Enter, I-B-VP-6-v2, 2026, pp. 1-39

I-B-VP-6 Longitudinally non-product self-adjoint extensions on the prescribed conical spacetime

Measurable boundary relations, exact spectral criteria, boost rigidity, and non-translation-invariant examples

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Version 1

Abstract

We develop the longitudinally non-product extension row of the I-B-VP ledger without changing its prescribed conical spacetime. If no transverse angular channel is limit circle, the full spatial minimal operator is essentially self-adjoint. Otherwise, all extensions strongly commuting with longitudinal translations are parametrized by measurable fields of finite-dimensional self-adjoint boundary relations. Product extensions are exactly the almost everywhere constant fields. We give an explicitly normalized cone Weyl function and prove the variable-boundary direct-integral resolvent formula, including the operator domain.

The essential spectrum is the union of the nonnegative continuum and the essential ranges of the momentum-dependent bound-channel energies. Every spectral point is essential, but point spectrum need not be absent: a tuned logarithmic relation produces an infinite-multiplicity eigenvalue at any prescribed negative energy, separated from the nonnegative continuum by a gap. Conversely, longitudinal momentum can compensate negative transverse eigenvalues, yielding nonnegative, non-Friedrichs untwisted extensions. These examples show precisely why product-class positivity and no-point-spectrum conclusions cannot be transferred.

We also prove an exact rigidity statement: within the declared time-independent spatial boundary class, covariance of all positive-transverse-energy boundary traces under longitudinal boosts holds if and only if the relation field is essentially constant. Beyond translation invariance, a rank-one change of the normalized deficiency-space unitary

gives a self-adjoint, longitudinally nonlocal extension with a rank-one resolvent difference from Friedrichs and unchanged essential spectrum.

The potentially useful contribution is this combination of spectral counterexamples, boost rigidity, and a domain-complete cone implementation. Symmetry reduction and boundary-extension theory themselves are established subjects. No physical preference among extensions, spectral ground state, covariance wavefront theorem, one-point observable, backreaction, or continuum acquisition by an Enter report is constructed.

Bibliographic record

Title	I-B-VP-6 Longitudinally non-product self-adjoint extensions on the prescribed conical spacetime
Author	Cheung, Ngo
Publication date	2026
Institution	Enter
Report number	I-B-VP-6-v2
Publisher	Cheung Ngo Medical Limited
First page	1
Last page	39
Language	en

Book edition note (t425u)

Standing Enter refusals, scope vocabulary (proved here/supplied input/conditional/ not constructed/not claimed), and Scholar bibliography policy are frozen in the companion chapter *I-B-VP Book Ledger* v1. O.1 remains NOT_CLOSED; XVI-N is not discharged; XV-B is not opened; no Host C, Page/island, or spacetime-emergence claim; wired=false; SOURCE_DATE_EPOCH=1792627200. This sheet is a companion manuscript record and is *not* claimed to be Google Scholar-indexed. The published Enter Calculus article is cited only for finite-instrument

/ operational interface discussions, never as a PDE premise.

Book expansion: product vs measurable boundary field

Start from the distinction between a product extension and a measurable boundary-relation field. State the direct-integral operator domain *before* the spectrum theorem. Boost-rigidity is tied to its on-shell trace-covariance definition—not a generic theorem about all boost-covariant boundary dynamics.

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Question, result, and inherited boundaries

The product assumption is a genuine restriction

The host is the prescribed regular product of I-B:

$$\begin{aligned} M &= \mathbb{R}_t \times (0,\infty)_r \times (\mathbb{R}/2\pi\mathbb{Z})_\varphi \times \mathbb{R}_z, \\ g &= -\mathrm{d}t^2 + \mathrm{d}r^2 + \alpha^2 r^2 \mathrm{d}\varphi^2 + \mathrm{d}z^2, \qquad 0 < \alpha < 1. \end{aligned}$$

The completion adds

$$\mathscr{W} = \mathbb{R}_t \times \{o\} \times \mathbb{R}_z.$$

The deficit $2\pi(1-\alpha)$, worldsheet, time orientation, and geometry are inputs, not conclusions [6].

Fix a flat Hermitian line-bundle character

$$u(r,\varphi+2\pi,z) = e^{2\pi i\beta} u(r,\varphi,z), \qquad 0 \le \beta < 1.$$

The spatial Hilbert space is

$$\mathscr{H}_\beta = L^2_\beta(N,\mathrm{d}V), \qquad N = (0,\infty)\times S^1 \times \mathbb{R}, \qquad \mathrm{d}V = \alpha r\,\mathrm{d}r\,\mathrm{d}\varphi\,\mathrm{d}z.$$

The lifted flat connection is ordinary differentiation.

I-B-VP's product class is

$$B = T_{\Lambda} \otimes I + I \otimes (-\partial_z^2),$$

where T_{Λ} is any self-adjoint transverse cone extension, including non-diagonal relations between deficient angular modes. Its transverse relation does not depend on longitudinal momentum [8]. I-B-G's rotation-diagonal product class is narrower [7].

This manuscript starts outside . It does not repeat the finite-dimensional non-diagonal transverse ledger or its product-class apex analysis [9].

Three corrections to the proposed wording

The following distinctions are needed for a correct proof.

1. A boundary relation and a Weyl function are not interchangeable pieces of arbitrary data. Once the minimal transverse operator, boundary traces, and Friedrichs reference are fixed, its Weyl function is fixed. The extension is selected by the relation; the resolvent combines the two.

2. A longitudinal boost mixes *t* and *z*. There is no canonical boost acting on the spatial Hilbert space alone with which one may simply ask *B* to commute. We state the boost question as covariance of spacetime on-shell boundary traces.

3. Raw apex coefficients need not belong to unweighted $L^2(\mathbb{R}_z)$ on the full adjoint domain. The general non-translation-invariant construction below uses normalized deficiency profiles, with their actual Hilbert topology. It does not postulate an unproved ordinary boundary triple for raw coefficients.

Analytical contribution and relation to the literature

Reduction of symmetry-preserving self-adjoint extensions to measurable families is established, including in the three-dimensional Aharonov–Bohm problem [1]. Explicit inverse-square radial realizations and their spectral analysis are also established [2]. Boundary relations and Weyl-function resolvent formulas are standard extension-theoretic tools [3].

Here these tools are implemented with a fixed cone, fixed holonomy, explicit trace normalization, and a full direct-integral domain. The principal consequences are:

• an exact spectrum and point-spectrum criterion for the translation-invariant non-product ledger;

• concrete flat-band and momentum-compensation counterexamples to product-only conclusions;

• boost rigidity in a precisely stated covariance class;

• an explicit finite-rank boundary perturbation outside translation invariance. No exhaustive priority claim is made for the general methods.

Enter and the philosophy of the interface

Enter is a destructive finite-register readout operation: measured quantum names are consumed and a classical record is returned. Its operational characterization is relative to an explicit class of strategies [4]. The philosophical companion emphasizes that reporting, quantum recovery, and physical identification are different interfaces [5].

In this manuscript, a finite Enter program may report an installed description of a boundary law and the scope of a proved statement. It neither selects that law nor acquires an unknown continuum extension. The exact finite channel is recorded in Section .

All spatial extension theorems below are independent of this reporting interface. Neither the interface nor the philosophical essay is an analytical premise of them.

Minimal operators and the product boundary coordinates

We use complex Hilbert spaces and inner products linear in the first argument. A real-field restriction, if wanted, requires the corresponding conjugation invariance in addition to self-adjointness.

Let

$$\mathfrak{h}_{\beta} = L^2_{\beta}(C^{\circ}_{\alpha}, \alpha r \, \mathrm{d}r \, \mathrm{d}\varphi).$$

Write

$$\psi_n(\varphi) = \frac{e^{\mathrm{i}(n+\beta)\varphi}}{\sqrt{2\pi\alpha}}, \qquad \nu_n = \frac{|n+\beta|}{\alpha}.$$

Angular decomposition identifies the transverse expression with the orthogonal sum of

$$\ell_\nu = -\partial_r^2 - \frac{1}{r}\partial_r + \frac{\nu^2}{r^2} \quad \text{in} L^2((0,\infty), r\,\mathrm{d}r).$$

Let $T_{\min}$ be the closure of the transverse operator on compactly supported smooth sections avoiding the apex, and let $T_{\max} = T_{\min}^*$. The transverse Friedrichs extension is T_F.

Set

$$J_\beta = \{n \in \mathbb{Z} : |n+\beta| < \alpha\}, \qquad d = d_\beta = |J_\beta|.$$

For the present range $0 < \alpha < 1$, $d \in \{0, 1, 2\}$. The strict inequality is important: $\nu = 1$ is not deficient.

The limit-circle count is the I-B-G deficiency result [7]. We give the trace conventions needed for the new calculation.

Boundary traces with a fixed reference length

Fix $\ell > 0$. For every deficient mode with $0 < \nu < 1$, define a, b by

$$u(r) = a(r/\ell)^{-\nu} + b(r/\ell)^{\nu} + o(r^{\nu}).$$

For $\nu = 0$, define them by

$$u(r) = a\log(r/\ell) + b + o(1).$$

Equivalently, the traces are limiting Wronskians with the two zero-energy solutions.

Define, in the finite boundary space $\mathbb{C}^d$,

$$\Gamma_0 u = (a_j)_{j \in J_\beta}, \qquad (\Gamma_1 u)_j = \begin{cases} 2\nu_j b_j, & 0 < \nu_j < 1, \\ -b_j, & \nu_j = 0. \end{cases}$$

Lemma (Finite transverse boundary system). *The maps in are graph-continuous on* $\operatorname{dom} T_{\max}$*, are jointly onto*

$\mathbb{C}^d \oplus \mathbb{C}^d$, *and satisfy*

$$\langle T_{\max} u, v\rangle - \langle u, T_{\max} v\rangle = \langle \Gamma_1 u, \Gamma_0 v\rangle - \langle \Gamma_0 u, \Gamma_1 v\rangle.$$

Their common kernel is $\operatorname{dom} T_{\min}$, *and*

$$T_F = T_{\max}|_{\ker \Gamma_0}.$$

Self-adjoint transverse extensions are parametrized by the self-adjoint linear relations

$$\Lambda \subset \mathbb{C}^d \oplus \mathbb{C}^d, \qquad \operatorname{dom} T_\Lambda = \{u \in \operatorname{dom} T_{\max} : (\Gamma_0 u, \Gamma_1 u) \in \Lambda\}.$$

Proof. The two zero-energy solutions are $r^{-\nu}, r^{\nu}$ for $\nu > 0$, and $\log r, 1$ for $\nu = 0$. Both are square integrable at zero exactly when $\nu < 1$. At infinity the radial expression is limit point.

Variation of constants for $\ell_\nu u = f$, with $f \in L^2(r\,\mathrm{d}r)$, gives the coefficients in –. For example, the power-channel remainder is controlled by integrals of $s^{1\pm\nu} f(s)$; Cauchy–Schwarz bounds the resulting remainder by $o(r)$, hence by $o(r^\nu)$. The logarithmic calculation uses $(ru')' = -rf$. Interior estimates on one fixed annulus and these integral identities bound the traces by the maximal graph norm.

For one power channel the inner endpoint term in integration by parts is

$$2\nu(b_u \overline{a_\nu} - a_u \overline{b_\nu}).$$

For the logarithmic channel it is $a_u \overline{b_\nu} - b_u \overline{a_\nu}$. These are exactly . The other angular modes contribute no endpoint term.

Cut off the two zero-energy solutions near zero. Their arbitrary finite coefficients prove joint surjectivity. Zero traces characterize the minimal domain. The finite-energy condition excludes the singular coefficient a, identifying Friedrichs.

Finally, the adjoint of a restriction specified by a boundary subspace is specified by its symplectic orthogonal for . Equality with that orthogonal is exactly self-adjointness of the boundary relation.

Remark (Coordinates, not an identification with another chart). In a scalar power channel, a finite condition $\Gamma_1 = h\Gamma_0$ is the condition $b = \lambda a$ with $\lambda = h/(2\nu)$, in the reference-length normalization . In the logarithmic channel it gives

$$u(r) = a\log(r/L) + o(1), \qquad L = \ell e^{h}.$$

The reference length and these normalizations are fixed throughout. No finite matrix from another sheet is identified with h without this change of coordinates.

Relations, including Friedrichs directions

Every self-adjoint relation can be written as

$$x \in W, \qquad P_W y = H_W x,$$

where $W \subset \mathbb{C}^d$ is a subspace and $H_W = H_W^*$ on W. The component of y in $W^\perp$ is unrestricted. The case $W = 0$ is Friedrichs.

Equivalently, a unique unitary matrix $\mathrm{U} \in U(d)$ specifies the relation by

$$y - \mathrm{i}x = \mathrm{U}(y + \mathrm{i}x).$$

Friedrichs is $\mathrm{U} = I$. For a finite Hermitian chart $y = Hx$, $\mathrm{U} = (H - \mathrm{i}I)(H + \mathrm{i}I)^{-1}$.

These relations are finite-dimensional transverse data. They are not yet the full longitudinal boundary space.

The normalized cone Weyl function

For $w \in \mathbb{C} \setminus [0, \infty)$, put

$$q = \sqrt{-w}, \qquad \operatorname{Re} q > 0.$$

For a deficient mode, the square-integrable solution at infinity is $K_{\nu_j}(qr)\psi_j(\varphi)$. Its Γ_0 coefficient is

$$A_j(q) = \begin{cases} 2^{\nu_j - 1}\Gamma(\nu_j)(q\ell)^{-\nu_j}, & 0 < \nu_j < 1, \\ -1, & \nu_j = 0. \end{cases}$$

Define

$$\gamma(w)c = \sum_{j \in J_\beta} c_j A_j(q)^{-1} K_{\nu_j}(qr)\psi_j(\varphi).$$

Thus $\Gamma_0\gamma(w) = I$.

Proposition (Explicit Weyl function and transverse resolvent). *The Weyl matrix $M(w) = \Gamma_1\gamma(w)$ is diagonal:*

$$M_j(w) = \begin{cases} -c_{\nu_j}(q\ell)^{2\nu_j}, & 0 < \nu_j < 1, \\ \log\dfrac{2}{q\ell} - \gamma_E, & \nu_j = 0, \end{cases} \qquad c_\nu = 2^{1-2\nu}\frac{\Gamma(1-\nu)}{\Gamma(\nu)} > 0.$$

For $\operatorname{Im} w \neq 0$,

$$\gamma(w)^*\gamma(w) = \frac{\operatorname{Im} M(w)}{\operatorname{Im} w} > 0.$$

For the relation , let $V : \mathbb{C}^{\dim W} \to W \subset \mathbb{C}^d$ be an isometry and set

$$C_\Lambda(w) = V[H_W - V^* M(w) V]^{-1} V^*,$$

with $C_\Lambda = 0$ for $W = 0$. Then

$$(T_\Lambda - w)^{-1} - (T_F - w)^{-1} = \gamma(w) C_\Lambda(w) \gamma(\overline{w})^*.$$

In particular, its rank is at most d.

Proof. For $0 < \nu < 1$,

$$K_\nu(qr) = 2^{\nu-1}\Gamma(\nu)(qr)^{-\nu} + 2^{-\nu-1}\Gamma(-\nu)(qr)^{\nu} + o(r^{\nu}).$$

Using $\Gamma(-\nu) = -\Gamma(1-\nu)/\nu$ and gives the power entry of . For $\nu = 0$,

$$K_0(qr) = -\log(r/\ell) + \log(2/(q\ell)) - \gamma_E + o(1),$$

which gives the logarithmic entry.

Apply to two defect solutions. It yields the usual polarized Weyl identity and, on taking equal spectral parameters,

. Strict positivity follows because $\gamma(w)$ is injective.

For a source *f*, write

$$u = (T_F - w)^{-1} f + \gamma(w) a.$$

The first term has $\Gamma_0 = 0$. Green's identity gives

$$\Gamma_1 (T_F - w)^{-1} f = \gamma(\overline{w})^{*} f.$$

The boundary condition is therefore

$$a \in W, \qquad H_W a = P_W \{ \gamma(\overline{w})^{*} f + M(w) a \}.$$

Solving it gives and . The matrix is invertible at nonreal *w* by .

The calculation is a cone specialization of the boundary-triple resolvent mechanism [3]. It is written out because its normalization and the later *k*-dependence must not be inferred from a constant product formula.

Lemma (Transverse spectrum needed below). *For every self-adjoint transverse relation,*

$$\operatorname{spec}_{\mathrm{ess}} (T_\Lambda) = [0, \infty).$$

There are no nonnegative transverse eigenvalues. Below zero there are at most d eigenvalues, counted with multiplicity. Their scales $q > 0$ *satisfy*

$$\det [H_W - V^{*} M(-q^2) V] = 0.$$

Proof. The Friedrichs operator is nonnegative. Radial cutoff Weyl sequences escaping to infinity show that every nonnegative energy belongs to its essential spectrum. The finite-rank resolvent difference preserves essential spectrum: equivalently, the bounded Fredholm tests obtained from two resolvents differ by a finite-rank operator.

At positive energy the Bessel solutions in each angular mode oscillate at infinity and are not in $L^2(r\,\mathrm{d}r)$. At zero the radial powers, or the logarithmic pair, cannot be square integrable at both ends. Angular orthogonality excludes cancellation of these norm divergences.

Every negative spectral point is consequently a discrete eigenvalue. If there were $d + 1$ independent negative

eigenvectors, their span would contain a nonzero vector with $\Gamma_0 = 0$. That vector would belong to dom T_F and have negative expectation value, contrary to $T_F \geq 0$. Thus there are at most d. The decaying negative-energy solutions are exactly the linear combinations in , and their boundary relation is .

For later examples, the scalar formulas are

$$\begin{aligned} ll0 < \nu < 1: \quad & h = -c_\nu (q\ell)^{2\nu} \quad \Leftrightarrow \quad -q^2 \text{is a bound eigenvalue,} \\ \nu = 0: \quad & L = 2e^{-\gamma_E}/q \quad \Leftrightarrow \quad -q^2 \text{is a bound eigenvalue.} \end{aligned}$$

These familiar Bessel facts are consistent with the radial analysis in [2].

The full minimal operator and the empty case

Let S be the closure in $\mathscr{H}_\beta$ of

$$-\Delta_q = -\partial_r^2 - \frac{1}{r}\partial_r - \frac{1}{\alpha^2 r^2}\partial_\varphi^2 - \partial_z^2$$

on $C_c^\infty(N; L_\beta)$. Use the unitary longitudinal Fourier convention

$$(\mathscr{F}_z u)(k) = (2\pi)^{-1/2} \int_{\mathbb{R}} e^{-ikz} u(z)\mathrm{d}z.$$

Lemma (Minimal direct integral). *Under $\mathscr{F}_z$,*

$$S = \int_{\mathbb{R}}^{\oplus} (T_{\min} + k^2)\mathrm{d}k, \qquad S^* = \int_{\mathbb{R}}^{\oplus} (T_{\max} + k^2)\mathrm{d}k.$$

Proof. The expression agrees with the indicated fibre expression on the initial compactly supported smooth domain. For the first equality, truncate momentum to $|k| \leq R$. On such a set, the graph norm of $T_{\min} + k^2$ is uniformly equivalent to that of $T_{\min}$.

A countable transverse graph core and simple-function approximation therefore approximate every truncated graph section by finite tensor sums. Approximate their momentum coefficients by compactly supported smooth functions. Their inverse Fourier transforms are Schwartz in z. Cutting those Schwartz functions off in z, and using transverse core vectors, gives initial-domain approximants in the full graph norm.

Momentum truncation converges in the direct-integral graph norm. This proves the first equality. The adjoint identity follows by testing against the same tensor core, or by the adjoint theorem for measurable closed operator fields.

This is the standard symmetry-reduction mechanism, with the core identification checked here [1, Theorem V.1 and Proposition V.3].

Theorem (Emptiness and infinite deficiency). *If $d_\beta = 0$, S is essentially self-adjoint on its initial smooth domain, and its unique extension is B_F. There is no non-product extension ledger.*

If $d_\beta > 0$, the deficiency indices of S are (∞, ∞). More precisely, each deficiency space is unitarily equivalent to

$$\mathscr{G} = L^2(\mathbb{R}_k; \mathbb{C}^d).$$

Proof. If $d = 0$, the transverse minimal closure is self-adjoint. The direct integral in is then self-adjoint and is the full closure.

For $d > 0$, the fibre kernel of $T_{\max} + k^2 \mp \mathrm{i}$ has dimension d. It has the orthogonal basis

$$K_{\nu_j}(\sqrt{k^2 \mp \mathrm{i}}\, r)\psi_j(\varphi), \qquad j \in J_\beta.$$

Each vector has finite nonzero norm, and its normalization depends continuously on k. The direct integral of these normalized bases identifies each full deficiency space with , which is infinite-dimensional.

Theorem proves, rather than enlarges, the corresponding I-B-G deficiency statement in the full spatial setting [7].

All translation-invariant extensions

Let

$$(\mathscr{T}_s u)(z) = u(z - s), \qquad s \in \mathbb{R}.$$

A self-adjoint operator strongly commutes with these translations when its resolvent commutes with every $\mathscr{T}_s$. This is equivalent to translation invariance of its domain and commutation on that domain.

Definition (Measurable relation field). A field $k \mapsto \Lambda(k)$ of self-adjoint relations in $\mathbb{C}^d$ is measurable when the entries of its Cayley matrix U(k) in are measurable. Fields are identified up to Lebesgue-null sets.

For such a field define

$$\widehat{B}_\Lambda = \int_{\mathbb{R}}^{\oplus} A_\Lambda(k)\mathrm{d}k, \qquad A_\Lambda(k) = T_{\Lambda(k)} + k^2.$$

Its domain is, explicitly,

$$\operatorname{dom}\widehat{B}_\Lambda = \{u \in L^2(\mathbb{R};\mathfrak{h}_\beta) : \quad u(k) \in \operatorname{dom} T_{\Lambda(k)} \text{a.e.}, \\ \int_{\mathbb{R}} \left\| (T_{\Lambda(k)} + k^2)u(k) \right\|_{\mathfrak{h}_\beta}^2 \mathrm{d}k < \infty\}.$$

In physical coordinates, $B_\Lambda = \mathscr{F}_z^{-1}\widehat{B}_\Lambda\mathscr{F}_z$.

Theorem (Translation-invariant extension ledger). *Assume $d > 0$.*

1. *Every measurable self-adjoint relation field defines a self-adjoint extension B_Λ of S by –. No boundedness or common lower bound on the fibre relations is required.*

2. *Every self-adjoint extension of S strongly commuting with all longitudinal translations is of this form.*

3. *Two fields define the same operator exactly when their relations agree almost everywhere.*

4. *The operator is of the product form if and only if $\Lambda(k)$ is almost everywhere constant.*

Proof. Fix $\zeta \notin \mathbb{R}$ and put $w = \zeta - k^2$. The fibre resolvent is given by , with $\Lambda = \Lambda(k)$.

Its measurability does not require a measurable choice of the subspace frame V in . Indeed, writing $M = M(w)$, define

$$D(k, w) = (I - \mathrm{U}(k))M - \mathrm{i}(I + \mathrm{U}(k)).$$

The relation equation for a correction coefficient a and source trace t is

$$D(k, w)a = -(I - \mathrm{U}(k))t.$$

The matrix $D(k, w)$ is invertible: a vector in its kernel would give a defect solution satisfying a self-adjoint

boundary relation at nonreal energy. Equivalently, the strict imaginary-part identity excludes it. Thus

$$C_{\Lambda(k)}(w) = -D(k,w)^{-1}(I - \mathrm{U}(k))$$

is measurable.

Each fibre resolvent has norm at most $|\mathrm{Im}\,\zeta|^{-1}$. Its direct integral is therefore a bounded resolvent on the whole Hilbert space. The fibre resolvent identity, adjoint identity, and dense range give the self-adjoint operator with domain . It extends the initial smooth operator, hence S.

Conversely, if an extension strongly commutes with translations, its resolvent commutes, after Fourier transformation, with all multipliers $e^{-\mathrm{i}ks}$. These generate the full scalar multiplication algebra. A bounded operator in its commutant is decomposable. The resolvent and adjoint identities give a measurable self-adjoint fibre operator field. The extension inclusion, tested on the countable tensor core of Lemma , implies that its fibres extend $T_{\min} + k^2$ almost everywhere. Lemma supplies the relations $\Lambda(k)$.

Uniqueness of the direct-integral resolvent field and uniqueness of the transverse boundary relation prove (iii). A constant relation gives the tensor sum. Conversely a tensor sum has constant transverse fibres, so (iii) proves (iv).

The decomposition step is a specialization of the general symmetry theorem [1]; the measurable cone resolvent and its normalization have been supplied above.

Remark (A domain issue that cannot be omitted). In , it is the *sum* $(T_{\Lambda(k)} + k^2)u(k)$ that must be square integrable. It is generally incorrect to replace the domain by the intersection of separate domains for a longitudinal operator and a fixed transverse operator. Momentum-dependent bound states can produce cancellations between the two terms.

The actual infinite-dimensional parameter space

In a relation-Cayley neighborhood of a fixed constant matrix U_0, consider

$$\mathrm{U}(k) = \mathrm{U}_0 e^{\mathrm{i}X(k)}, \qquad X \in L^\infty(\mathbb{R}; \mathrm{Herm}\,(d)), \qquad \|X\|_\infty < \pi.$$

This is an infinite-dimensional local parameter space of relation fields. Constant X's form a finite-dimensional complemented subspace: integrate X against any fixed probability density to obtain a bounded projection onto that subspace.

Consequently the constant-relation product family is an embedded finite-dimensional submanifold in these boundary-field charts. Its full relation manifold is $U(d)$, of real dimension d^2. It is not a chart of the translation-invariant non-product ledger, still less of all full spatial extensions.

This statement specifies the topology of the boundary parameter chart. It does not assert that this chart is a norm-resolvent chart for unbounded operators.

Operator-valued relation and the variable-boundary resolvent

On the boundary-profile space $\mathscr{G}$, the field defines the closed relation

$$\Lambda = \{(a, b) \in \mathscr{G} \oplus \mathscr{G} : (a(k), b(k)) \in \Lambda(k) \text{a.e.}\}.$$

Its self-adjointness follows directly by taking the direct integral of . The raw Weyl family is multiplication by

$$M_k(\zeta) = M(\zeta - k^2),$$

on its natural multiplication domain.

Corollary (Variable-boundary Krein formula). *For* $\zeta \notin \mathbb{R}$,

$$\mathscr{F}_z[(B_\Lambda - \zeta)^{-1} - (B_F - \zeta)^{-1}]\mathscr{F}_z^{-1} = \int_{\mathbb{R}}^{\oplus} \gamma(\zeta - k^2) C_{\Lambda(k)}(\zeta - k^2) \cdot \gamma(\overline{\zeta} - k^2)^* \mathrm{d}k.$$

The integrand has rank at most d, but the global difference need not be compact. The complete integrand is uniformly bounded by $2/|\mathrm{Im}\,\zeta|$.

Proof. Apply Proposition in each fibre and use Theorem . The norm bound follows from the difference of the two self-adjoint resolvents.

The factors in are not asserted to be separate bounded multiplication operators on an unweighted boundary space.

The formula is the bounded direct integral of their *complete fibre product*. This avoids an otherwise unjustified global trace-domain manipulation.

Exact essential spectrum and point spectrum

For each k, list the negative transverse eigenvalues, with multiplicity, as

$$\varepsilon_1(k) \le \cdots \le \varepsilon_{r(k)}(k) < 0, \qquad r(k) \le d.$$

They admit measurable ordered choices. For example, measurable spectral projections and rational spectral counting determine these ordered eigenvalues.

Set

$$D_j = \{k : r(k) \ge j\}, \qquad e_j(k) = k^2 + \varepsilon_j(k), \quad k \in D_j.$$

For a measurable function on $D \subset \mathbb{R}$, define

$$\operatorname{ess\,ran}_D e = \{E \in \mathbb{R} : |\{k \in D : |e(k) - E| < \delta\}| > 0 \text{for every} \delta > 0\}.$$

If D has measure zero, this set is empty.

We use the self-adjoint essential spectrum defined by failure of the Fredholm property, equivalently spectrum with isolated finite-multiplicity eigenvalues removed.

Theorem (Spectrum of the measurable-field ledger). *For every extension in Theorem ,*

$$\operatorname{boxed}\operatorname{spec}(B_\Lambda) = \operatorname{spec}_{\mathrm{ess}}(B_\Lambda) = [0,\infty) \ \cup ? \bigcup_{j=1}^{d} \operatorname{ess\,ran}_{D_j} e_j.$$

Moreover,

$$\operatorname{boxed} E \in \operatorname{spec}_{\mathrm{p}}(B_\Lambda) \quad \Leftrightarrow \quad \left|\{k : E - k^2 \in \operatorname{spec}_{\mathrm{p}}(T_{\Lambda(k)})\}\right| > 0.$$

Every eigenvalue has infinite multiplicity.

The lower bound and positivity criteria are

$$\inf \operatorname{spec}(B_\Lambda) = \min\left\{0,\ \min_{1 \le j \le d} \operatorname{ess\,inf}_{D_j} e_j\right\},$$

with empty sets omitted, and

$$B_\Lambda \ge 0 \quad \Leftrightarrow \quad k^2 + \varepsilon_j(k) \ge 0 \text{for a.e.} k \in D_j, \text{every} j.$$

The bottom in may be $-\infty$.

Proof. For a Borel interval I, functional calculus for the direct integral gives

$$E_{B_\Lambda}(I) \simeq \int_{\mathbb{R}}^{\oplus} E_{T_{\Lambda(k)} + k^2}(I)\mathrm{d}k.$$

A nonzero decomposable projection over Lebesgue measure has infinite-dimensional range. Indeed, choose a nonzero measurable range section from a countable fundamental family, restrict to a finite-measure set on which it has nonzero norm, and split that set into arbitrarily many disjoint positive-measure subsets. The resulting range vectors are orthogonal. Hence every nonzero spectral projection onto an open interval has infinite rank. There are no isolated finite-multiplicity spectral points, so spec = $\operatorname{spec}_{\text{ess}}$.

By Lemma , the fibre spectrum is

$$[k^2, \infty) \cup \{e_j(k) : 1 \le j \le r(k)\}.$$

The nonnegative continua contribute exactly $[0, \infty)$: every neighborhood of a nonnegative E meets $[k^2, \infty)$ on a positive-measure set of k's. The negative transverse eigenvalues contribute precisely the essential ranges in . Outside their union some neighborhood has zero spectral projection in almost every fibre. Inside their union every neighborhood has a nonzero projection on a positive-measure set. This proves .

Taking $I = \{E\}$ in proves ; nonnegative transverse energies are not transverse eigenvalues. The same nonatomic argument makes every nonzero full eigenprojection infinite rank.

Finally, the infimum and nonnegativity follow either from or by integrating the fibre quadratic expectations. If a negative fibre spectral region occurs on a positive-measure set, a measurable section supported there gives a negative full expectation.

Remark (What has not been inherited). Theorem is not the product-class band $[-\kappa_*^2, \infty)$, and it does not assert absence of point spectrum. Arbitrary measurable dispersions need not form one interval. The product-class argument in I-B-VP-3 uses constant transverse eigenvalues and is inapplicable before this new calculation [12].

Remark (Mass shift versus state threshold). Adding a fixed scalar mass term $m^2 I$ shifts the displayed spectral sets by m^2. This elementary operator statement is not a covariance existence theorem at the new spectral bottom, and proves no infrared threshold estimate for a quantum state.

Concrete non-product examples

A smooth nonnegative modulation

Choose

$$\alpha = \frac{3}{4}, \qquad \beta = \frac{1}{4}.$$

Then the only deficient mode is $n = 0$, with $\nu = 1/3$; the mode $n = -1$ has order 1 and is limit point.

Choose a nonconstant, nonnegative, even $\chi \in C_c^\infty(\mathbb{R})$, $h_0 > 0$, and $\delta > 0$. Declare

$$\Gamma_1 u(k) = h(k)\Gamma_0 u(k), \qquad h(k) = h_0 + \delta\chi(k\ell).$$

Proposition (Smooth, translation-invariant, non-product example). *The boundary law defines a self-adjoint, nonnegative, rotation-invariant, translation-invariant extension with*

$$\operatorname{spec}(B_h) = \operatorname{spec}_{\mathrm{ess}}(B_h) = [0, \infty), \qquad \operatorname{spec}_{\mathrm{p}}(B_h) = \varnothing.$$

It is not product. Its resolvent difference from the constant-h_0 product extension is not compact.

Proof. The relation is measurable, so self-adjointness follows from Theorem . For a power channel, $M(-q^2) < 0$. Since $h(k) > 0$, the bound-state equation has no solution. Lemma and Theorem prove the spectral and positivity claims. The nonconstant relation is not product. There is no angular mixing, so rotations commute with it.

The resolvent difference from the constant law is a nonzero decomposable bounded operator, nonzero on a

positive-measure momentum set. No such operator over a nonatomic measure space is compact: select a measurable unit section on which its norm has a positive lower bound, and restrict that section to disjoint positive-measure sets. Normalized input vectors then have mutually orthogonal outputs of uniformly positive norm.

Compact support of a change in momentum is thus not a compact operator perturbation.

Momentum compensation in the untwisted logarithmic channel

Now let $\beta = 0$, so $d = 1$. Fix $k_0 > 0$ and $0 < c < 1$. For $|k| > k_0$, impose the logarithmic condition

$$L(k) = \frac{2e^{-\gamma_E}}{c|k|}.$$

For $|k| \le k_0$, impose Friedrichs.

Proposition (A nonnegative untwisted non-Friedrichs extension). *The law defines a self-adjoint, non-product operator B_c with*

$$B_c \ge 0, \qquad \operatorname{spec}(B_c) = [0, \infty), \qquad \operatorname{spec}_{\mathrm{p}}(B_c) = \varnothing.$$

On every fibre $|k| > k_0$, its transverse operator has the negative eigenvalue $-c^2k^2$.

Proof. The relation is measurable. Formula gives the transverse eigenvalue. Its full energy is

$$e(k) = k^2 - c^2k^2 = (1 - c^2)k^2 > 0.$$

Thus gives nonnegativity. The continuous fibre spectra supply $[0, \infty)$. A level set of $(1 - c^2)k^2$ has measure zero, so gives no eigenvalues. The relation is not essentially constant.

This does not contradict the fact that Friedrichs is the only nonnegative untwisted extension *within the product class* [8]. It proves that the qualification is indispensable. The example is not asserted to arise from the monotone local cores of I-B-VP-5 [14].

An isolated essential flat band

Fix an arbitrary $E_0 < 0$. Choose a bounded symmetric measurable set $I \subset \mathbb{R}$ of positive measure. On I, prescribe

$$q(k) = \sqrt{k^2 - E_0}, \qquad L(k) = \frac{2e^{-\gamma_E}}{\sqrt{k^2 - E_0}}.$$

Outside I, prescribe Friedrichs.

Theorem (A negative isolated eigenvalue of infinite multiplicity). *The untwisted extension B_{flat} defined by is self-adjoint and non-product, and*

$$\text{boxed}\,\mathrm{spec}\,(B_{\mathrm{flat}}) = \mathrm{spec}_{\mathrm{ess}}\,(B_{\mathrm{flat}}) = \{E_0\} \cup [0, \infty).$$

The point E_0 is an eigenvalue of infinite multiplicity, and there is a genuine spectral gap $(E_0, 0)$.

An explicit isometric eigenspace embedding is

$$\begin{aligned} L^2(I) &\to \mathscr{H}_0, \\ f &\ \text{longmapsto}\ \mathscr{F}_z^{-1}[1_I(k) f(k) \chi_k(r, \varphi)], \\ \chi_k(r, \varphi) &= \sqrt{2}\, q(k) K_0(q(k) r) \psi_0(\varphi). \end{aligned}$$

Proof. Measurability gives self-adjointness. On I, the transverse bound energy is $-q(k)^2 = E_0 - k^2$. The full bound energy is therefore identically E_0. There are no transverse bound states outside I. Theorem gives .

The radial identity

$$\int_0^\infty r K_0(qr)^2 \mathrm{d}r = \frac{1}{2q^2}$$

normalizes χ_k. For completeness, it follows by scaling from the Bessel equation and the endpoint identity

$$\frac{\mathrm{d}}{\mathrm{d}x}\left[\frac{x^2}{2}(K_0(x)^2 - K_1(x)^2)\right] = x K_0(x)^2 :$$

the bracket tends to 0 at infinity and to $-1/2$ at zero.

Consequently is an isometry. For every $f \in L^2(I)$, its fibre sum $(T_{\Lambda(k)} + k^2) f(k) \chi_k$ is $E_0 f(k) \chi_k$. It belongs to the exact domain and is an eigenvector. Since $L^2(I)$ is infinite-dimensional, so is the eigenspace.

Translation invariance has not been broken. The eigenfunctions are square integrable because they are longitudinal wave packets in a continuum of fibre eigenvectors with the same *total* energy, not one fixed transverse eigenvector tensored with a plane wave.

The same construction works for any real target E_0, if I is chosen where $k^2 > E_0$. For $E_0 \geq 0$ it produces an embedded or threshold eigenvalue; the total spectrum remains $[0, \infty)$.

Remark (Physical interpretation is not supplied). The negative example is not a stable massless field construction. It is a spectral counterexample to transferring the product no-point-spectrum theorem. The nonnegative example likewise supplies no infrared covariance estimate and no preferred physical boundary law.

Longitudinal boosts: an exact boundary rigidity theorem

We now specify the meaning of boost covariance. It concerns the spacetime equation

$$(\partial_t^2 + B_\Lambda)\Phi = 0,$$

with a time-independent spatial boundary relation and no additional boundary time derivatives.

For a positive transverse energy $p^2 > 0$, a positive-frequency generalized mode has longitudinal momentum

$$\omega = \sqrt{k^2 + p^2}.$$

Its boundary pair belongs to $\Lambda(k)$. The radial positive-energy equation has two independent solutions in each deficient channel; their boundary traces span the full two-dimensional boundary pair. Thus every pair in $\Lambda(k)$ occurs in this generalized mode calculation.

A boost of rapidity s changes the longitudinal momentum to

$$k_s = k\cosh s - \sqrt{k^2 + p^2}\sinh s, \qquad \omega_s = \sqrt{k^2 + p^2}\cosh s - k\sinh s.$$

The transverse radius, angular variable, and scalar boundary coefficients are unchanged.

Definition (Scattering-trace boost covariance). The measurable boundary law is boost covariant in the present

sense if, for every $s \in \mathbb{R}$,

$$\Lambda(k_s) = \Lambda(k) \quad \text{for almost every} (p,k) \in (0,\infty) \times \mathbb{R}.$$

Null sets are understood with respect to the indicated Lebesgue measure class.

This is a necessary test for covariance of all scalar positive-transverse-energy solution boundary traces under geometric boosts. It makes no assumption about a quantum state or a unitary positive-energy boost representation.

Theorem (Boost rigidity). *For the translation-invariant, time-independent spatial boundary laws of Theorem , the following are equivalent:*

1. *scattering-trace boost covariance ;*

2. $\Lambda(k)$ *is almost everywhere constant;*

3. B_Λ *belongs to the product class. Hence every genuinely non-product relation field in this ledger fails the stated boost covariance test.*

Proof. The equivalence of (ii) and (iii) is Theorem . A constant relation plainly satisfies .

For the converse, let $P(k)$ be the orthogonal projection in $\mathbb{C}^{2d}$ onto $\Lambda(k)$. It is a bounded measurable matrix field. Apply for the countable set $s \in \mathbb{Q}$. By Fubini there exists some $p_0 > 0$ for which all those equalities hold for almost every k.

Set

$$k = p_0 \sinh\theta.$$

The measure change is locally equivalent to Lebesgue measure, and becomes $k_s = p_0 \sinh(\theta - s)$. Therefore

$$F(\theta) = P(p_0 \sinh\theta)$$

is almost everywhere invariant under every rational translation.

A bounded measurable function with this property is essentially constant. Indeed, convolve any matrix entry with

a compactly supported smooth approximate identity. Each convolution is continuous and invariant under rational, hence all real, translations, so it is constant. At Lebesgue points the convolutions converge to the original entry, showing that it is constant almost everywhere. Thus F, and therefore $P(k)$, is essentially constant. Its range gives a fixed relation Λ.

Remark (Exact scope of the no-go). The theorem is stronger than an undefined statement that a ``generic" field breaks boosts: every nonconstant field fails the precise test.

It does not classify boundary laws involving time derivatives, boundary dynamical fields, or a relation depending jointly on (ω, k), for example through $\omega^2 - k^2$. Those are different models. It also does not assert a unitary boost implementation for every product extension; negative transverse spectra and the choice of state raise separate questions.

Corollary (The examples are not boost covariant). *The extensions in Propositions and , and the flat-band extension in Theorem , fail scattering-trace boost covariance.*

Proof. Each relation field is nonconstant on sets of positive measure. Apply Theorem .

Beyond translations: a normalized longitudinal boundary space

This section constructs a restricted, explicit class outside translation invariance. It is not a classification of all geometrically local or differential boundary conditions in z.

Normalized deficiency coordinates

Assume $d > 0$, and write

$$\mathcal{N}_+ = \ker(S^* - \mathrm{i}), \qquad \mathcal{N}_- = \ker(S^* + \mathrm{i}).$$

For $j \in J_\beta$, put

$$w_j^{\pm}(k; r, \varphi) = \frac{K_{\nu_j}(\sqrt{k^2 \mp \mathrm{i}}\, r)\psi_j(\varphi)}{\left\| K_{\nu_j}(\sqrt{k^2 \mp \mathrm{i}}\, \cdot) \right\|_{L^2(r\mathrm{d}r)}}.$$

Define unitary identifications $J_{\pm} : \mathcal{G} \to \mathcal{N}_{\pm}$ by

$$\mathcal{F}_z J_{\pm} f = \sum_{j \in J_\beta} f_j(k) w_j^{\pm}(k).$$

They intertwine longitudinal translations with scalar momentum multipliers.

The von Neumann decomposition is

$$\operatorname{dom} S^* = \operatorname{dom} S \operatorname{dotplus} \mathcal{N}_+ \operatorname{dotplus} \mathcal{N}_-.$$

For the normalized boundary profiles f_+, f_-, the Green form is

$$2\mathrm{i}\{\langle f_+, g_+ \rangle - \langle f_-, g_- \rangle\}.$$

This is an infinite-dimensional symplectic boundary space after multiplication by the harmless constant 1/(2i).

Lemma (The Friedrichs deficiency unitary). *Let*

$$a_j^{\pm}(k) = \Gamma_0 w_j^{\pm}(k).$$

These coefficients are nonzero, and $a_j^-(k) = \overline{a_j^+(k)}$. *The multiplication operator* $\mathcal{U}_F$ *on* $\mathcal{G}$ *with diagonal entries*

$$u_{F,j}(k) = -\frac{a_j^+(k)}{a_j^-(k)}$$

is unitary. It specifies B_F *in the form*

$$\operatorname{dom} B_F = \operatorname{dom} S \operatorname{dotplus} \{J_+ f + J_- \mathcal{U}_F f : f \in \mathcal{G}\}.$$

Proof. The coefficients follow from . The two radial square roots and radial functions are complex conjugates, and their norms are equal. Thus $|u_{F,j}| = 1$.

The Γ_0 trace of $w_j^+ + u_{F,j} w_j^-$ is zero. In each fibre this is precisely the Friedrichs domain condition. The direct integral of those conditions is B_F, proving .

Remark (Two different Cayley descriptions). The matrix U(*k*) in parametrizes the *raw transverse trace relation*. The operator $\mathcal{U}_F$ in acts between *normalized full deficiency profiles*. A constant raw relation can have a

momentum-dependent normalized deficiency matrix. Confusing these coordinates would invalidate the product criterion.

A rank-one boundary mixing

Choose a normalized

$$h \in C_c^\infty(\mathbb{R};\mathbb{C}^d) \subset \mathscr{G}$$

and let

$$P_h f = \langle f, h\mathrm{rangleh}.$$

Fix $\theta \notin 2\pi\mathbb{Z}$, and define

$$V_\theta = I + (e^{\mathrm{i}\theta} - 1)P_h, \qquad \mathscr{U}_\theta = \mathscr{U}_F V_\theta.$$

The first formula is a unitary rank-one change of the identity. Define B_θ by

$$\begin{aligned} \operatorname{dom} B_\theta &= \operatorname{dom} S\mathrm{dotplus}\,\{J_+ f + J_- \mathscr{U}_\theta f : f \in \mathscr{G}\}, \\ B_\theta(u_0 + J_+ f + J_- \mathscr{U}_\theta f) &= S u_0 + \mathrm{i} J_+ f - \mathrm{i} J_- \mathscr{U}_\theta f. \end{aligned}$$

Theorem (A non-translation-invariant extension). *The operator is a self-adjoint extension of S. It does not commute with all longitudinal translations, and is therefore not product.*

Its resolvent satisfies

$$(B_\theta + \mathrm{i})^{-1} - (B_F + \mathrm{i})^{-1} = \frac{1}{2\mathrm{i}} J_- \mathscr{U}_F (V_\theta - I) J_+^*.$$

The difference has rank one, and

$$\operatorname{spec}_{\mathrm{ess}}(B_\theta) = [0, \infty).$$

No semiboundedness, state, or geometric propagation assertion is included.

Proof. Both $\mathscr{U}_F$ and V_θ are unitary. The graph $f_- = \mathscr{U}_\theta f_+$ is a maximal neutral subspace for : it is neutral by preservation of the inner product, and orthogonality to all graph vectors forces the same graph relation. By , this

proves self-adjointness of and its extension of S.

If B_θ commuted with translations, uniqueness of its normalized deficiency graph would force $\mathscr{U}_\theta$ to intertwine all scalar multipliers e^{-iks}. Since $\mathscr{U}_F$ already does so, P_h would commute with all those multipliers. Its one-dimensional range would then be invariant under all translations in momentum representation. This is impossible for a nonzero L^2 function: $e^{-iks}h(k)$ cannot be a scalar multiple of $h(k)$ for every s on a positive-measure set of k's. Hence B_θ is not translation invariant.

For the resolvent formula, use the Cayley transform

$$C_A = (A - \mathrm{i})(A + \mathrm{i})^{-1} = I - 2\mathrm{i}(A + \mathrm{i})^{-1}$$

of a self-adjoint extension A. It agrees with the Cayley transform of S on $\mathrm{ran}\,(S + \mathrm{i})$, and on $\mathscr{N}_+$ it equals $-J_-\mathscr{U}_A J_+^*$. Indeed,

$$(A + \mathrm{i})(J_+ f + J_-\mathscr{U}_A f) = 2\mathrm{i}J_+ f,$$

whereas applying $A - \mathrm{i}$ gives $-2\mathrm{i}J_-\mathscr{U}_A f$. Subtracting the two Cayley transforms proves . Its rank is one because $V_\theta - I$ has rank one.

Finally, for real E, $A - E$ is Fredholm exactly when

$$I - (E + \mathrm{i})(A + \mathrm{i})^{-1}$$

is Fredholm. For $A = B_\theta, B_F$, these bounded operators differ by rank one. Their Fredholm sets agree. Since $\mathrm{spec}_{\mathrm{ess}}(B_F) = [0, \infty)$, follows.

What ``a longitudinal function'' means here

For $d = 1$, a normalized deficiency profile is one L^2 function of k, equivalently one L^2 function of z after Fourier transformation. For $d = 2$, it is two-component.

The rank-one term acts by

$$f(k)\text{longmapsto}\, h(k)\int_{\mathbb{R}} h(k^{'})^{*} f(k^{'})\mathrm{d}k^{'}.$$

It mixes distinct longitudinal momenta. In the Fourier-transformed profile coordinates its kernel is

$$\text{check}\, h(z)\text{check}\, h(z^{'})^{*},$$

up to the fixed Fourier normalization. This is a longitudinally nonlocal boundary operation. It is not a claim that the *raw* traces $a(z), b(z)$ have this unweighted kernel: the maps $J_{\pm}$ contain momentum-dependent normalization.

Thus the example supplies an actual infinite-dimensional Lagrangian boundary relation with its topology fixed. It does not classify every differential boundary condition or every Lagrangian relation in unnormalized apex coefficients.

Evolution, state, and observable obligations

Self-adjointness is not every wave-evolution theorem

Every self-adjoint B constructed here generates a unitary Schrödinger group e^{-isB}. If $B \geq 0$, the spectral theorem also gives the usual finite-energy wave evolution for

$$\partial_t^2 u + Bu = 0.$$

A semibounded negative part can be treated by the corresponding hyperbolic functions, with exponential growth. For arbitrary measurable relation fields, the spectrum can be unbounded below; no unrestricted bounded wave propagator is claimed in that case.

The regular spacetime is not globally hyperbolic [7]. One can see the obstruction directly: a causal diamond between sufficiently separated points on a fixed radial ray contains a sequence approaching $r = 0$, with no convergent subsequence in M. Selecting an operator domain does not change that geometric fact and does not turn its regular slices into geometric Cauchy surfaces.

No state is constructed

The massless covariance candidate would involve the low-energy behavior of $B^{-1/2}$, together with its appropriate distributional realization. Neither nonnegativity nor self-adjointness alone supplies:

• infrared finiteness on the desired test space;

• a positive spectral ground-state covariance;

• a Hadamard or global wavefront theorem;

• a renormalized coincidence limit.

The product heat bound and the product massive threshold analysis are not used to supply these missing steps [8, 12]. The arrival time of a geometric or diffracted front is not the bottom of a non-product spatial spectrum.

No transfer of one-point formulas

The Friedrichs Wick square, stress, and charged-current formulas in the I-B-VP branch concern their declared product-class operators and states [8, 10]. The exact Friedrichs covariance wavefront theorem of I-B-VP-2 is likewise operator- and state-specific [11]. None becomes a formula for B_{Λ} or B_{θ} merely because a new resolvent has been defined.

The contact coefficients c, τ, w of the one-point extension problem are not the relation field $\Lambda(k)$, the logarithmic function $L(k)$, or the normalized boundary unitary $\mathscr{U}_{\theta}$ [9, 14]. The V1 class includes symmetry assumptions that a generic V6 choice need not preserve.

I-B-VP-4's Maxwell response uses a frozen current in its own declared sector [13]. No current for the present operators is constructed, and therefore no new Maxwell response is inferred.

No backreaction

The cone, deficit, holonomy, and all boundary laws are prescribed. No expectation value of stress is used to solve for any of them. The I-B refusal to infer Einstein dynamics from conservation and a trace condition remains

unchanged [6].

There is no distributional curvature coupling in the present construction. I-B-VP-5's smooth-core selection result and bare-curvature product problem concern different extra data [14]. A core radius is neither a longitudinal relation field nor a replacement for its operator domain.

Finite Enter reporting and the non-acquisition boundary

A supplied finite report may name:

$$\alpha, \beta, \ell, \ d_{\beta}, \text{ model identifier, ?formula or representation for} \Lambda(k),$$
$$\text{chosen example, spectral scope, ?boost-covariance verdict, ?evidence and failure labels.}$$

An arbitrary measurable relation field has no automatic finite encoding. A report contains only its installed representation and justified consequences.

Let $C \cong \mathbb{C}^{2^q}$, $q \geq 1$, have a calibrated orthonormal basis $\{|\tau_j\rangle\}$. Let A contain all surviving finite registers, and install finite words w_j, including unused and failure codes. Set

$$D_j = \langle \tau_j |_C \otimes I_A, \qquad \mathscr{M}(X) = \sum_j |j, w_j\rangle\langle j, w_j|_R \otimes D_j X D_j^{\dagger}.$$

Proposition (Complete supplied-report instrument). *The map is completely positive and trace preserving, and*

$$\mathrm{Tr}_R \mathscr{M}(X) = \mathrm{Tr}_C X.$$

These statements hold with every untouched finite reference. The quantum output contains A, not C. For a positive-probability branch, its normalized residual is its actual subnormalized output divided by its trace. No normalized state is assigned to a zero branch.

Proof. The recorded Kraus operators are $|j, w_j\rangle_R \otimes D_j$. Their squared moduli sum to

$$\sum_j |\tau_j\rangle\langle\tau_j|_C \otimes I_A = I_{CA}.$$

Writing X in clock matrix units shows that the output blocks are its diagonal clock blocks. Their sum is the partial trace. Tensoring the same calculation with a reference identity proves the complete assertion.

This is the existing I-B/En09-A finite interface, not a new continuum measurement theorem [6, 15]. No stationarity or clock-covariance premise for a relational unitary-time interpretation is imposed here.

Proposition (Finite evaluations do not specify an unrestricted field). *A finite family of linear evaluations on an unrestricted smooth compactly supported scalar modulation space cannot be injective. Consequently such evaluations do not, by themselves, determine an arbitrary momentum-dependent Hermitian boundary perturbation.*

If an independently supplied finite-dimensional class contains all admissible perturbations and the probe map is injective on that class, the evaluations do determine the perturbation within that class.

Proof. The smooth compactly supported modulation space is infinite-dimensional. A linear map from it to a finite-dimensional space has a nonzero kernel. Multiplying an invisible scalar modulation by a fixed Hermitian matrix gives an invisible boundary perturbation. The final assertion is injectivity on the independently specified class.

This is the coverage distinction used in the supplied Oai282-A audit [16]. It is not a claim that a finite-dimensional family can never be identified by measurements, nor that the finite probes are already an apparatus for acquiring boundary data.

Actual implementation still needs dimensions, permitted gates, locations, causal record wires, storage, and duration bounds. No such laboratory fit or continuum acquisition map is constructed here. Clock consumption does not consume the geometric coordinate t or z.

Release and exact remaining scope

Theorem (I-B-VP-6 analytical release). *For the prescribed host, character, and minimal operator:*

1. *The non-product ledger is empty when* $d_\beta = 0$.

2. *When* $d_\beta > 0$, *all translation-invariant self-adjoint extensions are the measurable relation fields of Theorem , with domain and resolvent . The product class is exactly the constant-field subfamily.*

3. *Their full spectrum, essential spectrum, point-spectrum criterion, and positivity criterion are –.*

4. *The smooth nonnegative modulation, momentum-compensated untwisted law, and isolated essential flat band are genuine non-product examples.*

5. *Scattering-trace covariance under all longitudinal boosts is equivalent to the product condition within the declared time-independent spatial boundary class.*

6. *The normalized rank-one boundary mixing is a self-adjoint extension outside translation invariance, with rank-one resolvent change and unchanged essential spectrum.*

7. *These results do not select a physical extension, construct a spectral ground state or observables, or source the host. The finite report has only the supplied-description interface.*

Proof. Theorem proves (i). Theorem and Corollary prove (ii). Theorem proves (iii). Section proves (iv). Theorem proves (v). Theorem proves (vi). Sections and specify and prove the interface separations in (vii).

begingroup 1.12 toprule Object Verdict Exact scope

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No deficient transverse channels Closed Unique full spatial realization; no extension freedom.

Translation-invariant spatial extension ledger Constructed All measurable finite relation fields, including angular mixing.

Constant transverse $U(2)$ relations Inherited product wall Not recounted as the new non-product result.

Variable-boundary resolvent Constructed Bounded direct integral of the complete fibre correction.

Full spatial spectrum Evaluated Essential ranges, possible flat bands, and positivity criterion.

Longitudinal boosts Rigidity proved On-shell scattering-trace covariance, not a spatial boost-commutator assertion.

General longitudinal boundary space One subclass constructed Normalized finite-rank unitary perturbations; no local differential classification.

Geometric finite propagation Not proved Self-adjointness and translation invariance are insufficient to import the product wave formula.

Spectral ground state and infrared estimates Deferred No covariance or state is released.

One-point observables and apex contacts Deferred No transfer of product formulas or normalization of c, τ, w.

Einstein or Maxwell backreaction Not undertaken Host and extension remain inputs.

endgroup

Documentary separations, not analytical inputs

The supplied II/II-B finite certificates, V/V-B fixed-time response classifications, VII-H/VII-I free-product results, IX sharp-product sheets, X supported-transfer and regulator sheets, XI gravitational sheets, and Em compatibility and lapse audits concern different operators or different maps. No identification with the present boundary space is used.

In particular, a Weyl algebra in a gravitational free-field construction is not the cone Weyl function . A supported equality does not supply a non-product wavefront or covariance theorem. A finite contact space does not replace the infinite-dimensional space $\mathscr{G}$.

The large supplied XIV–XV characteristic-composition material is not the analytical problem solved here. No characteristic graph map, joint section, quantum coefficient, or Ward admission is inferred from the self-adjoint extension results. The version-specific internal-amplitude construction recorded at I2 is not erased, and the later characteristic I3/J2 obligations are not discharged.

Standing stops

The following statuses are unchanged:

$$\text{nolinkurl}\, O.1 = NOT_C LOSED, \qquad \text{nolinkurl}\, bulk_s ymmetry_r estored = false.$$

XVI-N is not discharged. XV-B is not opened as a downstream quantum-admission exit. No OpenAI catalogue statement, family number, abstract, or formalization report is a premise. I-B-VP-AB and I-B-VP-SAE are not used as registered analytical premises.

There is no emergence, Einstein dynamics, Maxwell or Einstein backreaction, Page curve, de Sitter state, cosmological bounce, or admitted characteristic quantum coefficient.

In the requested programme reading, Problems 1, 2, and 8 remain unfinished; Problems 3–7 and 9–11 are not reached, with Problem 6 explicitly not the implication attempted here; Problem 12 remains unresolved.

appendix

Debugging audit

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Calling constant non-diagonal transverse data ``non-product'' Angular mixing is already permitted in I-B-VP's product class. V6 requires genuine longitudinal dependence or mixing.

Treating every finite matrix as a V6 chart Finite matrices chart transverse relations. V6 relation fields and normalized full boundary profiles are infinite-dimensional.

Equating a chosen relation with a Weyl function The minimal operator and trace normalization fix M. The extension is separate data Λ.

Dropping Friedrichs directions Use self-adjoint relations or their unitary Cayley matrices, not finite Hermitian matrices alone.

Demanding separately square-integrable summands in every non-product domain Use ; only the complete fibre

operator sum is required.

Importing the constant-boundary Krein correction Proposition and Corollary prove the variable-field formula.

Calling finite fibre rank a globally compact correction Nonzero decomposable operators over Lebesgue measure are not compact. Proposition gives the example.

Inheriting a single negative band or absence of point spectrum Use essential ranges and positive-measure level sets. The flat-band example has an isolated essential eigenvalue.

Inferring full negativity from a negative transverse fibre Longitudinal k^2 may compensate it. Proposition proves this explicitly.

Asking the spatial operator to commute with an undefined boost Use Definition , a spacetime trace-covariance condition.

Identifying normalized deficiency matrices with raw relation Cayley matrices They are different coordinate systems; see Lemma .

Assuming raw apex traces lie in unweighted L_z^2 The general example uses normalized deficiency profiles and the topology .

Calling abstract self-adjointness a state or finite-speed theorem Wave evolution, positivity, infrared behavior, and propagation have distinct hypotheses.

Applying a product current or stress formula to a new resolvent No state, coincidence-limit, or renormalization comparison has been constructed.

Identifying boundary functions with observable contacts $\Lambda(k)$, $L(k)$, $\mathscr{U}_\theta$, and (c, τ, w) act on different mathematical objects.

Treating a finite report as a continuum determination A representation and an acquisition or coverage theorem are additional inputs.

endgroup

Provenance and source-use ledger

The mathematical release is the stated collection of operator theorems, not a certification of an entire companion corpus.

• The online I-B and philosophy pages were consulted for the prescribed-host and destructive-interface background.

• The publisher record for the Enter article was consulted. It records acceptance on September 28, 2026 and states that the final formatted version is forthcoming in that consulted record.

• The supplied version-qualified I-B-G and I-B-VP passages fix the product boundary and its open non-product row. The required radial and full-operator calculations are given in this manuscript rather than delegated to an unavailable companion.

• The supplied I-B-VP-1 version 3 passages are used only to identify the existing product-class one-point work and its resonant-set boundary. This paper neither verifies that entire manuscript nor imports its unregistered cross-reference as a premise.

• Smirnov's primary paper was consulted for symmetry reduction; the Dereziński–Richard primary paper for inverse-square radial analysis; and the Behrndt–Malamud–Neidhardt primary paper for boundary relations and the Weyl resolvent framework. The present trace normalization, spectral examples, and boost test are explicitly derived.

• The supplied I-B-VP-5 manuscript is a comparison for core selection and the operator/contact distinction. It is not a source of a non-product boundary condition.

Failed retrieval of an online companion page is not evidence that a manuscript is absent from the registry. No registry installation, corpus-wide source scan, or independent validation of all named sheets is asserted.

Declarations

Authorship. The author field is left blank for submission preparation. Attribution of the companion manuscripts to N. Cheung does not assign authorship of this draft.

Novelty scope. The claimed contribution is the explicit cone implementation, essential-range spectral ledger, counterexamples to product-only conclusions, boost rigidity in the declared boundary class, and the normalized non-translation-invariant example. General symmetry reduction, von Neumann extension theory, and Weyl-function resolvent methods are not claimed as new.

Physical scope. All geometric and boundary data remain prescribed. No extension is declared physically preferred. No spectral ground state, interacting algebra, detector-field acquisition mechanism, or backreaction law is supplied.

Verification. No numerical experiment, successful pdflatex run, machine-checked proof, independent rebuild, independent review, repository installation, or journal acceptance is asserted. The source is self-contained for pdfLaTeX and contains the written proofs of the stated mathematical results.

References

[1] A. G. Smirnov, Reduction by symmetries in singular quantum-mechanical problems: general scheme and application to Aharonov–Bohm model, *Journal of Mathematical Physics* **56** (2015). DOI: 10.1063/1.4936305. Primary preprint, arXiv:1411.5351, version 3. https://arxiv.org/abs/1411.5351. Especially Theorem V.1, Corollary V.2, and Proposition V.3. The general reduction theorem is established prior work.

[2] J. Dereziński and S. Richard, On Schrödinger operators with inverse square potentials on the half-line, *Annales Henri Poincaré* **18** (2017), 869–928. DOI: 10.1007/s00023-016-0520-7. Primary preprint: https://arxiv.org/abs/1604.03340. Radial extension and Bessel spectral background.

[3] J. Behrndt, M. M. Malamud, and H. Neidhardt, Scattering matrices and Weyl functions, *Proceedings of the London Mathematical Society* **97** (2008), 568–598. Primary preprint: https://arxiv.org/abs/math-ph/0604013.

Boundary relations, gamma fields, Weyl functions, and the Krein resolvent formula. The normalization used here is computed explicitly.

[4] N. Cheung, ``The Enter Calculus: A Linear Quantum Lambda Calculus with an Operational Characterization of Measurement," *Frontiers in Physics* **14** (2026), article 1954943. DOI: 10.3389/fphy.2026.1954943. https://doi.org/10.3389/fphy.2026.1954943. Interface/operational background only; not a PDE premise.

[5] N. Cheung, *The Philosophy of Enter: Destructive Readout, Quantum Recovery, and the Refusal of Identification*, Cheung Ngo Medical Limited (2026). Framing essay; not a theorem premise. Programme note: https://enter.grok.me/enter.

[6] N. Cheung, *I-B*, I-B-v1.tex, version 1, Enter Calculus Companion. https://enter.grok.me/browse/I-B. Used for the prescribed conical product, def:experiment, thm:instrument, and prop:not-source. The finite interface supplies no extension-selection rule. Companion record; not a Scholar-indexed journal article.

[7] N. Cheung, *I-B-G*, I-B-G-v2.tex, version 2, Enter Calculus Companion. Supplied version-qualified excerpts. Especially lem:deficiency, ass:product, prop:nonGH, and the translation-invariant non-product open item. Its product Green and transfer theorems are not imported as non-product propagation results. https://enter.grok.me/browse/I-B-G. Companion record; not a Scholar-indexed journal article.

[8] N. Cheung, *I-B-VP*, I-B-VP-v4.tex, version 3, Enter Calculus Companion. Supplied version-qualified excerpts. Especially ass:product, def:relation, thm:krein, thm:spectrum, and the V6 row. Its product extension class includes finite-dimensional rotation-breaking transverse relations. https://enter.grok.me/browse/I-B-VP. Companion record; not a Scholar-indexed journal article.

[9] N. Cheung, *I-B-VP-1*, I-B-VP-1-v4.tex, version 3, Enter Calculus Companion. Supplied product-class one-point apex passages, including thm:uv and thm:u2ext. The non-diagonal transverse analysis is not a spatial longitudinally non-product classification. Its resonant-set analysis is not completed here. https://enter.grok.me/browse/I-B-VP-1. Companion record; not a Scholar-indexed journal article.

[10] N. Cheung, *I-B-VP-0*, I-B-VP-0-v2.tex, version 1, Enter Calculus Companion. Supplied thm:current. The charged Friedrichs current is not transferred to a new non-product extension. https://enter.grok.me/browse/I-B-VP-0. Companion record; not a Scholar-indexed journal article.

[11] N. Cheung, *I-B-VP-2*, I-B-VP-2-v2.tex, version 1, Enter Calculus Companion. Supplied thm:main. The global massless Friedrichs covariance wavefront theorem is not a theorem about the present operators. https://enter.grok.me/browse/I-B-VP-2. Companion record; not a Scholar-indexed journal article.

[12] N. Cheung, *I-B-VP-3*, I-B-VP-3-v2.tex, version 1, Enter Calculus Companion. Supplied def:class, thm:spectrum, and thm:threshold. The product band and state threshold are not inherited. https://enter.grok.me/browse/I-B-VP-3. Companion record; not a Scholar-indexed journal article.

[13] N. Cheung, *I-B-VP-4*, I-B-VP-4-v2.tex, version 1, Enter Calculus Companion. Supplied frozen-current Maxwell response. No source current for a V6 operator is constructed here. https://enter.grok.me/browse/I-B-VP-4. Companion record; not a Scholar-indexed journal article.

[14] N. Cheung, *I-B-VP-5: Curvature-core selection on the prescribed conical Enter spacetime*, I-B-VP-5-v2.tex, version 1. Supplied manuscript. Especially thm:selection, cor:measure, and prop:contactseparation. Core selection and observable normalization remain distinct from longitudinal boundary dependence. https://enter.grok.me/browse/I-B-VP-5. Companion record; not a Scholar-indexed journal article.

[15] N. Cheung, *En09-A*, version 1, Enter Calculus Companion. Supplied complete relational Enter theorem and no-return passages. https://enter.grok.me/browse/En09-A. Extraction is distinct from the additional stationarity and clock-covariance hypotheses for relational dynamics. Companion record; not a Scholar-indexed journal article.

[16] N. Cheung, *Oai282-A*, version 1, Enter Calculus Companion. Supplied prop:coverage and prop:invisible. https://enter.grok.me/browse/Oai282-A. Finite-probe coverage discipline only; no external catalogue theorem is imported. Companion record; not a Scholar-indexed journal article.

[17] N. Cheung, *Black-Hole Information as an Access Problem: Enter, Quantum Recovery, and Complete Residuals*, figshare (2026). DOI: 10.6084/m9.figshare.34335024.v1. https://doi.org/10.6084/m9.figshare.34335024.v1.

[18] N. Cheung, *Operational Wormhole Certificates—Existence, Dimension-Dependent Scaling, Operational Functions, and Composition*, figshare (2026). DOI: 10.6084/m9.figshare.34549575.v1. https://doi.org/10.6084/m9.figshare.34549575.v1.

[19] N. Cheung, *Enter, Quantum Magic, and Quantum Gravity: Readout, Recovery, and the Limits of Geometric Response*, figshare (2026). DOI: 10.6084/m9.figshare.34335582.v1. https://doi.org/10.6084/m9.figshare.34335582.v1.

[20] N. Cheung, *Operational Consistency Does Not Select Lorentzian Geometry*, figshare (2026). DOI: 10.6084/m9.figshare.34441758.v1. https://doi.org/10.6084/m9.figshare.34441758.v1.

[21] N. Cheung, *Realization of Relational Time by Destructive Enter on a Clock Register*, figshare (2026). DOI: 10.6084/m9.figshare.34442907.v1. https://doi.org/10.6084/m9.figshare.34442907.v1.