I-B-VP-5: Curvature-core selection on the prescribed conical Enter spacetime

A quantified Friedrichs limit, profile-dependent scattering lengths, and the non-selection of a bare apex curvature measure

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Abstract

We resolve a curvature-coupling selection problem in a specified untwisted sector of the prescribed conical spacetime of I-B. The additional geometric data are smooth rotational cores

$$q_\varepsilon = P_\varepsilon(r/\varepsilon)^2 \mathrm{d}r^2 + \alpha^2 r^2 \mathrm{d}\varphi^2 + \mathrm{d}z^2,$$

where $0 < \alpha < 1$, each profile increases from α to 1, and the exterior is exactly conical. For every fixed $\xi \geq 0$, the operators $-\Delta_{q_\varepsilon} + \xi R_\varepsilon$, after a declared unitary identification of their Hilbert spaces, converge in the strong resolvent sense to the Friedrichs cone operator. The theorem permits profiles varying with ε and requires no uniform bounds on their derivatives. Nevertheless, the curvature densities converge to the nonzero measure $4\pi(1-\alpha)\delta_{\mathscr{W}}$. Thus a concentrated curvature measure does not determine the limiting operator coupling by ordinary multiplication.

For a fixed profile, exact zero-energy matching gives $L_{\mathrm{eff}}(\varepsilon) = \varepsilon \exp[-1/d(P,\xi)]$. Its finite-radius value depends on the profile, whereas its boundary-relation limit is Friedrichs. We give quantitative bounds, an explicit profile functional in the weak-coupling expansion, and an exterior zero-mode resolvent estimate. These distinguish finite-core fitting from a zero-thickness operator limit.

For the alternative distributional model, the ordinary positive point-evaluation energy is not closable. On a

declared logarithmic maximal domain, however, a one-parameter family of well-defined renormalized curvature products realizes every finite logarithmic extension at the same curvature measure and coupling. The resulting non-selection theorem identifies the missing datum as a product prescription, not an observable contact. The contribution is a convention-complete selection criterion and obstruction within this ledger, not the discovery of two-dimensional scattering lengths. No state convergence, apex observable normalization, scalar or Einstein backreaction, or field acquisition by Enter reporting is inferred.

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Book expansion: operator convergence before scattering length

Reading order. (1) operator convergence theorem; (2) unitary identification and profile hypotheses; (3) scattering-length discussion. **Boxed scope.** Operator selection does *not* normalize one-point contacts. Object class of the main theorem: **core limit / operator**, not state and not contact.

The gap, the chosen alternative, and the scope of the result

The V5 question is whether additional curvature data can determine an operator extension at the conical apex. The answer must be attached to a class of models and a topology of convergence.

This paper proves the following two definite statements:

1. a **selection theorem**: the nonnegative monotone smooth-core class specified below selects the Friedrichs extension in the untwisted sector;

2. a **non-selection theorem for the other model**: a bare apex curvature measure, even with fixed nonzero ξ, does not determine a logarithmic extension without a multiplication prescription. The first statement is not a theorem about every smoothing. The second does not deny that a specified renormalized product can select an extension.

What is inherited and not recomputed

The regular host is the I-B product

$$M = \mathbb{R}_t \times (0,\infty)_r \times (\mathbb{R}/2\pi\mathbb{Z})_\varphi \times \mathbb{R}_z, \qquad g = -\mathrm{d}t^2 + \mathrm{d}r^2 + \alpha^2 r^2 \mathrm{d}\varphi^2 + \mathrm{d}z^2, \quad 0 < \alpha < 1.$$

The completion adds the worldsheet

$$\mathscr{W} = \mathbb{R}_t \times \{o\} \times \mathbb{R}_z.$$

The cone, deficit, product structure, and external time are prescribed data [9].

Three inherited distinctions are retained.

First, on M, $R = 0$. In I-B-VP-v4 the equation

$$(-\mathrm{Box}_g + \xi R)\Phi = (\partial_t^2 + B)\Phi = 0$$

contains no distributional curvature coupling. Its improvement identity

$$T_{\mu\nu}(\xi) - T_{\mu\nu}(1/6) = (\xi - 1/6)(g_{\mu\nu}\mathrm{Box}_g - \nabla_\mu\nabla_\nu)F_h$$

is an identity for observables at a fixed chosen operator and state, in the inherited normalization. It is not a boundary condition for B [11].

Second, the regular metric does not itself specify the self-adjoint boundary relation in I-B-G. Friedrichs denotes a particular finite-energy closure. A logarithmic length or a power-channel parameter is additional operator data [10]. The historical cone boundary analysis of Kay–Studer provides context, not a premise selecting a particular core [1].

Third, I-B-VP-1's constants c, τ, w are ambiguities in extensions of one-point observables, not the boundary coordinates of the spatial operator [12]. We do not identify these two sorts of freedom.

A deliberately quantified sector

The selection theorem concerns:

$$\text{boxed trivial holonomy} \beta = 0, \qquad \xi \geq 0, \qquad \text{the prescribed product class.}$$

It applies to a neutral scalar, or to the operator for either real component of a trivially twisted complex scalar. It includes minimal and conformal curvature coupling.

For $0 < \alpha < 1$, the untwisted cone has only one limit-circle angular channel, namely $n = 0$. Consequently selecting its logarithmic boundary relation selects the entire transverse operator in this sector.

Nontrivial holonomy, sign-changing curvature potentials, negative ξ, power-channel extensions, and matrix-valued boundary mixing are not covered by this smooth-core theorem. They are not silently resolved by an untwisted calculation.

Enter, spacetime, and the meaning of a report

Enter is a destructive finite-register readout interface: the measured quantum names are not returned as quantum output. The calculus distinguishes classical records from quantum residuals, and characterizes measurement only within its stated operational class [7]. The philosophical companion emphasizes that a record, a recovery map, and a physical identification require different interfaces [8].

Here Enter may report a supplied profile identifier, coupling, boundary parameter, and theorem scope. It does not choose a profile, manufacture apex curvature, or acquire an unknown continuum operator. The finite report is constructed in Section .

A retained analytical width is not automatically physical thickness. X-L-ST makes that distinction explicitly [13]. Our ε is instead the radius of an additional prescribed geometric family; it is not a reinterpretation of an earlier ramp regulator.

The untwisted extension coordinates

Let

$$N = (0,\infty)\times S^1\times\mathbb{R}, \qquad \mathrm{d}V_0 = \alpha r\,\mathrm{d}r\,\mathrm{d}\varphi\,\mathrm{d}z, \qquad \mathscr{H}_0 = L^2(N,\mathrm{d}V_0).$$

The nonnegative Friedrichs form is the closure of

$$\mathfrak{q}_F[u] = \int_N \left(|\partial_r u|^2 + \frac{|\partial_\varphi u|^2}{\alpha^2 r^2} + |\partial_z u|^2 \right) \mathrm{d}V_0$$

initially defined on $C_c^\infty(N)$. Its operator is denoted B_F.

For Fourier mode $e^{\mathrm{i}n\varphi}$, the transverse expression is

$$h_n = -\frac{\mathrm{d}^2}{\mathrm{d}r^2} - \frac{1}{r}\frac{\mathrm{d}}{\mathrm{d}r} + \frac{n^2}{\alpha^2 r^2}, \qquad \nu_n = \frac{|n|}{\alpha}.$$

Fix a reference length $\ell > 0$. For the zero mode, write

$$u(r) = a(u) + b(u)\log(r/\ell) + o(1), \qquad ru'(r) = b(u) + o(1).$$

The notation a, b refers to operator boundary traces, not to one-point counterterms.

Lemma (Logarithmic boundary traces and self-adjoint relations). *On the radial maximal domain*

$$\mathscr{D}_{\max} = \left\{ u \in L^2((0,\infty), r\,dr) : u, ru' \in AC_{\mathrm{loc}}(0,\infty), ?h_0 u \in L^2((0,\infty), r\,dr) \right\},$$

the limits in exist and are continuous in the maximal graph norm.

The zero-mode self-adjoint extensions are precisely

$$\mathscr{D}(h_F) : b = 0, \qquad \mathscr{D}(h_L) : a = \log(\ell/L)b, \quad 0 < L < \infty.$$

For $n \neq 0$, the endpoint 0 is limit point, so there is no additional angular boundary parameter.

Proof. Put $f = h_0 u$. Since

$$(ru')' = -rf,$$

Cauchy–Schwarz shows that rf is integrable near zero. Thus ru' has a limit b, and

$$ru'(r) - b = -\int_0^r sf(s)\mathrm{d}s, \qquad \left| \int_0^r sf(s)\mathrm{d}s \right| \le \frac{r}{\sqrt{2}} \|f\|_{L^2((0,r),\,s\mathrm{d}s)}.$$

After division by r, the remainder is integrable at zero. This gives a and . Interior one-dimensional elliptic estimates on a fixed annulus, followed by these integral identities, bound both traces by the maximal graph norm.

For unnormalized angular-constant functions, the boundary Green form is

$$2\pi\alpha\big(\overline{a(u)}b(v) - \overline{b(u)}a(v)\big).$$

Both independent zero-energy solutions $1, \log r$ are square integrable near zero. At infinity there is one square-integrable deficiency solution, represented by a modified Bessel function with decaying complex argument. Hence this radial problem has one boundary pair. Its maximal isotropic complex lines are $b = 0$ and $a = \theta b$, $\theta \in \mathbb{R}$. Writing $\theta = \log(\ell/L)$ yields . Finite radial energy excludes $b \neq 0$, identifying the Friedrichs relation.

For $n \neq 0$, the irregular behavior $r^{-\nu_n}$ is not square integrable because $\nu_n \geq 1$. The endpoint is therefore limit point.

This is the logarithmic subledger of I-B-G, with its normalization made explicit for the matching and distributional calculations. The classification is not a new general cone extension theorem [10, 1].

A length Ldownarrow0 is not an interior value of the finite logarithmic family. It approaches the Friedrichs boundary line in the space of boundary relations. This distinction will be proved directly for the matching data.

Additional smooth-core geometry

Assumption (Monotone flat-center profiles). Fix $s_0 \in (0,1)$. For each $\varepsilon > 0$, choose independently a profile $P_\varepsilon \in C^\infty([0,\infty))$ satisfying

$$P_\varepsilon(s) = \alpha \quad (0 \leq s \leq s_0), \qquad P_\varepsilon(s) = 1 \quad (s \geq 1), \qquad P_\varepsilon'(s) \geq 0.$$

No uniform derivative bound is required. Put

$$p_\varepsilon(r) = P_\varepsilon(r/\varepsilon)$$

and prescribe, on $\overline{N} = \mathbb{R}^2_x \times \mathbb{R}_z$,

$$q_\varepsilon = p_\varepsilon(r)^2 \mathrm{d}r^2 + \alpha^2 r^2 \mathrm{d}\varphi^2 + \mathrm{d}z^2, \qquad g_\varepsilon = -\mathrm{d}t^2 + q_\varepsilon, \qquad x = (r\cos\varphi, r\sin\varphi).$$

The class is nonempty: integrate any nonnegative smooth bump supported in $(s_0, 1)$ and normalize its integral to $1 - \alpha$.

These metrics are a flat-center subclass of the rounded-cone metrics used by Allen–Kay–Ottewill, after translating their angular period to 2π [2]. The family is additional geometry; it is not determined by the regular host.

Proposition (Smoothness, curvature, and curvature density). *Every metric in Assumption is smooth, positive definite, and complete on $\overline{N}$. It equals the cone metric for $r \geq \varepsilon$. With the curvature sign for which the sphere has positive scalar curvature,*

$$R_\varepsilon(r) = \frac{2p_\varepsilon'(r)}{r p_\varepsilon(r)^3} = \frac{2}{\varepsilon^2 s P_\varepsilon(s)^3}\frac{P_\varepsilon'(s)}{}, \qquad s = r/\varepsilon.$$

It is nonnegative and supported in $s_0\varepsilon \le r \le \varepsilon$, and

$$\int_{\mathbb{R}^2} R_\varepsilon \, \mathrm{d}V_{\varepsilon,\perp} = 4\pi(1-\alpha) =: \mathcal{K}_\alpha.$$

On the Cartesian completion, as curvature densities,

$$R_\varepsilon \, \mathrm{dvol}_{g_\varepsilon} \to \mathcal{K}_\alpha \delta_{\mathcal{W}}.$$

Here the right side means evaluation on tests along $\mathcal{W}$; equivalently its pairing with ψ is $\mathcal{K}_\alpha \int \psi(t,z,0)\mathrm{d}t\,\mathrm{d}z$. The convergence error on a fixed compact test support is bounded by $C\varepsilon^2 p_2(\psi)$, uniformly over the profile class.

Proof. Near the origin the transverse metric is $\alpha^2[(\mathrm{d}x^1)^2 + (\mathrm{d}x^2)^2]$. Away from the origin its coefficients are smooth. Its radial and angular eigenvalues relative to the Euclidean metric lie between α^2 and 1. This proves smooth positive definiteness and completeness.

Let ρ denote interior geodesic radius, $\mathrm{d}\rho = p_\varepsilon \mathrm{d}r$. The transverse warping function is $f(\rho) = \alpha r(\rho)$, so

$$f' = \frac{\alpha}{p_\varepsilon}, \qquad f'' = -\frac{\alpha p_\varepsilon'}{p_\varepsilon^3}.$$

The formula $R = -2f''/f$ gives . Since

$$\mathrm{d}V_{\varepsilon,\perp} = \alpha p_\varepsilon r \, \mathrm{d}r \, \mathrm{d}\varphi,$$

its integral is

$$4\pi\alpha \int_0^\infty \frac{p_\varepsilon'}{p_\varepsilon^2}\mathrm{d}r = 4\pi\alpha(\alpha^{-1} - 1).$$

For a Cartesian test, angular averaging gives

$$\frac{1}{2\pi}\int_0^{2\pi} \psi(y, r\omega)\mathrm{d}\varphi = \psi(y,0) + O(r^2 p_2(\psi)).$$

The curvature density is positive, has total transverse mass $\mathcal{K}_\alpha$, and is supported in $r \le \varepsilon$. Integrating the remainder

proves the uniform estimate and .

Remark (The density is essential). Relative to the fixed smooth density

$$\mathrm{d}\Omega = \alpha\,\mathrm{d}t\,\mathrm{d}z\,\mathrm{d}^2x,$$

the generalized functions in are $p_\varepsilon R_\varepsilon$, not R_ε alone. No multiplication by an undefined value of the limiting conical metric at the origin has been used. The completion convention agrees with the explicitly additional Cartesian convention in I-B-VP-1 [12].

Proposition (An incompatible geodesic-radius shortcut). *For $\alpha < 1$, there is no smooth nonnegatively curved cap of the form*

$$\mathrm{d}r^2 + f_\varepsilon(r)^2\mathrm{d}\varphi^2$$

that has $f_\varepsilon(0) = 0$, $f_\varepsilon'(0) = 1$, and matches $f_\varepsilon(r) = \alpha r$ at the same exterior coordinate radius $r = \varepsilon$.

Proof. Nonnegative curvature implies $f_\varepsilon'' \le 0$. Matching gives $f_\varepsilon'(\varepsilon) = \alpha$, hence $f_\varepsilon'(r) \ge \alpha$ on $[0, \varepsilon]$, with strict inequality near zero. Consequently $f_\varepsilon(\varepsilon) = \int_0^\varepsilon f_\varepsilon'(r)\mathrm{d}r > \alpha\varepsilon$, a contradiction.

Our construction avoids this incompatibility because r is not geodesic radius inside the core.

A profile-independent Friedrichs selection theorem

Fix $\xi \ge 0$. Let

$$\mathscr{H}_\varepsilon = L^2(\overline{N}, \mathrm{d}V_\varepsilon), \qquad \mathrm{d}V_\varepsilon = p_\varepsilon\,\mathrm{d}V_0.$$

Define the closed nonnegative form

$$\mathfrak{q}_\varepsilon[u] = \int_N \left(p_\varepsilon^{-2}|\partial_r u|^2 + \frac{|\partial_\varphi u|^2}{\alpha^2 r^2} + |\partial_z u|^2 + \xi R_\varepsilon |u|^2 \right) p_\varepsilon \mathrm{d}V_0.$$

For each fixed ε, R_ε is smooth and bounded. The form domain is the ordinary Cartesian $H^1(\mathbb{R}^3)$, with equivalent norms. Its associated operator is denoted B_ε. Thus the smooth problem uses the usual finite-energy realization on

the completed smooth geometry, not an independently chosen inner boundary condition.

The unitary identification is

$$U_\varepsilon : \mathscr{H}_\varepsilon \to \mathscr{H}_0, \qquad U_\varepsilon u = p_\varepsilon^{1/2} u.$$

It is necessary to declare this identification before speaking of strong resolvent convergence.

Lemma (Zero capacity of the worldsheet in the energy form). *The form domain of* $\mathfrak{q}_F$ *is* $H^1(\mathbb{R}^3)$ *under the Cartesian identification, with an equivalent norm. In particular, compactly supported smooth functions avoiding the axis are form dense.*

Proof. The conical energy and measure are uniformly comparable to the Euclidean H^1 energy and measure. It remains to remove the axis from smooth compactly supported functions.

For small dimensionless δ, choose a radial cutoff η_δ that is zero for $r \le \ell\delta^2$, one for $r \ge \ell\delta$, and transitions on the logarithmic radial scale. It can be chosen so that

$$\int_0^\infty |\eta_\delta'|^2 r \, \mathrm{d}r \le \frac{C}{|\log \delta|}.$$

For fixed smooth compactly supported u, $(1 - \eta_\delta)u \to 0$ in L^2. The energy of the differentiated cutoff is bounded by the displayed estimate times a bound on u and its longitudinal support. All other terms converge by dominated convergence. Smooth approximation within the annulus preserves the estimates. Ordinary density of $C_c^\infty(\mathbb{R}^3)$ completes the proof.

Theorem (Selection of the Friedrichs product operator). *For every family in Assumption and every fixed* $\xi \ge 0$*, put*

$$\widetilde{B}_\varepsilon = U_\varepsilon B_\varepsilon U_\varepsilon^{-1}.$$

Then, for every $s > 0$ *and* $f \in \mathscr{H}_0$*,*

$$(\widetilde{B}_\varepsilon + s)^{-1} f \to (B_F + s)^{-1} f \quad \text{strongly in } \mathscr{H}_0.$$

Equivalently, the transported closed forms converge to $\mathfrak{q}_F$ *in the sense of Mosco.*

The selected I-B-G boundary relation is

$$\text{boxed}\, b = 0 \quad \text{in the zero angular channel.}$$

All nonzero angular channels already have their unique limit-point realization. The result is independent of the profiles, including profiles depending on ε.

Proof. Write

$$Q_\varepsilon[v] = \mathfrak{q}_\varepsilon[U_\varepsilon^{-1} v].$$

We prove the weak lower bound and strong recovery properties directly.

Weak lower bound. Suppose $v_\varepsilon \text{rightharpoonup}\, v$ in $\mathscr{H}_0$ and

$$\sup_\varepsilon \left(Q_\varepsilon[v_\varepsilon] + \|v_\varepsilon\|^2\right) < \infty.$$

Put $u_\varepsilon = p_\varepsilon^{-1/2} v_\varepsilon$. Since $\alpha \le p_\varepsilon \le 1$, multiplication by $p_\varepsilon^{\pm 1/2}$ is uniformly bounded and converges strongly to the identity on $\mathscr{H}_0$. It follows that $u_\varepsilon \text{rightharpoonup}\, v$ in $\mathscr{H}_0$.

The kinetic coefficients in give

$$\mathfrak{q}_\varepsilon[u] \ge \alpha \mathfrak{q}_F[u].$$

Thus u_ε is bounded in $H^1(\mathbb{R}^3)$. Its weak H^1 limit is v. For every fixed $a > 0$, once $\varepsilon < a$ the forms coincide with the conical kinetic form on $r > a$. Nonnegativity of every omitted term and weak lower semicontinuity imply

$$\liminf_{\varepsilon \text{downarrow}\, 0} Q_\varepsilon[v_\varepsilon] \ge \int_{r>a} \left(|\partial_r v|^2 + \frac{|\partial_\varphi v|^2}{\alpha^2 r^2} + |\partial_z v|^2 \right) \mathrm{d}V_0.$$

Letting $a \text{downarrow}\, 0$ proves

$$\liminf Q_\varepsilon[v_\varepsilon] \ge \mathfrak{q}_F[v].$$

Strong recovery. For $v \in \mathscr{D}(\mathfrak{q}_F)$, choose $\varphi_j \in C_c^\infty(N)$ converging to v in form norm, using Lemma . For each fixed

j, the support avoids the core when ε is sufficiently small. Then

$$U_\varepsilon \varphi_j = \varphi_j, \qquad Q_\varepsilon[\varphi_j] = \mathfrak{q}_F[\varphi_j].$$

A diagonal choice $j = j(\varepsilon) \to \infty$ gives a strong recovery sequence. For v outside the form domain the recovery upper bound is vacuous. These are precisely the Mosco conditions [5].

Resolvent convergence without an unstated limit theorem. Let v_ε minimize

$$J_\varepsilon[w] = Q_\varepsilon[w] + s\|w\|^2 - 2\mathrm{Re}\,\mathrm{pair}\, fw,$$

and let v minimize the corresponding functional J_F. The preceding lower bound and recovery sequence show that every weak cluster point of v_ε is v and that $J_\varepsilon[v_\varepsilon] \to J_F[v]$. If w_ε is a recovery sequence for v, the minimizer equation gives

$$J_\varepsilon[w_\varepsilon] - J_\varepsilon[v_\varepsilon] = Q_\varepsilon[w_\varepsilon - v_\varepsilon] + s\|w_\varepsilon - v_\varepsilon\|^2.$$

The left side tends to zero. Thus $v_\varepsilon \to v$ strongly. These minimizers are the two resolvents in .

Lemma identifies the selected relation and the other angular channels.

Remark (What the topology does not imply). The theorem is a strong resolvent statement on the entire spatial Hilbert space after , not merely a finite set of exterior tests.

It does not assert norm resolvent convergence on that entire space, convergence of arbitrary spectral derivatives, or convergence of a massless covariance involving the unbounded function $B^{-1/2}$ near zero. In particular, no convergence theorem for renormalized one-point observables follows merely from .

Corollary (The curvature measure is not an operator limit rule). *The nonzero curvature-density limit does not survive as an ordinary nonzero point-evaluation term in the limiting Friedrichs energy form.*

Proof. The limiting form in Theorem is precisely $\mathfrak{q}_F$, with no extra term. Its recovery sequences avoid the core with vanishing energy cost, by Lemma . This is compatible with the nonzero weak limit of the curvature density because multiplication by a varying form-domain sequence is not continuous in the topology of curvature

measures.

This corollary concerns the specified untwisted codimension-two problem. It is not a general rule that concentrating positive potentials always disappear.

Exact matching and finite-radius profile dependence

Now fix a single profile P satisfying , and use $p_{\varepsilon}(r) = P(r/\varepsilon)$. The following matching calculation is separate from the operator-limit proof.

The regular zero-energy, angular-constant, longitudinally constant solution, normalized at the center, is $u_{\varepsilon}(r) = U(r/\varepsilon)$, where

$$\left(\frac{s}{P(s)}U'(s)\right)' = 2\xi\frac{P'(s)}{P(s)^2}U(s), \qquad U(0) = 1.$$

Regularity sets $U' = 0$ on the central flat neighborhood. The equation is ordinary and nonsingular after that neighborhood.

Theorem (Effective logarithmic length and quantitative bounds). *For $\xi = 0$, $U \equiv 1$ and the exterior zero-energy solution has $b = 0$.*

For $\xi > 0$, define

$$d(P,\xi) = \frac{U'(1)}{U(1)}, \qquad G = 2\xi(\alpha^{-1} - 1) = \frac{\xi\mathcal{K}_{\alpha}}{\sigma_{\alpha}}, \qquad \sigma_{\alpha} = 2\pi\alpha.$$

Then

$$0 < Gs_0^G \le d(P,\xi) \le G.$$

For $r \ge \varepsilon$,

$$u_{\varepsilon}(r) = U(1)[1 + d(P,\xi)\log(r/\varepsilon)] = U(1)d(P,\xi)\log\frac{r}{L_{\mathrm{eff}}(\varepsilon)},$$

where

$$\text{boxed}\, L_{\text{eff}}(\varepsilon) = \varepsilon \exp\left[-1/d(P,\xi)\right].$$

In particular,

$$\varepsilon \exp\left[-1/(G s_0^{G})\right] \text{leq} L_{\text{eff}}(\varepsilon) \le \varepsilon \exp\left[-1/G\right] < \varepsilon.$$

For $\varepsilon < \ell$, the associated boundary line has

$$\frac{b}{a} = \frac{d(P,\xi)}{1 + d(P,\xi)\log(\ell/\varepsilon)} \le \frac{1}{\log(\ell/\varepsilon)}.$$

Consequently these fitted boundary lines converge to $b = 0$, uniformly over the profile class.

Proof. Set

$$H(s) = \alpha^{-1} - P(s)^{-1}, \qquad H'(s) = \frac{P'(s)}{P(s)^{2}} \ge 0.$$

Integrating gives

$$\frac{s}{P(s)} U'(s) = 2\xi \int_0^s H'(t) U(t) \mathrm{d}t.$$

The solution remains positive and nondecreasing: starting from $U = 1, U' = 0$, the right side is nonnegative for as long as $U > 0$, and therefore a first loss of positivity is impossible. Since $H(1) > 0$, $U'(1) > 0$.

By monotonicity,

$$\frac{U'(s)}{U(s)} \le \frac{2\xi P(s)}{s} H(s) \le \frac{G}{s} \quad (s \ge s_0).$$

Hence $1 \le U(1) \le s_0^{-G}$. At $s = 1$, yields

$$d = \frac{2\xi \int_0^1 H'(t) U(t) \mathrm{d}t}{U(1)}.$$

Using $1 \le U(t) \le U(1)$ proves .

Outside the core, $P = 1, R = 0$, so the zero-energy solution is constant plus logarithm. Matching its value and derivative at $r = \varepsilon$ gives –. The length bounds follow from . Expanding in the coordinates gives .

The matching length in is the two-dimensional zero-energy scattering length, with the convention $L = 0$ for the constant solution at $\xi = 0$. Its fitting formula is established in Kay–Studer and Allen–Kay–Ottewill [1, 2]. The point of Theorems and is that operator convergence and fitted boundary-line convergence have been proved separately.

Proposition (Exact profile dependence at fixed outer cutoff). *Let $P_c(s) = P(s/c)$, $0 < c < 1$. Whenever both profiles belong to the declared class,*

$$d(P_c, \xi) = \frac{d(P, \xi)}{1 + d(P, \xi)\log(1/c)}, \qquad L_{\mathrm{eff}}(P_c, \varepsilon) = c\, L_{\mathrm{eff}}(P, \varepsilon).$$

Thus fixed deficit and fixed $\xi > 0$ do not determine the finite-ε matching length. Nevertheless, all these lengths have the same limiting Friedrichs boundary relation.

Proof. The normalized solution for P_c is $U_c(s) = U(s/c)$. At $s = 1$, the original solution is already in its exterior logarithmic region:

$$U(1/c) = U(1)[1 + d(P, \xi)\log(1/c)], \qquad c^{-1}U'(1/c) = U(1)d(P, \xi).$$

This proves both formulas.

For two examples with a common s_0, take $s_0 < 1/4$, choose P constant on $[0, 2s_0]$, and use $c = 1/2$. Both are admissible. Their curvatures have the same integrated value $\mathcal{K}_\alpha$ by Proposition , but their matching lengths differ.

The examples move curvature within the same prescribed outer radius bound. They prove dependence within the stated class; they do not identify two distinct radii of one physical string.

A profile functional beyond the leading exponent

Proposition (Weak-coupling expansion). *For a fixed profile, let*

$$H_1 = \alpha^{-1} - 1, \qquad Q[P] = \int_0^1 \frac{P(s)}{s}\left(\alpha^{-1} - P(s)^{-1}\right)^2 \mathrm{d}s.$$

As ξdownarrow0,

$$d(P,\xi) = 2\xi H_1 - 4\xi^2 Q[P] + O(\xi^3),$$

$$\log\frac{L_{\mathrm{eff}}(\varepsilon)}{\varepsilon} = -\frac{1}{2\xi H_1} - \frac{Q[P]}{H_1^2} + O(\xi).$$

The error constants may depend on the fixed profile.

Proof. On $[0,1]$, is a Volterra equation with a bounded kernel because P' vanishes near zero. Its solution is analytic in ξ near zero. If

$$W(s) = \int_0^s \frac{P(t)}{t} H(t)\mathrm{d}t,$$

the first variation is $U(s) = 1 + 2\xi W(s) + O(\xi^2)$. Using at 1,

$$d = 2\xi H_1 + 4\xi^2\left(\int_0^1 H'(s)W(s)\mathrm{d}s - H_1 W(1)\right) + O(\xi^3).$$

Integration by parts changes the expression in parentheses to

$$-\int_0^1 H(s)W'(s)\mathrm{d}s = -Q[P].$$

Inverting the expansion for d and using proves .

The leading exponent is $-[2\xi(\alpha^{-1}-1)]^{-1}$, agreeing with the established weak-potential expression after the angular convention is translated [2, Eq. (3.22)]. No priority claim is made for that exponent or for the fitting formula. The explicit $Q[P]$ records the profile information that the total curvature integral does not contain.

An exterior resolvent estimate and a finite-core warning

The zero-energy matching length is not an exact energy-independent replacement of a smooth core.

For this section, factor out the normalized angular zero mode and use the radial Hilbert space $L^2((0,\infty), r\mathrm{d}r)$. Let $k > 0$. The regular interior solution at spectral parameter $-k^2$ obeys, in scaled radius,

$$\left(\frac{s}{P_\varepsilon(s)}U_k'(s)\right)' = \left[2\xi\frac{P_\varepsilon'(s)}{P_\varepsilon(s)^2} + (k\varepsilon)^2 s P_\varepsilon(s)\right]U_k(s).$$

It is normalized by $U_k(0) = 1$ and regularity. Set

$$D_\varepsilon(k) = U_k'(1)/U_k(1).$$

Positivity of the coefficients in gives $D_\varepsilon(k) \geq 0$.

Proposition (Exterior zero-mode norm estimate). *For $r, r' > \varepsilon$, the radial resolvent kernel is*

$$G_\varepsilon(r, r'; k) = I_0(kr_<)K_0(kr_>) + c_\varepsilon(k)K_0(kr)K_0(kr'),$$

with respect to $r'\mathrm{d}r'$, where

$$c_\varepsilon(k) = \frac{xI_0'(x) - D_\varepsilon(k)I_0(x)}{D_\varepsilon(k)K_0(x) - xK_0'(x)}, \qquad x = k\varepsilon.$$

For fixed $a, k > 0$, and sufficiently small $\varepsilon < a$, the exterior compression to $r > a$ satisfies

$$\left\| 1_{r>a}[(h_{\varepsilon,0} + k^2)^{-1} - (h_F + k^2)^{-1}]1_{r>a} \right\| \leq \frac{C_{a,k}}{|\log(k\varepsilon)|}.$$

The comparison uses the natural identical measures on the exterior. It is uniform over the profile class. For $\xi = 0$, the right side can instead be replaced by $C_{a,k}(k\varepsilon)^2$.

Proof. The exterior regular solution has the form $I_0(kr) + cK_0(kr)$. Matching its logarithmic derivative gives . The Wronskian with K_0 is unchanged by adding cK_0, which proves .

For fixed $x > 0$, the fractional expression in decreases with $D \geq 0$, by the Bessel Wronskian. Its two endpoint values give

$$-\frac{I_0(x)}{K_0(x)} \leq c_\varepsilon(k) \leq \frac{I_1(x)}{K_1(x)}.$$

As $x$downarrow0, the absolute values of these bounds are $O(|\log x|^{-1})$ and $O(x^2)$, respectively. On $r > a$, the kernel difference is rank one, with norm

$$|c_\varepsilon(k)| \int_a^\infty K_0(kr)^2 r\mathrm{d}r.$$

The integral is finite, proving .

If $\xi = 0$, integrate and use $P_{\varepsilon} \le 1$ and $U_k(s) \le U_k(1)$:

$$0 \le D_{\varepsilon}(k) \le (k\varepsilon)^2 \int_0^1 s\,ds = \frac{(k\varepsilon)^2}{2}.$$

The numerator in is then $O(x^2)$, while its denominator is bounded below by $xK_1(x)$, which tends to 1.

This is a norm estimate for a specified exterior compression and a specified angular channel. It is not a norm resolvent theorem for the entire spatial operator.

Proposition (The fitted finite-length operator is not the core). *Every finite logarithmic extension h_L has a negative eigenvalue*

$$-k_L^2, \qquad k_L = \frac{2e^{-\gamma_E}}{L}.$$

No h_L, $0 < L < \infty$, is the exact smooth-core transverse operator in the nonnegative class. For fixed $L > 0$, its zero-energy boundary derivative at $r = \varepsilon < L$ would be negative:

$$\frac{\varepsilon u'(\varepsilon)}{u(\varepsilon)} = \frac{1}{\log(\varepsilon/L)} < 0.$$

This cannot match with $\xi > 0$.

Proof. The function $K_0(kr)$ is square integrable at zero and infinity and solves $h_0 u = -k^2 u$. Its small-radius expansion is

$$K_0(kr) = -\log(r/\ell) - \log(k\ell/2) - \gamma_E + o(1).$$

Condition gives exactly . The smooth-core form is nonnegative, so the operators cannot be equal. The derivative comparison follows from $u = b\log(r/L)$ and the positivity in Theorem .

For the fitted $L = L_{\text{eff}}(\varepsilon)$, the formal bound-state scale diverges as the core shrinks. That does not create a negative eigenvalue in B_{ε}. The distinction between a finite-core fit and an exact massless logarithmic theory is already important in the classical literature [2].

On the worldsheet product, a literal h_L has mode values $p_z^2 - k_L^2$. Its unstable band $|p_z| < k_L$ is not removed by calling the boundary condition a core fit. No massless ground state for that unstable operator is constructed here.

The alternative model: a bare apex curvature measure

We now keep the conical metric on the regular part and work on the additional Cartesian completion

$$\overline{M} = \mathbb{R}^{1+1}_{t,z} \times \mathbb{R}^2_x, \qquad \mathrm{d}\Omega = \alpha \, \mathrm{d}t \, \mathrm{d}z \, \mathrm{d}^2x.$$

Normalize

$$\langle \delta_{\mathscr{W}}, \psi \rangle = \int_{\mathbb{R}^{1+1}} \psi(t, z, 0) \mathrm{d}t \, \mathrm{d}z, \qquad \delta_{\mathscr{W}} = \alpha^{-1} \delta^{(2)}_{\mathrm{Leb}}(x)$$

relative to this density. The candidate curvature distribution is

$$\mathcal{R} = \mathscr{K}_\alpha \delta_{\mathscr{W}}.$$

Equation agrees with our curvature-density limit on smooth tests. It does not yet define $\mathcal{R}u$ for a logarithmically singular u. This distinction is not a denial of distributional or generalized-function constructions of cone curvature [3, 4]. The missing multiplication is a separate question.

Failure of the ordinary positive point-evaluation form

Theorem (Nonclosability of the naive curvature energy). *Let $g_\xi = \xi \mathscr{K}_\alpha > 0$. On $C_c^\infty(\mathbb{R}^2)$, consider*

$$\mathfrak{t}_{\mathrm{naive}}[u] = \int_{\mathbb{R}^2} |\nabla u|^2_{q_0} \alpha \, \mathrm{d}^2x + g_\xi |u(0)|^2,$$

where the regular conical energy is evaluated off the origin. This densely defined nonnegative form is not closable in $L^2(\mathbb{R}^2, \alpha \mathrm{d}^2x)$.

The corresponding worldsheet form with $g_\xi \int |u(0,z)|^2 \mathrm{d}z$ is likewise not closable. Thus the ordinary positive curvature measure and pointwise multiplication do not define a self-adjoint operator by closing this energy.

Proof. Choose smooth radial functions ζ_j equal to 1 on $r \le \ell\delta_j^2$, zero on $r \ge \ell\delta_j$, and transitioning logarithmically,

where $\delta_j \text{downarrow } 0$. They can be chosen with

$$\|\zeta_j\|_{L^2} \to 0, \qquad \mathfrak{q}_F[\zeta_j] \to 0, \qquad \zeta_j(0) = 1.$$

The construction is the complement of the cutoff used in Lemma . For all j, m, $(\zeta_j - \zeta_m)(0) = 0$, hence

$$\mathfrak{t}_{\text{naive}}[\zeta_j - \zeta_m] = \mathfrak{q}_F[\zeta_j - \zeta_m] \to 0.$$

But $\mathfrak{t}_{\text{naive}}[\zeta_j] \to g_\xi \neq 0$. This violates the defining criterion for closability.

For the worldsheet form, use $u_j(x, z) = \zeta_j(x)\chi(z)$ with fixed nonzero $\chi \in C_c^\infty(\mathbb{R})$. The additional longitudinal kinetic term tends to zero, whereas the point term tends to $g_\xi \|\chi\|_{L^2}^2$.

The theorem does not exclude renormalized point interactions. It excludes precisely the naive form . Restricting its initial domain to functions avoiding the origin instead removes the point term entirely and gives the Friedrichs form; that is a different prescription.

A well-defined logarithmic product family

It suffices to display the product issue in the angular zero mode, since that is the only boundary channel in the selected untwisted sector.

On the transverse completion use $d\Omega_\perp = \alpha d^2x$, and normalize $\langle \delta_o, \psi \rangle = \psi(0)$. For $u \in \mathscr{D}_{\max}$, its logarithmic expansion is . Its ordinary Cartesian gradient is locally integrable. In the radial sector, the inverse conical metric acts on that gradient as the identity because the radial eigenvalue is 1. Accordingly define the distributional kinetic expression by the ordinary divergence of this locally integrable radial flux.

Lemma (The kinetic apex term). *If $u \in \mathscr{D}_{\max}$ and $f = h_0 u$ on $r > 0$, the declared kinetic distribution is*

$$\Re u = f - \sigma_\alpha b(u)\delta_o, \qquad \sigma_\alpha = 2\pi\alpha.$$

This operation is well defined by integration against smooth Cartesian tests. It does not require multiplying a supported distribution by the singular metric.

Proof. The flux has leading behavior $b\,x/r^2$, which is locally integrable in two dimensions. Its divergence has the usual logarithmic flux contribution. Equivalently, integrate by parts on $r > \rho$ and let ρdownarrow 0. The inner boundary term is $-2\pi\alpha b\,\psi(0)$. The remainder tends to the regular L^2 function f. The trace estimate in Lemma justifies this limit.

Definition (Renormalized curvature products). For a fixed real number η, define on $\mathscr{D}_{\max}$

$$(\mathcal{R} \star_\eta u) := \mathscr{K}_\alpha [a(u) + \eta b(u)]\delta_o.$$

The worldsheet version tensors this transverse rule with longitudinal variables on its declared product domain.

For smooth radial functions, $b = 0, a = u(0)$, so agrees with ordinary multiplication by the curvature measure. For logarithmic functions it is an additional rule.

Theorem (Product-dependent selection and bare-measure non-selection). *The product in Definition is linear and continuous from the maximal graph domain to distributions. It is real on real functions and agrees with ordinary measure multiplication on smooth radial functions.*

For $\xi > 0$, requiring

$$\Re u + \xi(\mathcal{R} \star_\eta u) \in L^2(\mathbb{R}^2, \alpha \mathrm{d}^2 x)$$

selects the self-adjoint logarithmic boundary condition

$$a = \left(\frac{\sigma_\alpha}{\xi \mathscr{K}_\alpha} - \eta\right) b, \qquad \text{boxed}\, L_\eta = \ell \exp\left(\eta - \frac{\sigma_\alpha}{\xi \mathscr{K}_\alpha}\right).$$

The operator acts as h_0 on the regular part.

For the same fixed $(\alpha, \xi, \mathcal{R})$, every finite $L \in (0, \infty)$ occurs for exactly one η. Therefore those three data alone do not select an extension within this product class. For $\xi = 0$, the same no-apex-residual condition is $b = 0$, independently of η.

Proof. Continuity follows from the graph continuity of a, b and the continuity of test evaluation at the origin. The

reality and smooth-function statements are immediate.

Using Lemma , the left side of is the regular L^2 function f plus

$$[-\sigma_\alpha b + \xi \mathcal{K}_\alpha (a + \eta b)]\delta_o.$$

A nonzero delta is not an L^2 function, so its coefficient must vanish. This gives the real boundary line in , which is self-adjoint by Lemma . Solving for its logarithmic length proves the formula.

Conversely, for any $L > 0$, set

$$\eta = \log(L/\ell) + \frac{\sigma_\alpha}{\xi \mathcal{K}_\alpha}.$$

It yields that extension, and the value is unique. When $\xi = 0$, only the kinetic delta coefficient remains.

Proposition (Completeness within the declared trace-product class). *Suppose a candidate radial curvature product:*

1. *is supported at the apex and has no transverse delta derivatives;*

2. *depends linearly and only on the two traces a, b;*

3. *is real and agrees with $\mathcal{K}_\alpha u(0)\delta_o$ for every smooth radial u. Then it is exactly $\mathcal{R} \star_\eta$ for a real η.*

Proof. Such a product has coefficient $\mathcal{K}_\alpha(Aa + Bb)$ for constants A, B. Smooth-function agreement forces $A = 1$, and reality gives $B \in \mathbb{R}$. Put $\eta = B$.

This completeness is relative to the stated trace-local, order-zero class. More elaborate nonlocal or derivative-valued defect interactions are not classified.

Remark (Reference length is not an invariant extra parameter). If ℓ' replaces ℓ, the same function has

$$a' = a + b\log(\ell'/\ell), \qquad b' = b.$$

The same product is represented by

$$\eta' = \eta - \log(\ell'/\ell).$$

Consequently L_η in is unchanged. The physical boundary length and the arbitrary coordinate reference must not be counted twice.

Remark (The two models have not been identified). Theorem does not assert convergence to $\mathcal{R}\star_\eta$ for any finite η. It selects Friedrichs, whereas every finite-η product at $\xi > 0$ in Theorem selects a finite L. The products are distinct additional prescriptions. A curvature-density limit on smooth tests cannot be substituted for convergence of its products with singular or varying operator-domain functions.

Holonomy, observables, and the absence of backreaction

Why a charged nontrivial character needs more core data

Proposition (Flat-bundle filling obstruction). *A smooth flat $U(1)$ connection on a transverse disk has trivial meridian holonomy. Consequently a nontrivial character $e^{2\pi i\beta}$, $\beta \neq 0 \bmod 1$, cannot be extended over the smooth core as a smooth flat connection.*

Proof. A flat connection on a simply connected disk is gauge equivalent to the trivial connection, so its holonomy around a contractible loop is 1.

A smooth nonflat flux core could realize nontrivial exterior holonomy, but its connection profile would be additional data. We have not supplied it. In particular, the untwisted selection theorem is not a selection theorem for the power-channel or $U(2)$ parts of the charged I-B-G ledger.

Operator parameters are not one-point contacts

In the notation of I-B-VP-1, the previously constructed Friedrichs one-point extensions may, under that paper's full stationary, longitudinal-covariant, rotation-covariant, scaling, parity, and Ward prescription, be changed by

$$\begin{aligned} F &= \widehat{F} + c\delta_{\mathscr{W}}, \\ T^{\mu}{}_{\nu} &= \widehat{T}^{\mu}{}_{\nu} + \tau\mathscr{H}^{\mu}{}_{\nu} + n(\xi - 1/6)c\,\mathscr{I}[\delta_{\mathscr{W}}]^{\mu}{}_{\nu}, \\ J^{i} &= \widehat{J}^{i} + w\varepsilon^{i}{}_{k}\partial_{k}\delta_{\mathscr{W}}. \end{aligned}$$

This is cited as a version-qualified inherited classification, not reproved here [12].

For generic charged sectors it has three parameters. For a neutral real field there is no charged-current parameter, and the stated class has two. Those dimensions depend on that paper's complete assumptions; they are not universal apex-counterterm counts.

Proposition (Selection does not normalize the observables). *The selection theorem fixes the limiting operator boundary relation. It does not fix c, τ, w in .*

If the limiting Friedrichs state and the V1 one-point extension contract are independently adopted, the allowed changes in preserve that operator. Conversely, changing L or η changes the operator domain and cannot be represented merely by choosing those observable contacts.

Proof. Neither the forms , the unitary , nor the boundary traces involve c, τ, w. Under the independently supplied V1 contract, its classification provides the different one-point extensions at a fixed operator. Thus the operator construction supplies no equation fixing their coefficients.

The parameter L instead imposes a relation between field boundary traces in $\mathscr{D}(h_L)$. It changes spectral properties, including . A supported change of one-point observables is not such a domain change.

We have not proved convergence of smooth-core expectation values to any member of . A core action, a state-limit theorem, and a renormalization normalization might impose further conditions, but none is deduced from strong resolvent convergence alone. The distinction between field equations, preparations, and coincident observable normalization is also explicit in VII-H [16].

No Einstein equation or stress feedback

The metrics g_ε, the limiting cone, and $\mathcal{R}$ are prescribed. No expectation value of stress is used to determine them. The regular improvement identity does not turn into an equation for P_ε.

The non-sourcing distinction of I-B and the corresponding fixed-host boundaries of X-A-ST and XI-A-ST remain intact [9, 14, 15]. We do not solve a semiclassical Einstein system, or infer one from stress conservation.

The construction also does not:

• update the Maxwell response of V4;

• prove a massive-state threshold theorem for V3;

• construct a quantum field algebra or covariance at the completed apex;

• identify the renormalized product $\star_\eta$ with an interacting product of quantum fields.

The finite Enter report and its coverage boundary

A possible installed report describes:

$$\text{model identifier, } \alpha,\ \xi,\ \beta = 0, \text{?profile formula or certificate, ?}\varepsilon,\ \ell,$$
$$\text{Hilbert identification, ?}d,\ L_{\text{eff}}, \text{?boundary relation, ?}\eta\text{if used, ?scope and evidence grade.}$$

Arbitrary smooth profiles and exact real numbers are not thereby encoded by finite words without a representation convention. The analytic theorem quantifies over the whole class; a particular report contains only the supplied finite representation and certified consequences.

Let $C \cong \mathbb{C}^D$, $D = 2^q$, have calibrated orthonormal basis $\{|\tau_j\rangle\}$, and let A contain every surviving finite register. Install finite words w_j, including failure and unused labels, and put

$$D_j = \langle \tau_j|_C \otimes I_A, \qquad \mathscr{M}(X) = \sum_j |j, w_j\rangle\langle j, w_j|_R \otimes D_j X D_j^\dagger.$$

Theorem (Complete supplied-report instrument). *The map is completely positive and trace preserving, and*

$$\mathrm{Tr}_R \mathscr{M}(X) = \mathrm{Tr}_C X.$$

The statements hold after adjoining any untouched finite reference. The quantum output contains A, not C. A positive-probability branch retains its actual conditional A-state; no normalized state is assigned to a zero-probability branch.

Proof. The Kraus maps are $|j, w_j\rangle_R \otimes D_j$, and

$$\sum_j D_j^\dagger D_j = \sum_j |\tau_j\rangle\langle\tau_j|_C \otimes I_A = I_{CA}.$$

Expansion in the calibration basis identifies the output blocks with the diagonal clock blocks of X, whose sum is the partial trace. Tensoring this calculation with a reference identity proves the complete statements.

This is the supplied-report interface of I-B, I-B-G, and the ST reporting sheets, not a new measurement theorem [9, 10, 14]. The instrument does not sample unknown apex curvature, calculate an unknown Dirichlet-to-Neumann map by readout, or establish the profile inequalities.

Finite samples of profiles or residuals cannot replace the quantified lower-bound and recovery arguments. An independently specified finite-dimensional residual class and an injective evaluation map can make finite tests complete within that class; the class membership and injectivity require their own proof [17]. Nothing here claims that every finite set of measurements is incapable of determining a boundary parameter once a valid finite-dimensional model has already been supplied.

The evidence labels are kept separate:

\texttt{ANALYTIC} The displayed mathematical proofs, or an explicitly identified inherited analytical input.

\texttt{SUPPLIED} A version-qualified description or computational report not independently executed here.

\texttt{UNCERTIFIED} A numerical approximation without the artifacts and total error bounds needed for certification. No numerical core simulation, ODE integration, or physical acquisition is claimed in this manuscript.

Release, literature boundary, and standing stops

Theorem (V5 release in the declared sector). *For the model and scope :*

1. *the smooth-core geometry, curvature, volume density, and spatial realization are well defined;*

2. *every admissible shrinking family selects B_F in the strong resolvent sense after ;*

3. *zero-energy matching gives the explicit profile-dependent finite-radius length , whose boundary line converges to Friedrichs;*

4. *the ordinary positive point-evaluation curvature energy is not closable;*

5. *a fixed curvature measure and fixed* $\xi > 0$*, without a product prescription, do not select a finite logarithmic extension in the trace-product class;*

6. *operator selection neither fixes V1 contacts nor constructs a state limit or backreaction;*

7. *the finite report preserves the complete unmeasured residual but acquires no continuum field.*

Proof. Proposition and give (i). Theorem gives (ii). Theorems and Proposition give (iii). Theorem gives (iv). Theorem and Proposition give (v). Section gives the exact boundaries in (vi). Theorem and its declared interfaces give (vii).

What is and is not a novelty claim

The rounded-cone metric family, logarithmic fitting formula, and leading weak-coupling exponential are established cosmic-string tools [2, 1]. The rigorous contribution claimed here is the specified profile-uniform selection statement, its independent matching analysis, and its explicit separation from a classified distributional multiplication choice.

The results may be useful because they prevent a nonzero curvature-density limit, a successful finite-core fit, or a stress improvement formula from being mistaken for a unique zero-thickness operator law. No exhaustive priority theorem is asserted.

More recent work on massive scalar observables with prescribed cone boundary conditions addresses a different, lower-dimensional state problem [6]. It is not imported as a curvature-selection or core-limit theorem.

Remaining scope

begingroup 1.12 toprule Question Result here Boundary

midrule endfirsthead toprule Question Result here Boundary

midrule endhead bottomrule endfoot

Untwisted monotone nonnegative curvature cores, $\xi \geq 0$ Selection proved Friedrichs, full spatial strong resolvent convergence.

Fixed-radius zero-energy fitting Computed and bounded Profile dependent; not the exact smooth operator.

Weak curvature density Computed $\mathscr{K}_{\alpha}\delta_{\mathscr{W}}$; does not determine its singular-field product.

Ordinary positive delta energy Obstructed The displayed point-evaluation form is not closable.

Renormalized radial trace product Defined and classified Each η selects L_{η}; the bare measure does not.

Nontrivial holonomy and power channels Not selected here A smooth flux-core connection and further analysis are required.

Negative ξ, sign-changing cores, tuned attractive limits Not analyzed The positivity proof does not cover them.

V1 contacts and core observable limits Not normalized Operator convergence is insufficient.

Self-consistent geometry, state, or Maxwell response Not constructed The entire geometric family is prescribed.

endgroup

The V5 omission is therefore replaced by theorems in the declared neutral/untwisted core class and the stated distributional product class. It is not replaced by a universal assertion about every possible apex model.

No statement from an unretrieved VII-D, Em01-A, Em01-D, XVI-N, or XV-B construction is used as an analytical premise. Shared words such as ``smoothing," ``finite part," or ``origin germ" do not identify domains or transfer a theorem.

O.1 remains outside any independently established, version-specific scope. XVI-N is not discharged, XV-B is not opened as a downstream quantum-admission exit, and no OpenAI catalogue claim is a premise. These are unchanged programme boundaries, not consequences of the smooth-core limit.

appendix

Source-use and access ledger

This ledger concerns the sources actually used here. It is not an audit of the whole companion corpus. ``Inspected'' refers only to the stated passages or primary formulas, not to verification of an entire paper.

begingroup 1.12 toprule Source Access and use Explicit non-import

midrule endfirsthead toprule Source Access and use Explicit non-import

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Cheung, I-B and philosophical companion [9, 8] Online host, reporting, non-sourcing, and interpretive passages consulted. No emergence, core geometry, or extension-selection theorem borrowed.

Cheung, I-B-G and I-B-VP-v4 [10, 11] Supplied version-qualified excerpts and source descriptions. No full-source revalidation or unprovided theorem asserted.

Cheung, I-B-VP-1 [12] Supplied completion and contact-classification passages. No scalar-contact parameter identified with L or η.

Kay–Studer [1] Primary publisher record and historical extension context. No uninspected numbered equation is used as a proof step.

Allen–Kay–Ottewill [2] Primary PDF inspected: (2.1)–(2.2), (3.8), (3.15), (3.20)–(3.22), and the associated bound-state discussion. No finite-core expectation value, principal-part covariance, or state-limit result imported.

Geroch–Traschen [3] Primary publisher abstract and its singular-metric framework. No claim that it supplies our scalar product or that it forbids every generalized curvature construction.

Clarke–Vickers–Wilson [4] Primary paper on generalized cone curvature. No generalized-function algebra imported to define $\star_\eta$.

Mosco [5] Primary convergence framework cited for terminology. The necessary lower bound, recovery, and resolvent argument are proved directly.

Other named companion methods Only supplied, identified passages used where cited. No theorem inferred from an absent source or from a catalogue relationship.

endgroup

The targeted consultation is not a novelty search proving absence of comparable core-limit results elsewhere.

Debugging and acceptance checklist

begingroup 1.12 toprule Potential error Correction

midrule endfirsthead toprule Potential error Correction

midrule endhead bottomrule endfoot

Treating ξ-dependent regular stress as an operator boundary law Keep the inherited improvement identity separate from B.

Using an impossible nonnegative cap in the same geodesic radius Proposition explains the obstruction; the core uses a radial lapse.

Taking resolvents in different Hilbert spaces without identification Use $U_{\varepsilon} = p_{\varepsilon}^{1/2}$.

Claiming profile independence from one example Theorem quantifies over every admissible profile family.

Confusing curvature with curvature density The weak limit uses $R_{\varepsilon}\mathrm{dvol}_{g_{\varepsilon}}$.

Replacing a curvature-density limit by a product limit Theorem and Definition keep the missing operation explicit.

Calling $L\mathrm{downarrow}\,0$ an ordinary finite logarithmic parameter Convergence is to the boundary line $b = 0$.

Equating a zero-energy fit with the exact core operator Their spectra and energy-dependent matching differ.

Interpreting an exterior rank-one estimate as full norm resolvent convergence Proposition is explicitly compressed and zero-mode only.

Claiming a massless covariance limit from strong resolvent convergence The function $B^{-1/2}$ is unbounded at the

spectral threshold; a separate theorem is required.

Extending nontrivial holonomy over a flat disk Proposition ; a flux-core connection is additional data.

Identifying L, η with c, τ, w They act on different objects: operator domains and observable extensions.

Calling the product family a canonical distributional curvature coupling Its trace dependence and renormalization parameter are declared, not geometrically forced.

Deriving geometry from an Enter report The report has no geometric update or continuum acquisition port.

endgroup

Declarations

Authorship. The author field is left blank for submission preparation. Attribution of the companion records to N. Cheung does not assign authorship of this draft.

Analytical dependence. The main selection, matching, nonclosability, and product theorems have the displayed proofs. The regular-observable and contact results are inherited, version-qualified inputs and are not presented as newly proved.

Physical scope. No realistic string core is identified. No microscopic detector model, stress-sourced geometry, self-consistent quantum state, or interacting field product is supplied.

Execution and verification. The manuscript is a single pdfLaTeX source with an internal bibliography. No successful compilation, numerical execution, proof-assistant verification, independent rebuild, independent review, or journal acceptance is asserted.

References

[1] B. S. Kay and U. M. Studer, Boundary conditions for quantum mechanics on cones and fields around cosmic strings, *Communications in Mathematical Physics* **139** (1991), 103–139. DOI: 10.1007/BF02102731. Primary record: https://link.springer.com/article/10.1007/BF02102731. Historical extension and scattering-length context;

the restricted boundary calculation needed here is given explicitly.

[2] B. Allen, B. S. Kay, and A. C. Ottewill, Long-range effects of cosmic string structure, *Physical Review D* **53** (1996), 6829–6841. DOI: 10.1103/PhysRevD.53.6829. Inspected preprint: https://arxiv.org/pdf/gr-qc/9510058v1. Especially the rounded-cone metrics, zero-energy fitting formula, weak-potential exponent, and the distinction between a nonnegative core and the bound state of a fitted logarithmic extension.

[3] R. Geroch and J. Traschen, Strings and other distributional sources in general relativity, *Physical Review D* **36** (1987), 1017–1031. DOI: 10.1103/PhysRevD.36.1017. Primary record: https://journals.aps.org/prd/abstract/10.1103/PhysRevD.36.1017. Singular-metric framework and its limitations; not a multiplication prescription for logarithmic scalar fields.

[4] C. J. S. Clarke, J. A. Vickers, and J. P. Wilson, Generalised functions and distributional curvature of cosmic strings, *Classical and Quantum Gravity* **13** (1996), 2485–2498. DOI: 10.1088/0264-9381/13/9/013. https://arxiv.org/abs/gr-qc/9605060. Generalized-function curvature context only. Its algebra is not silently imported into the present trace product.

[5] U. Mosco, Composite media and asymptotic Dirichlet forms, *Journal of Functional Analysis* **123** (1994), 368–421. DOI: 10.1006/jfan.1994.1093. Convergence framework; the particular variational and resolvent arguments used in this manuscript are proved in Section .

[6] J. P. M. Pitelli, R. A. Mosna, V. H. M. Ramos, and J. C. A. Barata, *Vacuum fluctuations and the renormalized stress-energy tensor on a cone with arbitrary boundary conditions*. arXiv:2509.20231. https://arxiv.org/abs/2509.20231. Context for prescribed cone boundary conditions and massive scalar observables, not a theorem selecting a core limit in the present four-dimensional product.

Cheung articles and companion records. The unpublished records below are cited by identifier and supplied version. Their appearance does not certify an entire corpus, a successful build, or a formal proof.

[7] N. Cheung, ``The Enter Calculus: A Linear Quantum Lambda Calculus with an Operational Characterization

of Measurement," *Frontiers in Physics* **14** (2026), article 1954943. DOI: 10.3389/fphy.2026.1954943. https://doi.org/10.3389/fphy.2026.1954943. Interface/operational background only; not a PDE premise.

[8] N. Cheung, *The Philosophy of Enter: Destructive Readout, Quantum Recovery, and the Refusal of Identification*, Cheung Ngo Medical Limited (2026). Framing essay; not a theorem premise. Programme note: https://enter.grok.me/enter.

[9] N. Cheung, *I-B-v1: A Finite Enter Spacetime: Calibrated Destructive Time Readout on a Prescribed Conical Product*, version 1, Enter Calculus Companion. Used: prescribed geometry, def:experiment, thm:instrument, and prop:not-source. https://enter.grok.me/browse/I-B. Companion record; not a Scholar-indexed journal article.

[10] N. Cheung, *I-B-G: Intrinsic and holonomy-twisted Green operators on a prescribed conical spacetime: Self-adjoint extensions, diffraction, finite propagation, and supported transfer from flat-completion laboratories*, version 2, October 2026. Supplied version-qualified record I-B-G-v2.tex, Enter Calculus Companion. Used for the extension ledger, its non-selection boundary, and finite-report coverage discipline. No unprovided full-source verification is asserted. https://enter.grok.me/browse/I-B-G. Companion record; not a Scholar-indexed journal article.

[11] N. Cheung, *I-B-VP: Vacuum polarization of the Friedrichs ground state on a prescribed conical spacetime: Local Hadamard subtraction, holonomy, and the renormalized stress tensor*, version 3. Supplied excerpts from I-B-VP-v4.tex, Enter Calculus Companion. Used: ass:host, the fixed-operator improvement identity, and the V5 question. The regular scalar observables are not recomputed here. https://enter.grok.me/browse/I-B-VP. Companion record; not a Scholar-indexed journal article.

[12] N. Cheung, *I-B-VP-1: Prescribed apex finite parts on a conical Enter spacetime: Wick square, stress tensor, and charged current: strong extension, Ward identities, and classified contact freedom*, version 1, October 2026. Supplied record I-B-VP-1-v1.tex, Enter Calculus Companion. Used: ass:completion, ass:class, thm:classification, and the non-selection of a physical normalization. The observable contacts are not operator boundary parameters. https://enter.grok.me/browse/I-B-VP-1. Companion record; not a Scholar-indexed journal article.

[13] N. Cheung, *X-L-ST: Restricted classical BRST domains, exact free recovery, and complete residual reports*, version 1, Enter Calculus Companion. Supplied host and width-versus-defect passages. Its fixed analytical ramp width is not the core radius of the present model. https://enter.grok.me/browse/X-L-ST. Companion record; not a Scholar-indexed journal article.

[14] N. Cheung, *X-A-ST*, version 1, Enter Calculus Companion. Supplied prescribed-host, non-sourcing, complete-residual, and finite-report passages only. https://enter.grok.me/browse/X-A-ST. Companion record; not a Scholar-indexed journal article.

[15] N. Cheung, *XI-A-ST*, version 1, Enter Calculus Companion. Supplied host, width, conservation-versus-dynamics, and reporting passages. No gravitational operator or Einstein jet is imported into the curvature-selection proof. https://enter.grok.me/browse/XI-A-ST. Companion record; not a Scholar-indexed journal article.

[16] N. Cheung, *VII-H*, version 1, VII-H.tex, Enter Calculus Companion. Supplied physical-matching and normalization ledger. Used only to distinguish a free equation and preparation from a coincident observable normalization. https://enter.grok.me/browse/VII-H. Companion record; not a Scholar-indexed journal article.

[17] N. Cheung, *Oai282-A*, version 1, Enter Calculus Companion. Supplied prop:coverage and prop:invisible. Finite evaluations require a declared residual-space coverage theorem; no catalogue theorem is imported. https://enter.grok.me/browse/Oai282-A. Companion record; not a Scholar-indexed journal article.

[18] N. Cheung, *Black-Hole Information as an Access Problem: Enter, Quantum Recovery, and Complete Residuals*, figshare (2026). DOI: 10.6084/m9.figshare.34335024.v1. https://doi.org/10.6084/m9.figshare.34335024.v1.

[19] N. Cheung, *Operational Wormhole Certificates—Existence, Dimension-Dependent Scaling, Operational Functions, and Composition*, figshare (2026). DOI: 10.6084/m9.figshare.34549575.v1. https://doi.org/10.6084/m9.figshare.34549575.v1.

[20] N. Cheung, *Enter, Quantum Magic, and Quantum Gravity: Readout, Recovery, and the Limits of Geometric Response*, figshare (2026). DOI: 10.6084/m9.figshare.34335582.v1. https://doi.org/10.6084/m9.figshare.34335582.v1.

[21] N. Cheung, *Operational Consistency Does Not Select Lorentzian Geometry*, figshare (2026). DOI: 10.6084/m9.figshare.34441758.v1. https://doi.org/10.6084/m9.figshare.34441758.v1.

[22] N. Cheung, *Realization of Relational Time by Destructive Enter on a Clock Register*, figshare (2026). DOI: 10.6084/m9.figshare.34442907.v1. https://doi.org/10.6084/m9.figshare.34442907.v1.