_{\{r \geq a\}}$*, where* $a > 0$*. For* $\varepsilon < a$*, after the declared unitary transport,*

$$\begin{aligned}\left\| P_a\left[(U_\varepsilon h_{\varepsilon,n} U_\varepsilon^{-1} + k^2)^{-1} - (h_{\nu,\mathrm{F}} + k^2)^{-1}\right]P_a\right\| \\ = |c_{\varepsilon,n}(k)|\int_a^\infty K_\nu(kr)^2 r\,\mathrm{d}r \leq C_{\nu,k,a}\varepsilon^{2\nu}.\end{aligned}$$

For a finite collection of angular modes, the analogous estimate follows by taking the largest of the channel bounds. No all-angular norm-resolvent rate on the entire spatial Hilbert space is asserted.

Proof. The unitary multiplier is one outside the core. The difference of exterior kernels is the rank-one operator in . Its norm is the coefficient magnitude times the squared norm of $K_\nu(kr)$ on $r \geq a$. That norm is finite by exponential decay.

Scope boundary (Matching is not contact normalization). The coefficient $\eta_{\varepsilon,n}$, the effective scale in , and concern the spatial operator. They do not determine the one-point contacts c, τ, w of I-B-VP-1 [16, 17, 18]. They also impose no Maxwell or Einstein source equation [10, 21].

Dirac channels: the first-order problem

Spin structure and differential expression

Use the spin structure that extends over the smooth disk. In the polar orthonormal frame its components satisfy

$$\psi(\varphi + 2\pi) = -\psi(\varphi),$$

so the angular labels are

$$j \in \mathbb{Z} + \frac{1}{2}.$$

This is the bounding spin structure; the magnetic connection remains the separate a_ε.

Let $\sigma_1, \sigma_2, \sigma_3$ be the Pauli matrices. The transverse minimally coupled Dirac expression is

$$\mathscr{D}_\varepsilon = -\mathrm{i}\sigma_1 p_\varepsilon^{-1}\left(\partial_r + \frac{1}{2r}\right) - \frac{\mathrm{i}\sigma_2}{\alpha r}(\partial_\varphi + \mathrm{i}a_\varepsilon).$$

This follows from the spin connection of the orthonormal coframe $p_\varepsilon \mathrm{d}r, \alpha r \mathrm{d}\varphi$. No independently adjustable scalar curvature coupling ξ is inserted into the Dirac expression.

In angular basis $e^{\mathrm{i}j\varphi}/\sqrt{2\pi\alpha}$, multiply the radial component by

$$(\mathscr{V}_\varepsilon f)(r) = \sqrt{p_\varepsilon(r) r}\, f(r).$$

The radial Hilbert space becomes $L^2(\mathbb{R}_+, \mathrm{d}r; \mathbb{C}^2)$, and the transported expression is

$$d_{\varepsilon,j} = -\mathrm{i}\sigma_1 p_\varepsilon^{-1}\left(\partial_r - \frac{p_\varepsilon'}{2p_\varepsilon}\right) + \sigma_2\frac{j+a_\varepsilon}{\alpha r}.$$

Outside the core this is

$$d_j = -\mathrm{i}\sigma_1\partial_r + \sigma_2\frac{\gamma_j}{r}, \qquad \gamma_j = \frac{j+\beta}{\alpha}.$$

The exact Dirac window

Lemma (First-order deficiency and boundary form). *For $|\gamma_j| \geq 1/2$, the minimal exterior operator d_j is essentially self-adjoint. For $|\gamma_j| < 1/2$, its deficiency indices are $(1,1)$. There is at most one such channel:*

$$j_* = -\frac{1}{2}, \qquad \gamma := \gamma_{j_*} = \frac{\beta - \frac{1}{2}}{\alpha},$$

and it is critical exactly when

$$\frac{1-\alpha}{2} < \beta < \frac{1+\alpha}{2}.$$

In the critical channel, every maximal-domain vector has boundary traces

$$A(u) = \lim_{r\,\mathrm{downarrow}\,0} r^{-\gamma}u_+(r), \qquad B(u) = \lim_{r\,\mathrm{downarrow}\,0} r^{\gamma}u_-(r),$$

with

$$u_+ = A(u)r^\gamma + o(r^{1/2}), \qquad u_- = B(u)r^{-\gamma} + o(r^{1/2}).$$

The endpoint Green form is

$$\mathrm{i}\mathrm{p}\, u d_j v - \mathrm{i}\mathrm{p}\, d_j u v = \mathrm{i}(\overline{A(u)}B(v) + \overline{B(u)}A(v)).$$

The minimal closed domain has $A = B = 0$. All self-adjoint boundary lines are

$$A = \mathrm{i}tB, \quad t \in \mathbb{R}, \qquad \text{or} \qquad B = 0.$$

In particular $A = 0$ is self-adjoint.

Proof. The zero-energy equations are diagonal after multiplication by $\mathrm{i}\sigma_1$; their solutions are

$$(r^{\gamma_j}, 0)^T, \qquad (0, r^{-\gamma_j})^T.$$

Both are square integrable near zero exactly when $|\gamma_j| < 1/2$. Infinity is limit point because the $1/r$ coefficient is bounded on tails and tends to zero. This gives the deficiency count. For $0 < \beta < 1$, all $j \neq -1/2$ have $|j + \beta| > 1/2$, hence cannot be critical when $\alpha < 1$.

For $d_j u = f \in L^2$, variation of constants gives

$$u_+(r) = r^{\gamma}\Big(A + \mathrm{i}\int_0^r s^{-\gamma} f_-(s)\mathrm{d}s \Big),$$

$$u_-(r) = r^{-\gamma}\Big(B + \mathrm{i}\int_0^r s^{\gamma} f_+(s)\mathrm{d}s \Big).$$

Cauchy–Schwarz and $|\gamma| < 1/2$ give the stated remainders and continuity of the traces in the local graph norm. Integration by parts gives $-\mathrm{i}[u^\dagger \sigma_1 v]_0^\infty$, namely . The two cutoff zero solutions realize arbitrary traces. The adjoint of a maximal-domain restriction is therefore determined by the orthogonal boundary line for this nondegenerate form. Its self-adjoint lines are exactly those listed. The zero-trace domain is the minimal closure.

The first-order extension setting is established prior work [3, 4, 6]. The proof above fixes the present sign, spin structure, and trace convention.

Remark (Scale invariance at the equal-weight point). Under $u(r) \mapsto \lambda^{1/2} u(\lambda r)$,

$$A \mapsto \lambda^{1/2+\gamma} A, \qquad B \mapsto \lambda^{1/2-\gamma} B.$$

For $\gamma \neq 0$, the only scale-invariant boundary lines are $A = 0$ and $B = 0$. At $\gamma = 0$, every self-adjoint line is dilation invariant. Thus the phrase ``the scale-invariant Dirac extension" would be ambiguous without a boundary convention. The smooth cores will select $A = 0$, also at $\gamma = 0$.

Dirac strong-resolvent selection by an explicit graph core

For each fixed smooth core, the radial operator $d_{\varepsilon,j}$ has its unique smooth-centre self-adjoint realization. Near zero its leading exponents are $r^{\pm j}$ in the flattened radial space. Since $|j| \geq 1/2$, the centre is limit point. Infinity is also

limit point. Thus no inner boundary parameter is added.

Lemma (A graph-core criterion for strong resolvent convergence). *Let T_ε, T be self-adjoint on one Hilbert space. Suppose $\mathscr{C}$ is an operator core for T, and for every $u \in \mathscr{C}$ there are $u_\varepsilon \in \operatorname{Dom} T_\varepsilon$ such that*

$$u_\varepsilon \to u, \qquad T_\varepsilon u_\varepsilon \to Tu.$$

Then $T_\varepsilon \to T$ in the strong-resolvent sense.

Proof. Fix $z \notin \mathbb{R}$. For $f = (T - z)u$, $u \in \mathscr{C}$,

$$(T_\varepsilon - z)^{-1} f - u_\varepsilon = (T_\varepsilon - z)^{-1}((T - z)u - (T_\varepsilon - z)u_\varepsilon) \to 0.$$

The set $(T - z)\mathscr{C}$ is dense because $\mathscr{C}$ is an operator core. The uniform bound $\|(T_\varepsilon - z)^{-1}\| \le |\operatorname{Im} z|^{-1}$ extends convergence to every f.

Continuation of the singular boundary generator

Assume . In the critical channel set

$$b_\varepsilon(r) = \frac{p_\varepsilon(r)(-\frac{1}{2} + a_\varepsilon(r))}{\alpha}.$$

Define a lower-component zero mode by

$$z_\varepsilon(r) = \begin{pmatrix} 0 \\ \sqrt{p_\varepsilon(r)}\, \varepsilon^{-\gamma} \exp\left(-\int_\varepsilon^r b_\varepsilon(s)\frac{\mathrm{d}s}{s}\right) \end{pmatrix}.$$

For $r \ge \varepsilon$,

$$z_\varepsilon(r) = (0, r^{-\gamma})^T.$$

Lemma (Uniform core mass of the continued boundary mode). *The spinor solves $d_{\varepsilon, j_*} z_\varepsilon = 0$. It is smooth at the core centre after returning to the smooth spin frame.*

Let $\gamma_+ = \max(\gamma, 0)$. Then

$$\int_0^{\varepsilon} |z_{\varepsilon}(r)|^2 \mathrm{d}r \le \frac{\varepsilon^{1-2\gamma}}{1-2\gamma_+}.$$

In particular this mass tends to zero, uniformly over the admissible profiles.

Proof. The lower zero-mode equation from is

$$\left(\partial_r - \frac{p_{\varepsilon}'}{2p_{\varepsilon}} + \frac{p_{\varepsilon}(j_* + a_{\varepsilon})}{\alpha r}\right) z_{\varepsilon,-} = 0,$$

which gives . Near zero $p_{\varepsilon} = \alpha$ and $a_{\varepsilon} = O(r^2)$, so $z_{\varepsilon,-}$ is a constant times $r^{1/2}$ times a smooth function of r^2. The physical radial component divides by $\sqrt{p_{\varepsilon} r}$; together with $j_* = -1/2$, this is a smooth spinor in the Cartesian spin frame.

Since $0 \le a_{\varepsilon} \le \beta$ and $\alpha \le p_{\varepsilon} \le 1$,

$$b_{\varepsilon}(r) \le \max(\gamma, 0) = \gamma_+.$$

For $0 < r < \varepsilon$,

$$|z_{\varepsilon}(r)| \le \varepsilon^{-\gamma} (\varepsilon/r)^{\gamma_+}.$$

Integrating its square proves . The denominator is positive because $\gamma < 1/2$.

Theorem (Profile-independent Dirac-window selection). *Let d_{β}^{sel} be the angular direct sum of the unique exterior realizations in all noncritical channels and, when the window is present, the realization*

$$\mathrm{boxed}\, A(u) = 0 \quad \mathrm{in} j = -\tfrac{1}{2}.$$

Then the transported smooth-core transverse Dirac operators converge strongly in the resolvent sense to d_{β}^{sel}, for every family in Assumption .

No self-similarity or convergence of the rescaled profiles is required. If the window is absent, the exterior first-order operator is essentially self-adjoint and its unique realization is selected.

In the critical channel, if χ is smooth, compactly supported, and one near zero, then the boundary generator

$$u = (0, \chi(r) r^{-\gamma})^{T}$$

has graph approximants $u_{\varepsilon} = \chi z_{\varepsilon}$ *with*

$$\mathrm{lVertu_}\varepsilon - u\| \le C_{\gamma} \varepsilon^{1/2 - \gamma}, \qquad d_{\varepsilon, j_*} u_{\varepsilon} = d_{j_*} u \quad \text{for sufficiently small} \varepsilon.$$

The constants may depend on the fixed cutoff and radial units, but not on the core profiles.

Proof. In a noncritical channel, $C_c^{\infty}(\mathbb{R}_+; \mathbb{C}^2)$ is a core of the unique exterior realization. On each such function the core and exterior expressions agree for all sufficiently small ε.

In the critical channel, the selected domain is the minimal closed domain plus one boundary dimension. Consequently

$$C_c^{\infty}(\mathbb{R}_+; \mathbb{C}^2) + \mathrm{span}\,\{(0, \chi r^{-\gamma})^{T}\}$$

is a core. For its additional generator use $u_{\varepsilon} = \chi z_{\varepsilon}$. It is a smooth-core domain vector by Lemma . The vectors agree outside $r \le \varepsilon$, and

$$\int_0^{\varepsilon} r^{-2\gamma} \mathrm{d}r = \frac{\varepsilon^{1 - 2\gamma}}{1 - 2\gamma}.$$

Together with , this gives the norm estimate in .

Both zero modes are annihilated where $\chi = 1$. Where $\chi' \neq 0$, sufficiently small cores have $p_{\varepsilon} = 1$ and $z_{\varepsilon} = (0, r^{-\gamma})^{T}$. Their operator images are therefore exactly equal.

Lemma proves channel convergence. Finite angular sums of these cores form a core of the selected direct sum. The same graph criterion, or the uniform nonreal resolvent bound and angular truncation, gives the full transverse result.

Remark (Why the singular generator matters). Convergence on tests supported away from zero alone would not choose a boundary line in a deficient channel. The added graph approximant in supplies that missing information. It identifies the selected line rather than merely reproducing the exterior differential expression.

Remark (Opposite flux). For actual flux $-1 < \beta < 0$, with $\beta \le a_{\varepsilon} \le 0$ and the corresponding monotonicity, the repeated argument uses $j = +1/2$ and an upper-component regular zero mode. In its critical channel it selects $B = 0$, with graph error $O(\varepsilon^{1/2+\gamma})$, where $\gamma = (\beta + 1/2)/\alpha$. The change follows from the first-order equation, not from an unqualified identification of flux conventions.

The selected Dirac square and the origin of a spinor E_0

Theorem (Selected square in the Dirac window). *In the critical positive-flux channel,*

$$d_{j_*}^2 = \begin{pmatrix} -\partial_r^2 + \gamma(\gamma-1)r^{-2} & 0 \\ 0 & -\partial_r^2 + \gamma(\gamma+1)r^{-2} \end{pmatrix}.$$

Define

$$\nu_+ = \frac{1}{2} - \gamma, \qquad \nu_- = \frac{1}{2} + \gamma, \qquad 0 < \nu_{\pm} < 1.$$

After returning from $L^2(\mathrm{d}r)$ to $L^2(r\,\mathrm{d}r)$, the selected square is

$$\operatorname{boxed}(d_{j_*}^{\mathrm{sel}})^2 = h_{\nu_+,\mathrm{F}} \oplus h_{\nu_-,\mathrm{E}_0}.$$

All squared noncritical channels have their Friedrichs Bessel realizations; at a zero-order threshold component this means the no-logarithm realization.

The transported smooth-core squares converge strongly in the resolvent sense to $(d_{\beta}^{\mathrm{sel}})^2$. Thus a singular nonnegative Bessel realization does have a smooth minimally coupled spinor-core origin in the declared Dirac window. It is not the scalar limit of Theorem .

Proof. Squaring gives the displayed diagonal expression. In the critical channel its possible leading powers are

$$u_+ = a_+ r^{\gamma} + b_+ r^{1-\gamma} + \cdots, \qquad u_- = a_- r^{-\gamma} + b_- r^{1+\gamma} + \cdots.$$

Membership in $\operatorname{Dom} d^{\mathrm{sel}}$ imposes $a_+ = 0$. For membership in the square domain, du must satisfy the same boundary condition. The upper component of du is

$$-\mathrm{i}(\partial_r + \gamma/r)u_-.$$

It annihilates $a_- r^{-\gamma}$, while the next term has coefficient $-\mathrm{i}(1+2\gamma)b_- r^{\gamma}$. Hence $A(du) = 0$ requires $b_- = 0$.

Dividing by $\sqrt{r}$, the upper component therefore has only its regular r^{ν_+} boundary power, and the lower component only its singular $r^{-\nu_-}$ boundary power. These are exactly the two conditions in . For $|\gamma_j| \geq 1/2$, the alternative deficient squared-component power, when it exists, makes $d_j u$ non-square-integrable. At equality, the excluded alternative is the logarithmic solution. Thus the squared noncritical conditions are Friedrichs.

Finally, for every $s > 0$,

$$(d_\varepsilon^2 + s)^{-1} = \frac{(d_\varepsilon - \mathrm{i}\sqrt{s})^{-1} - (d_\varepsilon + \mathrm{i}\sqrt{s})^{-1}}{2\mathrm{i}\sqrt{s}}.$$

Theorem proves convergence of both terms. Nonnegativity of the limiting square follows directly from self-adjointness of d^{sel}.

Remark (The scalar–spinor distinction is structural). For the smooth minimally coupled Dirac operator, the Lichnerowicz expression contains the spinor connection Laplacian, $R_\varepsilon/4$, and a magnetic term with opposite signs in the two spin components. In the convention , the latter is $\sigma_3 \mathscr{B}_\varepsilon$. The lower component can therefore feel an attractive magnetic contribution even though the full square is nonnegative.

The scalar form has no such Pauli term. It is not legitimate to apply its positive-potential selection theorem separately to the components of the Dirac square. The general distinction between Dirac and Pauli extensions is also discussed in [5, 6].

Scope boundary (No parent branch-name identification). The boundary condition and square are explicit. We do not identify them by an unverified ``branch A'' or ``branch B'' convention from a different spinor polarization calculation. In particular, the disclosed Dirac coefficient discrepancy in I-B-VP-1 is not resolved by this domain theorem [16, 17, 18].

Longitudinal products and four-component Dirac operators

The scalar selection theorem was proved directly on the full spatial product. For spinors, the transverse result extends to a precise four-component product realization as follows.

Let τ_1, τ_2, τ_3 be a second set of Pauli matrices, let $M \geq 0$ be fixed, and Fourier transform in z. For longitudinal momentum k, define

$$H_\varepsilon(k) = \tau_1 \otimes d_\varepsilon + k\,\tau_2 \otimes I + M\,\tau_3 \otimes I,$$

and define $H_{\mathrm{sel}}(k)$ using d_β^{sel}. This is a fixed Clifford representation of the four-component minimally coupled spatial Dirac Hamiltonian.

Corollary (Strong-resolvent product selection). *The direct-integral Hamiltonians associated with converge strongly in the resolvent sense to the selected direct integral.*

For every fixed k,

$$H_{\mathrm{sel}}(k)^2 = I \otimes (d_\beta^{\mathrm{sel}})^2 + (k^2 + M^2)I.$$

No longitudinal-momentum-dependent apex relation is introduced.

Proof. For fixed k, the last two terms of are bounded. Tensor the graph approximants in Theorem with $\mathbb{C}^2$. Their bounded-term images converge as well, so Lemma proves fiberwise strong-resolvent convergence. The nonreal resolvents are bounded by $|\mathrm{Im}\, z|^{-1}$, independently of k, ε. Dominated convergence proves convergence of their direct integrals. The square identity follows from anticommutation of the τ-matrices.

Scope boundary (No arbitrary helicity or gauge-field transfer). The spinor construction uses the coupled first-order Clifford operator and the bounding spin structure. It is not a statement about independently assigned helicity-twisted scalar channels.

The source-test Maxwell and de Donder compatibility criteria of I-B-G-S-v2 concern different fields, ghost domains, and intertwiners [13]. Its curvature-regular tensor branch is not certified as a physical smooth-core limit

by the present theorem.

What is selected, what is excluded, and what remains separate

The operator classification within the declared class

toprule Object Selected limit Exact qualification

midrule endfirsthead toprule Object Selected limit Exact qualification

midrule endhead bottomrule endfoot

Charged scalar, $\xi \geq 0$ Twisted Friedrichs in every deficient channel All families in Assumption ; no self-similarity required.

Scalar E_0 or finite non-Friedrichs relation Not selected Exclusion is relative to the positive scalar-core class, not all possible regularizations.

Minimal Dirac, no critical window Unique exterior first-order realization Bounding spin structure; actual flux $0 < \beta < 1$.

Minimal Dirac, critical window $A = 0$ in $j = -1/2$ Explicit traces ; all admissible profiles select the same line.

Other Dirac boundary lines in that channel Not selected by this core class They remain mathematically available exterior extensions. At $\gamma = 0$, dilation invariance alone does not distinguish them.

Square of selected critical Dirac channel $h_{v_+,\mathrm{F}} \oplus h_{v_-,\mathrm{E}_0}$ A singular nonnegative component has a spinor-core origin; this is not a scalar ξR-core theorem.

Different spin structure, electric potentials, anomalous magnetic moments, tuned short-range terms Outside the class A new selection problem is required.

The classification answers the smooth-core-origin question for its specified scalar and spinor models. It is neither an assertion that all non-Friedrichs domains are unphysical nor an assertion that every Dirac-window domain has a realization in this class.

One-point contacts are not fixed

The one-point completion of I-B-VP-1 has, under its declared symmetries and Ward prescription, finite contact coordinates c, τ, w [16, thm:classification]. Changing those c-number completion contacts changes none of the operators in this paper. Thus operator selection cannot distinguish their values.

Nor does an exterior resolvent estimate by itself control:

• an unbounded covariance weight near spectral zero;

• coincidence subtraction and its uniformity;

• a thin-core stress counterterm;

• an apex distributional product or an interacting field algebra. The strengthened contact and boundary-domain distinctions in Ex1 and Ex2 remain relevant [17, 18].

Proposition (Strong resolvent convergence alone does not control an inverse covariance weight). *There exist strictly positive T_n converging strongly in the resolvent sense to I, and a fixed vector v, such that*

$$\mathrm{ip}\, v T_n^{-1/2} v \to +\infty.$$

Proof. On $\ell^2(\mathbb{N})$, let T_n equal I except in the n-th basis direction, where it equals 2^{-8n}. For fixed $s > 0$, the resolvent difference applied to f has norm at most $C_s|f_n| \to 0$.

Take $v_j = 2^{-j}$. Its n-th contribution to the inverse-weight quadratic form is

$$2^{-2n}(2^{-8n})^{-1/2} = 2^{2n}.$$

Every individual form value is finite, but the sequence diverges.

This is a logical counterexample, not a model of the present physical core. The fixed positive-mass case can admit additional bounded-functional-calculus arguments, but those must be stated and do not automatically prove renormalized one-point convergence.

Scope boundary (Non-sourcing after contact separation). Neither the selected domain nor any subsequently chosen c, τ, w imposes Maxwell or Einstein dynamics. A physical source equation and its solution would be additional data and analysis [10, 21].

Reliability, finite coverage, and the mathematical release

Evidence classes

Following the supplied XVI-O1-ST integrity discipline [22], use:

\texttt{ANALYTIC} A result proved by the displayed argument on its stated domain, or an explicitly identified analytical input.

\texttt{SUPPLIED} A version-qualified source statement or historical computational report not independently executed here.

\texttt{UNCERTIFIED} A numerical approximation without the complete artifacts and justified input, truncation, quadrature, and roundoff bounds needed for certification.

No numerical or symbolic program was executed for this draft. The matching estimates, scalar Mosco argument, and Dirac graph construction are written proofs, not accounts of runs.

A future resolvent-rate report must retain the actual profiles or their certified representation, flux lift, spin structure, charge convention, unitary identification, channel, spectral parameter, external cutoff, raw resolvent comparison, precision, and total error bounds. Sampling several profiles does not establish the quantifier ``every profile'' in the theorems.

Proposition (Finite coverage is class relative). *Let a proved finite-dimensional space Z contain every admissible residual for a declared claim. A finite linear probe map $P : Z \to \mathbb{R}^m$ detects every residual exactly when it is injective. Without proved membership in Z and injectivity, vanishing finite tests need not prove an operator or distributional identity.*

Proof. Within Z, the assertion is precisely $\ker P = \{0\}$. A nonzero element of its kernel is otherwise invisible.

Without the containment hypothesis, that conclusion has no stated domain of application.

This is the coverage distinction attributed to the supplied Oai282-A record [23]. No finite numerical sampling is used in the selection proofs.

Debugging audit

toprule Potential error in the proposal Correction used here

midrule endfirsthead toprule Potential error in the proposal Correction used here

midrule endhead bottomrule endfoot

Copy the untwisted global form comparison Use exterior lower semicontinuity and the magnetic Hardy-domain lemma.

Call the twisted form domain ordinary H^1 Use $H^1 \cap L^2(r^{-2}\mathrm{d}V_0)$.

Treat a Dirac operator as semibounded Use the first-order boundary form and graph-core resolvent argument.

Choose an arbitrary punctured-cone spin structure while claiming a smooth disk origin Use the bounding structure, antiperiodic in the polar frame.

Assign independent scalar conditions to the Dirac square Derive the square domain from $u \in \operatorname{Dom} d$ and $du \in \operatorname{Dom} d$.

Infer scalar E_0 exclusion also excludes spinor E_0 The selected Dirac square explicitly contains E_0; its magnetic Pauli term changes the problem.

Use one positive logarithmic matching length at $0 < v < 1$ Retain the signed power-law parameter and the pair $(\sigma, L_{\mathrm{eff}})$.

Turn exterior kernel convergence into full norm-resolvent convergence The norm rate is localized and channelwise; the full result is strong resolvent convergence.

Turn the graph-generator rate into a global Dirac norm-resolvent rate Equation is only the stated graph approximation.

Use ``positive measure of profiles" without a measure Quantify over all admissible profiles instead.

Identify the actual core flux with its fractional exterior character Fix $0 < \beta < 1$ as an actual flux lift; other lifts require a separate statement.

Normalize one-point contacts from the operator limit Keep domain selection, covariance control, and contact normalization separate.

Theorem (I-B-VP-5-Ex2 release). *For Assumption , actual flux* $0 < \beta < 1$*, and the explicitly stated scalar and spinor identifications:*

1. *every scalar core with fixed* $\xi \geq 0$ *selects the twisted Friedrichs product operator in the strong-resolvent sense;*

2. *the scalar deficient channels and selected boundary powers are those of Lemma ;*

3. *scalar zero-energy matching is exactly , with* $|B_{\varepsilon}/A_{\varepsilon}| \leq \varepsilon^{2\nu}$*;*

4. *the finite-energy exterior coefficient and localized channel estimate are –;*

5. *the minimally coupled Dirac cores select the unique exterior realization outside the window , and* $A = 0$ *inside it;*

6. *the critical boundary generator has the uniform graph approximation ;*

7. *the selected critical Dirac square is , and both the squares and the declared longitudinal Dirac products have strong-resolvent limits;*

8. *none of these results normalizes one-point contacts or proves renormalized one-point convergence. Negative scalar curvature coupling, additional electric or magnetic-moment interactions, other actual flux lifts, non-product extensions, interacting products, and a complete physical core-contact limit are not classified here.*

Proof. Items (i)–(ii) are Theorem and Lemma . Items (iii)–(iv) are Theorem , Theorem , and Corollary . Items

(v)–(vi) are Theorem . Item (vii) is Theorem and Corollary . Item (viii) is the object-level separation established in Section , including the explicit inverse-weight counterexample. The final sentence identifies hypotheses not included in the released theorems.

Companion relationships and unchanged programme boundaries

I-B-VP-5 provides the untwisted variational template and the operator/contact separation [11]. The present scalar proof extends the question to flux, but replaces the untwisted global comparison by a different local lower-bound argument.

I-B-G and the I-B-VP sources provide the exterior Bessel-extension and holonomy comparison [12, 14, 15]. Their Green-operator transfer statements are not core-selection or state-transfer theorems.

The minimally coupled Dirac result is independent of the vacuum-stress and current formulas in I-B-VP-1. It resolves a specified smooth-core domain question, not the disclosed numerical or polynomial discrepancy in that source [16, 17, 18].

The longitudinally non-product extensions of I-B-VP-6 are a different class [19]. The limits proved here retain a single transverse domain independent of longitudinal momentum.

The supplied historical XIV-H record has its H2 ending. The later XIV-I record constructs its named I2 internal amplitude but does not construct the full I3 joint map. XIV-J retains its unresolved J2 full-map obligation [24]. These version-qualified statements are not conflated: the I2 advance is not withdrawn. This paper discharges none of the full characteristic map, section, transverse-estimate, contact-gluing, output-wavefront, or quantum-admission obligations.

For the present programme reading:

$$\text{nolinkurl}\, O.1 = NOT_C LOSED, \qquad \text{texttt}\, bulk_symmetry_restored = \text{texttt}\, false.$$

XVI-N and Route B are not discharged. XV-B is not opened as a downstream quantum-admission exit. Problems 1, 2, and 8 remain unfinished. The other supplied problem numbers are not advanced by these results. No

version-specific separately scoped closure elsewhere is verified, revoked, or imported.

No Einstein dynamics, Maxwell or Einstein backreaction, spacetime emergence, Page curve, de Sitter state, cosmological bounce, or ultraviolet completion is constructed. OpenAI catalogue entries remain external records, not theorem premises.

Conclusion. For the declared smooth positive scalar cores, Friedrichs selection is universal even with nontrivial holonomy; profile dependence survives only in vanishing matching coefficients. For the minimally coupled Dirac problem, the smooth centre instead selects a definite first-order boundary line, whose square contains a singular nonnegative Bessel component in the Dirac window.

The same smooth geometric and flux data therefore need not select the same componentwise scalar realization for different field equations. That distinction is the principal mathematical and physical lesson of the construction. It identifies specified smooth-core origins without turning operator selection into a vacuum normalization, a finite acquisition map, or a source law for spacetime.

appendix

Boundary and normalization checks

Scalar boundary-form check

For

$$u = A_u r^{\nu} + B_u r^{-\nu}, \qquad v = A_v r^{\nu} + B_v r^{-\nu},$$

direct expansion gives

$$r(\overline{u}v' - \overline{u'}v) = 2\nu(\overline{B_u}A_v - \overline{A_u}B_v).$$

Thus the use of $B = 0$ for Friedrichs throughout this paper follows its displayed power convention. It is not a claim that every companion uses that letter for the singular coefficient.

Dirac square check

The Pauli identities imply

$$\left(-\mathrm{i}\sigma_1\partial_r + \sigma_2\frac{\gamma}{r}\right)^2 = -\partial_r^2 + \frac{\gamma^2}{r^2} - \sigma_3\frac{\gamma}{r^2}.$$

The upper radial Bessel order is $|\gamma - 1/2|$ and the lower order is $|\gamma + 1/2|$ after dividing by $\sqrt{r}$.

In the critical window these become $\nu_+ = 1/2 - \gamma$, $\nu_- = 1/2 + \gamma$. The two selected conditions are

$$u_+ = O(r^{1-\gamma}), \qquad u_- = a_- r^{-\gamma} + o(r^{1+\gamma})$$

in the boundary-coefficient sense. They are precisely Friedrichs above and E_0 below. The singular lower term is annihilated by $\partial_r + \gamma/r$, explaining why it belongs to the first-order graph domain.

Curvature and flux measure limits

For a smooth compact spatial test f, nonnegativity, support concentration, and give

$$\int R_\varepsilon f \,\mathrm{d}V_\varepsilon \to 4\pi(1-\alpha)\int_{\mathbb{R}} f(0,z)\mathrm{d}z,$$

$$\int \mathscr{B}_\varepsilon f \,\mathrm{d}V_\varepsilon \to 2\pi\beta\int_{\mathbb{R}} f(0,z)\mathrm{d}z.$$

These measure limits do not specify a limiting operator domain by themselves. In particular, the scalar form theorem and Dirac square theorem treat the same concentrating flux through different differential expressions. Multiplication by varying domain sequences is not continuous in the weak topology of these measures.

Declarations

Source scope. The I-B-VP-5, I-B-G, I-B-G-S, I-B-VP, and I-B-VP-6 items used here are supplied version-qualified passages, not an asserted audit of their full sources. The complete I-B-VP-1 and its supplied corrective companions are distinguished from those excerpts. No unavailable I-B-VP-5-Ex1 result is assumed.

The philosophy page was consulted as introductory background. Direct retrieval of the I-B browse page and the I-B-VP-5 browse page was unavailable through the consulted interface; their use rests on the supplied

version-qualified material. Primary literature records and the indicated Bessel formulas were consulted for attribution. No entire-corpus search or verification is claimed.

Analytical dependency. The smooth core class, unitary transports, magnetic Hardy-domain characterization, scalar Mosco argument, matching identities, Dirac traces, graph approximation, and selected square are stated and proved here. The general background of self-adjoint extension theory, Bessel functions, and variational convergence is cited without claiming novelty.

Novelty scope. The claimed contribution is the explicit profile-uniform conical implementation and its scalar–Dirac comparison, including the normalized matching estimates and singular graph generator. It is not a priority claim for Aharonov–Bohm extensions, Dirac regularization, Pauli core selection, or Mosco convergence.

Operational and physical scope. A prescribed smooth core is a mathematical model, not a derived cosmic-string interior. Its operator limit is a realization theorem for that model. No physical preference among different enlarged core models is inferred. The finite Enter interface supplies no operator or contact-selection rule.

Verification. No successful pdfLaTeX execution, numerical quadrature, symbolic-program run, proof-assistant verification, independent rebuild, independent referee approval, repository installation, or journal acceptance is asserted.

Submission format. This is one pdfLaTeX source with an internal bibliography and no external data, figures, code, or bibliography file. Two runs are intended for references and the contents. The author name follows the supplied series. Submission requires the author's approval of the text, attribution, and mathematical scope.

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