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I-B-VP-7 Spin structures and first-order Dirac domains on the prescribed conical spacetime

A spinor-domain release, with constrained gauge systems deferred

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Version 1

Abstract

We address the spin-structure and first-order operator stages of I-B-VP's open spinor and constrained-field row on its unchanged, prescribed regular conical product. The regular host has exactly two spin structures. Their polar-frame signs are independent of the prescribed flat line-bundle character, although the charged spinor equation sees their combined monodromy. We derive the spin connection and the radial Dirac expression explicitly.

For a two-component transverse spinor, the deficient channels are exactly those satisfying

$$|n + s + \beta| < \frac{\alpha}{2}, \qquad s \in \{0, \tfrac{1}{2}\}.$$

There is at most one such channel. We prove the endpoint criterion, including equality cases, construct its boundary traces, and state all self-adjoint radial domains. Two coordinate-axis domains preserve the transverse grading. Either gives a longitudinal product realization of the massless four-component Hamiltonian, with its full operator domain and an exact graph-norm identity. The unrestricted spatial minimal operator has infinite deficiency indices when a critical channel is present; the finite boundary condition belongs to the chosen transverse product construction.

The principal debugging result is a square-domain obstruction: in a critical channel, no self-adjoint realization of

the first-order radial Dirac expression has the componentwise Friedrichs realization of its formal square as its operator square. Thus independently choosing regular scalar squares does not define a Dirac domain. Explicit examples also show that scalar essential self-adjointness does not determine the spinor verdict.

The potential usefulness is a domain-complete interface for later spinor analysis, separating topology, flux, first-order boundary data, and one-point contacts. The underlying spin and Aharonov–Bohm extension theory is established; no general priority claim is made. No vacuum state, current or stress expectation value, quantum Maxwell system, Proca field, twisted tensor sector, backreaction, or inherited characteristic quantum coefficient is constructed.

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Book expansion: spin structures diagram

toprule Structure Role

midrule Spin structure A/B Two structures kept distinct

Independent line character Organizing datum with flux

Radial first-order form Before squaring

Squared second-order operator Separate; no-Friedrichs + square-domain obstruction prominent

Domain ⇒ contacts? **No**: domain does not determine one-point contacts

bottomrule

Scope, host, and analytical question

The stage completed here

This manuscript implements the suggested first V7 stage:

1. the spin-structure question on the regular host;

2. a first-order Dirac domain calculation and one explicitly declared product-type realization in each supplied spin/flux sector. The constrained Maxwell, Proca, and geometrically twisted tensor stages remain separate successor problems. Completing the present stage does not complete the whole V7 row.

The long supplied XIV–XV characteristic-composition material is not the operator problem studied here. None of its missing maps is supplied by a spin structure or a self-adjoint Dirac Hamiltonian.

Prescribed geometry

Set $c = \hbar = 1$ as a choice of units, not as a derived speed or normalization. The regular host is

$$\begin{aligned} M &= \mathbb{R}_t \times (0,\infty)_r \times (\mathbb{R}/2\pi\mathbb{Z})_\varphi \times \mathbb{R}_z, \\ g &= -\mathrm{d}t^2 + \mathrm{d}r^2 + \alpha^2 r^2 \mathrm{d}\varphi^2 + \mathrm{d}z^2, \qquad 0 < \alpha < 1. \end{aligned}$$

Its spatial slice is

$$\Sigma = (0,\infty) \times S^1 \times \mathbb{R}, \qquad q = \mathrm{d}r^2 + \alpha^2 r^2 \mathrm{d}\varphi^2 + \mathrm{d}z^2, \qquad \mathrm{d}V_q = \alpha r \, \mathrm{d}r \, \mathrm{d}\varphi \, \mathrm{d}z.$$

The completion adds the worldsheet

$$\mathscr{W} = \mathbb{R}_t \times \{o\} \times \mathbb{R}_z.$$

The cone, deficit, orientation, future direction, and worldsheet are prescribed as in I-B [11]. No Einstein equation is used to determine them.

The regular host is not globally hyperbolic. For example, choose two points on one radial ray at times sufficiently far apart. Their causal diamond contains a sequence with $r \mathrm{downarrow} 0$ and has no corresponding limit point in M. An operator extension does not change this geometry or make the regular slices geometric Cauchy surfaces [12].

Three different extension questions

The following objects must not be identified:

1. a lift of the oriented frame bundle to a spin bundle;

2. a self-adjoint domain for a first-order spinor Hamiltonian;

3. a distributional extension of a numerical current or stress. The first is topological input. The second is operator-domain input unless essential self-adjointness removes its ambiguity. The third requires an independently constructed state and normalization prescription.

In particular, the Dirac one-point results recorded in I-B-VP-1 version 3 do not supply the first two objects [14]. The scalar relation fields of the attached I-B-VP-6 are not first-order spinor domains [19].

Literature and novelty boundary

Spin structures and their effect on Dirac operators are standard spin geometry [1]. Self-adjoint extension issues for fermions in Aharonov–Bohm and cosmic-string backgrounds were studied by de Sousa Gerbert [2]. Cone Dirac extensions and evolution estimates also occur in the work of D'Ancona, Yin, and Zhang [3]. Homogeneous half-line Dirac realizations are part of the more general analysis of Dereziński and Ruba [4].

Our contribution is the explicit implementation of these questions in the fixed V7 input ledger, with:

• a polar-frame spin/line-character convention;

• an endpoint proof and actual boundary traces;

• a domain-complete longitudinal construction;

• a proof that componentwise Friedrichs squaring cannot replace a critical first-order domain. We do not claim that spin structures, the half-unit indicial criterion, or Aharonov–Bohm extension theory originate here.

Spin structures and the independent line character

A global frame and two spin lifts

Use the global oriented, future-oriented orthonormal frame

$$e_0 = \partial_t, \qquad e_1 = \partial_r, \qquad e_2 = \frac{1}{\alpha r}\partial_\varphi, \qquad e_3 = \partial_z.$$

The one-form dφ is globally defined on the punctured transverse space, although φ is not a global single-valued real function.

Theorem (Spin structures on the regular product). *The oriented, time-oriented host admits spin structures. Up to equivalence it has exactly two.*

Relative to the polar frame , they are represented on the angular cover by

$$\psi(t, r, \varphi + 2\pi, z) = e^{2\pi i s}\psi(t, r, \varphi, z), \qquad s \in \{0, \tfrac{1}{2}\}.$$

The contractible t, r, z factors add no obstruction and no additional spin choices.

Proof. The global frame trivializes the oriented, time-oriented orthonormal frame bundle. Pulling back the standard spin double cover along this trivialization gives a spin structure.

For completeness, let $\widetilde{M}$ be the angular universal cover. For each $\eta \in \{1, -1\}$, form

$$P_\eta = (\widetilde{M} \times \mathrm{Spin}^+(1,3))/\sim, \qquad (\varphi + 2\pi, h) \sim (\varphi, \eta h).$$

The covering map to the frame bundle is well defined because η is central and projects to the identity. The two choices have different lift monodromy around the meridian and cannot be equivalent as lifts.

There are no further choices: M deformation retracts onto its angular circle, so

$$\pi_1(M) = \mathbb{Z}, \qquad H^1(M; \mathbb{Z}_2) = \mathbb{Z}_2.$$

After pulling any spin structure to the simply connected cover, the deck generator can act on a lift only by one of the two central signs. These exhaust the possibilities. The spin representation sends the central minus sign to minus the identity, giving .

This direct construction is the present specialization of the usual $H^1(\,\cdot\,; \mathbb{Z}_2)$-torsor description [1, Sections 2–3].

What a transverse filling would additionally require

Proposition (Bounding spin structure). *Relative to the polar frame, the spin structure with $s = 1/2$ is the one that extends over a smooth transverse disk. The structure with $s = 0$ does not.*

This is a statement about a smooth topological filling with a smooth metric supplied on it. It is not a smooth extension of the singular conical metric, its connection, or an operator domain across the apex.

Proof. For a Euclidean disk, the polar frame makes one full rotation relative to a Cartesian frame along the boundary. A lift of that rotation to the spin group ends at minus its initial value. Thus components of a spinor that is single valued in a Cartesian spin frame are antiperiodic in the polar spin frame.

The disk has a unique spin structure because it is contractible. Its restriction therefore gives precisely $s = 1/2$. The other restriction cannot extend.

Changing the positive angular scale α on the punctured collar identifies its frame bundle with the reference frame bundle by an orientation-preserving rescaling and does not change this winding calculation. No extension of the conical Levi–Civita connection at $r = 0$ has been assumed.

Flat line bundle: a separate datum

Independently prescribe the scalar-ledger character parameter $\beta \in [0, 1)$. We use its lifted-section convention:

$$u(\varphi + 2\pi) = e^{2\pi i\beta} u(\varphi).$$

On the angular cover choose a parallel frame for this flat line connection. Its local connection form is then zero; its nontrivial datum is the seam character.

A spinor coupled to this line has total lifted condition

$$\Psi(\varphi + 2\pi) = e^{2\pi i(s+\beta)} \Psi(\varphi).$$

We retain s and β as separate inputs and set

$$\delta = s + \beta \bmod 1$$

only as a convenient label for their combined monodromy.

Remark (Flux-sign convention and double counting). The definition of β is . If $\Psi = e^{i\beta\varphi}\chi$, then χ has the spin-only condition , and differentiation of Ψ becomes $\partial_\varphi + i\beta$ on χ.

Thus a constant angular gauge potential and the lifted character are alternative representations of the same flat line input. They must not both be inserted. Identifying β with a signed quantity $e\Phi/(2\pi)$ requires an additional convention for the charged covariant derivative and loop orientation. We do not identify it by name with the flux convention of another paper.

Proposition (Separate factors and combined equivalence). *Changing the spin structure is not the same operation as changing the line character. Nevertheless, at the level of the charged spinor bundle with its connection, simultaneous replacement*

$$(s,\beta)\text{longmapsto}\,(s+\tfrac{1}{2},\beta+\tfrac{1}{2})\ \text{mod}\ 1$$

leaves the combined lifted description unchanged, up to angular mode relabeling.

Consequently the twisted spinor differential operator need not determine the two factors separately.

Proof. The spin lift is a double cover of the frame bundle, while the line factor is a Hermitian line with a flat connection. They have different defining projections and structure groups.

Tensoring a spinor bundle by the nontrivial flat sign line changes its spin-lift sign. Tensoring the charged line by the same sign line changes its character by a half unit. Their product sign is unchanged. Equivalently, changes $s+\beta$ by an integer, and the local line connection is still zero in the chosen parallel description. The differential expressions and total seam conditions are therefore identified.

Remark (Filling the line factor). A nontrivial flat meridian character cannot extend over a disk as a smooth *flat* connection: every loop in a disk is contractible and flat parallel transport there has trivial holonomy. A nonflat flux core is extra input [18]. The combined equivalence in Proposition does not extend either factor separately over a core.

The spin connection and the first-order operator

We first use two-component spatial spinors. A four-component massless Dirac Hamiltonian will be constructed by chiral doubling in Section .

All Hilbert inner products are linear in the first argument. Let

$$\sigma_1=\begin{pmatrix}0&1\\1&0\end{pmatrix},\qquad \sigma_2=\begin{pmatrix}0&-\mathrm{i}\\\mathrm{i}&0\end{pmatrix},\qquad \sigma_3=\begin{pmatrix}1&0\\0&-1\end{pmatrix}.$$

The spatial Hilbert space is

$$\mathscr{H}^{(2)}_{s,\beta} = L^2(\Sigma; S_s \otimes L_\beta), \qquad \|\Psi\|^2 = \int_\Sigma \Psi^\dagger \Psi \, \mathrm{d}V_q.$$

Proposition (Connection and Hamiltonian expression). *In the polar spin frame and the parallel line frame, the spatial spin connection is*

$$\nabla_r = \partial_r, \qquad \nabla_\varphi = \partial_\varphi - \frac{\mathrm{i}\alpha}{2}\sigma_3, \qquad \nabla_z = \partial_z.$$

The massless two-component Hamiltonian expression is

$$\mathscr{H} = -\mathrm{i}\sigma_1\left(\partial_r + \frac{1}{2r}\right) - \frac{\mathrm{i}\sigma_2}{\alpha r}\partial_\varphi - \mathrm{i}\sigma_3\partial_z.$$

It is symmetric on smooth compactly supported sections on Σ, with the seam condition .

Proof. For the spatial coframe

$$\theta^1 = \mathrm{d}r, \qquad \theta^2 = \alpha r \, \mathrm{d}\varphi, \qquad \theta^3 = \mathrm{d}z,$$

Cartan's equation gives

$$\omega^2{}_1 = \alpha \, \mathrm{d}\varphi, \qquad \omega^1{}_2 = -\alpha \, \mathrm{d}\varphi.$$

The spin representation sends the corresponding connection to $-\mathrm{i}\alpha\sigma_3 \mathrm{d}\varphi/2$. This proves .

Contracting with $-\mathrm{i}\sigma^a e_a$, and using $\sigma_2\sigma_3 = \mathrm{i}\sigma_1$, gives . In particular, the radial $1/(2r)$ term is produced by the spin connection, not by an extra flux.

Integration by parts with measure $\alpha r \, \mathrm{d}r \, \mathrm{d}\varphi \, \mathrm{d}z$ proves symmetry. The angular endpoint terms cancel because the seam multiplier is unitary.

The same cone spin-connection mechanism appears in [3]. Here its frame and monodromy have been fixed explicitly.

Angular and radial unitaries

Write

$$q_n = n + s + \beta, \qquad \lambda_n = \frac{q_n}{\alpha}, \qquad e_n(\varphi) = \frac{e^{\mathrm{i} q_n \varphi}}{\sqrt{2\pi\alpha}}.$$

Then

$$\Psi(r, \varphi, z) = \sum_{n \in \mathbb{Z}} e_n(\varphi) f_n(r, z),$$

with $f_n \in L^2(r\,\mathrm{d}r\,\mathrm{d}z; \mathbb{C}^2)$. Set

$$u_n(r, z) = r^{1/2} f_n(r, z).$$

The combined transformation is unitary to

$$\mathrm{bigoplus}_{n \in \mathbb{Z}} L^2((0, \infty) \times \mathbb{R}, \mathrm{d}r\,\mathrm{d}z; \mathbb{C}^2).$$

Under it, the transverse expression becomes

$$d_\lambda = -\mathrm{i}\sigma_1 \partial_r + \frac{\lambda}{r}\sigma_2 = \begin{pmatrix} 0 & -\mathrm{i}(\partial_r + \lambda/r) \\ -\mathrm{i}(\partial_r - \lambda/r) & 0 \end{pmatrix}.$$

The longitudinal term is $-\mathrm{i}\sigma_3 \partial_z$.

The parameter λ in is an angular indicial parameter. It is not a scalar extension parameter or the Coulomb coupling denoted by similar letters elsewhere.

Radial deficiency and actual boundary traces

Let $d_{\lambda,\min}$ be the closure of on $C_c^\infty((0, \infty); \mathbb{C}^2)$. Its adjoint is $d_{\lambda,\max}$, with domain

$$\mathrm{dom}\, d_{\lambda,\max} = \{u \in L^2 : u \in AC_{\mathrm{loc}}((0, \infty); \mathbb{C}^2), ?d_\lambda u \in L^2\}.$$

The characterization follows by distributional integration by parts; on each compact interval avoiding zero, the coefficient λ/r is bounded.

Defect solutions, including the endpoints

Fix a reference length $\ell > 0$, and put $\mu = \ell^{-1}$.

Lemma (Radial endpoint criterion). *For real λ, the deficiency indices of $d_{\lambda,\min}$, at the parameters ±iμ, are*

$$(n_+, n_-) = \begin{cases} (1,1), & |\lambda| < \tfrac{1}{2}, \\ (0,0), & |\lambda| \geq \tfrac{1}{2}. \end{cases}$$

In particular, the equality cases are essentially self-adjoint, not critical.

Proof. At the spectral parameter +iμ, a solution square integrable at infinity is

$$v_+(r) = \sqrt{r}\begin{pmatrix} K_{\lambda-1/2}(\mu r) \\ K_{\lambda+1/2}(\mu r) \end{pmatrix}.$$

For −iμ, replace the lower component by its negative. The derivative identities

$$\left(\partial_r - \frac{\lambda}{r}\right)\sqrt{r}K_{\lambda-1/2}(\mu r) = -\mu\sqrt{r}K_{\lambda+1/2}(\mu r)$$

and its companion with the other sign verify the equations.

The other independent solution contains an exponentially growing modified Bessel function. Thus the square-integrable solution space at infinity has dimension one at each of these spectral parameters.

Near zero, $\sqrt{r}K_\nu(\mu r)$ is square integrable with measure dr exactly when $|\nu| < 1$. The logarithmic case $\nu = 0$ satisfies this condition. At $|\nu| = 1$, its leading term is a nonzero multiple of $r^{-1/2}$, which is not square integrable. Both components of are therefore square integrable exactly when

$$|\lambda - \tfrac{1}{2}| < 1, \qquad |\lambda + \tfrac{1}{2}| < 1,$$

or $-1/2 < \lambda < 1/2$. This proves .

This elementary calculation is consistent with the general half-line Dirac extension analysis [2, 4]. It is not the scalar limit-circle test.

Critical-channel traces

Assume now $|\lambda| < 1/2$. For $u \in \operatorname{dom} d_{\lambda,\max}$, define

$$a(u) = \lim_{r \operatorname{downarrow} 0} (r/\ell)^{-\lambda} u_1(r), \qquad b(u) = \lim_{r \operatorname{downarrow} 0} (r/\ell)^{\lambda} u_2(r).$$

Lemma (Trace existence and Green form). *The limits in exist and are continuous for the maximal graph norm.*

More precisely,

$$\begin{aligned} u_1(r) &= a(u)(r/\ell)^{\lambda} + o(r^{1/2}),\\ u_2(r) &= b(u)(r/\ell)^{-\lambda} + o(r^{1/2}). \end{aligned}$$

The trace map (a, b) *is onto* $\mathbb{C}^2$*, and*

$$\langle d_{\lambda,\max} u, v\rangle - \langle u, d_{\lambda,\max} v\rangle = \mathrm{i}(a(u)\overline{b(v)} + b(u)\overline{a(v)}).$$

Its common trace kernel is $\operatorname{dom} d_{\lambda,\min}$.

Proof. Write $f = d_\lambda u$. The component equations give

$$\left((r/\ell)^{-\lambda} u_1\right)' = \mathrm{i}(r/\ell)^{-\lambda} f_2, \qquad \left((r/\ell)^{\lambda} u_2\right)' = \mathrm{i}(r/\ell)^{\lambda} f_1.$$

Both right sides are locally integrable down to zero because $|\lambda| < 1/2$ and $f \in L^2(\mathrm{d}r)$. Hence the limits exist.

For example,

$$\left| u_1(r) - a(u)(r/\ell)^{\lambda} \right| \le C_\lambda r^{1/2} \|f_2\|_{L^2(0,r)} = o(r^{1/2}).$$

The other component is identical with λ replaced by $-\lambda$. On a fixed annulus, the equation bounds the H^1 norm of u by its maximal graph norm. Evaluating there and integrating the displayed identities to zero proves graph continuity of a, b.

A smooth cutoff of

$$(r/\ell)^{\lambda} \frac{1}{0}, \qquad (r/\ell)^{-\lambda} \frac{0}{1}$$

belongs to the maximal domain and has arbitrary prescribed trace. This proves surjectivity.

On (ε, R), integration by parts gives

$$-\mathrm{i}[u(r)^{\mathsf{T}} \sigma_1 \overline{v(r)}]_{\varepsilon}^{R}.$$

On a tail, maximal-domain spinors belong to H^1, since λ/r is bounded there. Their values tend to zero at infinity.

At zero use ; all remainder products vanish because $1/2 \pm \lambda > 0$. This gives .

The minimal domain has zero traces by graph continuity. Conversely, a maximal-domain vector with zero traces has zero Green pairing with every maximal-domain vector, so belongs to the adjoint of $d_{\lambda,\max}$, namely $d_{\lambda,\min}$.

Self-adjoint radial domains

Theorem (All radial domains in a critical channel). *If $|\lambda| < 1/2$, all self-adjoint restrictions of $d_{\lambda,\max}$ extending $d_{\lambda,\min}$ have the boundary conditions*

$$b = \mathrm{i}ta, \quad t \in \mathbb{R}, \qquad \text{or} \qquad a = 0.$$

The latter is denoted $t = \infty$.

In particular,

$$\begin{aligned} \operatorname{dom} d_\lambda^{(b)} &= \{u \in \operatorname{dom} d_{\lambda,\max} : b(u) = 0\}, \\ \operatorname{dom} d_\lambda^{(a)} &= \{u \in \operatorname{dom} d_{\lambda,\max} : a(u) = 0\} \end{aligned}$$

are self-adjoint domains. For $|\lambda| \geq 1/2$, write $d_\lambda^{(b)} = d_\lambda^{(a)} = \overline{d_{\lambda,\min}}$.

Proof. By Lemma , the adjoint of a restriction specified by a trace subspace is specified by its orthogonal complement for the boundary form . Self-adjointness is therefore maximal isotropy for the Hermitian form

$$(a, b), (a', b') \mathrm{longmapsto} a\overline{b'} + b\overline{a'}.$$

This form has signature $(1, 1)$, so its maximal isotropic subspaces are complex lines. A line with $a \neq 0$ is isotropic exactly when $\operatorname{Re}(a\overline{b}) = 0$, or $b = \mathrm{i}ta$ with $t \in \mathbb{R}$. The remaining isotropic line is $a = 0$. The noncritical assertion is Lemma .

The condition that both traces vanish gives the critical *minimal* domain, not a self-adjoint ``regular in both components'' Dirac domain.

The transverse ledger and its grading-compatible choice

Define

$$J_D(s,\beta) = \{n \in \mathbb{Z} : |n + s + \beta| < \alpha/2\}, \qquad m_D = |J_D(s,\beta)|.$$

Theorem (Spin-dependent transverse deficiency). *For the two-component transverse operator:*

1. $m_D \in \{0, 1\}$*;*

2. *its minimal deficiency indices are* (m_D, m_D)*;*

3. *it is essentially self-adjoint precisely when*

$$\operatorname{dist}(s + \beta, \mathbb{Z}) \geq \alpha/2;$$

4. *if* $m_D = 1$*, its self-adjoint extension freedom is the one critical boundary line . For the four-component transverse direct sum* $D_\perp \oplus (-D_\perp)$*, the minimal indices are* $(2m_D, 2m_D)$*. We do not classify chirality-mixing four-component domains here.*

Proof. The interval defining J_D has length $\alpha < 1$, so contains at most one integer. Angular decomposition is unitary and the transverse minimal operator is the orthogonal sum of the radial minimal operators. Finite angular truncation and radial graph-core approximation prove this closure assertion. Lemma then gives the indices and . If a critical summand exists, all other summands are already self-adjoint and have no boundary quotient. Theorem gives its extension freedom. Direct-sum addition gives the four-component count.

Corollary (Bounding spin structure and the flux window). *For* $s = 1/2$ *and* $\beta \in [0, 1)$*, a critical channel exists exactly when*

$$\left|\beta - \frac{1}{2}\right| < \frac{\alpha}{2}.$$

In particular, the bounding, untwisted spinor $(s,\beta) = (1/2, 0)$ *has a unique transverse realization.*

For the nonbounding, untwisted spinor $(s,\beta) = (0, 0)$*, there is always one critical channel, with* $\lambda = 0$*.*

Proof. Substitute $s = 1/2$ or $s = 0$ into . The inequalities remain strict.

The transverse grading

Let $\Gamma_\perp = \sigma_3$, acting on each radial summand. The formal identity is

$$d_\lambda \Gamma_\perp = -\Gamma_\perp d_\lambda.$$

Proposition (Which radial domains preserve the grading). *In a critical channel, the only self-adjoint domains in invariant under $\Gamma_\perp$ are the two axes . On either of those domains, anticommutation holds as an operator identity.*

Under radial unitary dilations

$$(\mathscr{V}_\rho u)(r) = \rho^{1/2} u(\rho r), \qquad \rho > 0,$$

the boundary parameter transforms as

$$t \operatorname{longmapsto} \rho^{-2\lambda} t.$$

Thus for $\lambda \neq 0$, the axes are also the only dilation-invariant radial domains in this family. For $\lambda = 0$, every line in is dilation invariant, but only the axes preserve $\Gamma_\perp$.

Proof. The grading sends (a, b) to $(a, -b)$. A line $b = \mathrm{i}ta$ is invariant under this map only when $t = 0$, or when the line is $a = 0$. The formal anticommutation identity then acts on the same domain. Noncritical domains are invariant by closure from the initial smooth domain.

The two traces of $\mathscr{V}_\rho u$ are

$$a \operatorname{longmapsto} \rho^{1/2+\lambda} a, \qquad b \operatorname{longmapsto} \rho^{1/2-\lambda} b.$$

Taking their ratio gives the asserted transformation and its fixed lines.

One declared transverse realization

For the rest of the construction choose the $b = 0$ axis whenever there is a critical channel, and define

$$D_{\perp}^{(b)} = \mathrm{bigoplus}_{n \in \mathbb{Z}}\, d_{\lambda_n}^{(b)}.$$

Its domain is the orthogonal-sum graph domain:

$$\mathrm{dom}\, D_{\perp}^{(b)} = \{(u_n) : \quad u_n \in \mathrm{dom}\, d_{\lambda_n}^{(b)}, \quad \sum_n (\|u_n\|^2 + \left\|d_{\lambda_n}^{(b)} u_n\right\|^2) < \infty\}.$$

This choice is declared, not selected by a physical principle.

A concrete graph core consists of finite angular sums of radial compactly supported smooth spinors away from zero, augmented, in the critical channel, by

$$\Phi_b(r) = \chi(r)(r/\ell)^{\lambda} \begin{matrix} 1 \\ 0 \end{matrix},$$

where $\chi = 1$ near zero and has compact radial support. Indeed, subtracting $a(u)\Phi_b$ leaves zero traces and hence the minimal domain. This proves that the boundary condition is an actual graph-domain prescription.

We have

$$\Gamma_{\perp}\mathrm{dom}\, D_{\perp}^{(b)} = \mathrm{dom}\, D_{\perp}^{(b)}, \qquad D_{\perp}^{(b)}\Gamma_{\perp} = -\Gamma_{\perp} D_{\perp}^{(b)}.$$

Replacing every critical $b = 0$ condition by $a = 0$ gives a second construction with the same properties.

Why a Friedrichs square is not a Dirac domain

The proposal's phrase ``Friedrichs-type Dirac domain'' requires correction. The first-order operator is not semibounded.

Proposition (No first-order Friedrichs extension). *Every self-adjoint extension of the transverse Dirac minimal operator is unbounded both above and below. In particular, there is no Friedrichs extension of that first-order symmetric operator.*

Proof. Fix one angular summand and a real normalized radial $\chi \in C_c^{\infty}((a, b))$, $0 < a < b < \infty$. Let $v_{\pm}$ be normalized eigenvectors of σ_1 with eigenvalues ± 1, and set

$$u_R^{\pm}(r) = \chi(r) e^{iRr} v_{\pm}.$$

These vectors belong to the initial domain and hence to every extension. The derivative of χ has zero integrated expectation, and $v_{\pm}^{\dagger}\sigma_2 v_{\pm} = 0$. Therefore

$$\left\langle d_\lambda u_R^{\pm}, u_R^{\pm} \right\rangle = \pm R.$$

Letting $R \to \infty$ excludes both a lower and an upper bound.

The absence of a first-order Friedrichs choice in cone Dirac analysis is also emphasized in [3]. More importantly, taking Friedrichs extensions after formal squaring does not repair the missing first-order domain.

The radial formal square is

$$d_\lambda^2 = \begin{pmatrix} -\partial_r^2 + \lambda(\lambda-1)/r^2 & 0 \\ 0 & -\partial_r^2 + \lambda(\lambda+1)/r^2 \end{pmatrix}.$$

In the original $L^2(r\,\mathrm{d}r)$ radial coordinates, its component Bessel orders are

$$|\lambda - \tfrac{1}{2}|, \qquad |\lambda + \tfrac{1}{2}|.$$

These expressions do not state the operator-square domain.

Theorem (Critical square-domain obstruction). *Assume $|\lambda| < 1/2$. Let A be any self-adjoint extension of $d_{\lambda,\min}$. Let L_F be the componentwise Friedrichs realization of , initially defined on $C_c^\infty((0,\infty);\mathbb{C}^2)$. Then*

$$A^2 \neq L_F.$$

Thus the componentwise Friedrichs square is not the square of any self-adjoint first-order radial extension in a critical channel.

Proof. Put

$$\nu_1 = \tfrac{1}{2} - \lambda \in (0,1), \qquad \nu_2 = \tfrac{1}{2} + \lambda \in (0,1).$$

The two scalar expressions in are

$$L_{\nu_j} = -\partial_r^2 + (\nu_j^2 - \tfrac{1}{4})r^{-2}.$$

On compactly supported smooth functions, integration by parts gives the nonnegative factorization

$$\langle L_\nu f, f\rangle = \int_0^\infty \left| f'(r) - \frac{\nu + 1/2}{r} f(r) \right|^2 \mathrm{d}r.$$

Consequently the Friedrichs form domain is the closure of this graph norm. In particular, its elements have the distributional expression inside the absolute value in L^2.

The function $\chi(r)(r/\ell)^\lambda$ fails this condition for $\nu = \nu_1$: near zero the expression in is a nonzero multiple of $r^{\lambda-1}$, which is not in L^2. Similarly, $\chi(r)(r/\ell)^{-\lambda}$ fails it for $\nu = \nu_2$.

The domain of A corresponds to one nonzero isotropic trace line. Choose a nonzero pair (a, b) in that line, and set

$$u(r) = \chi(r)\begin{pmatrix} a(r/\ell)^\lambda \\ b(r/\ell)^{-\lambda} \end{pmatrix}.$$

This vector belongs to $\operatorname{dom} A$. Since the two displayed powers solve $d_\lambda u = 0$ near zero, $Au = d_\lambda u$ is smooth and compactly supported away from zero. It therefore belongs to $\operatorname{dom} d_{\lambda,\min} \subset \operatorname{dom} A$. Thus $u \in \operatorname{dom} A^2$.

At least one of a, b is nonzero, so at least one component fails the corresponding Friedrichs form condition. Hence $u \notin \operatorname{dom} L_F$, proving .

The factorization proof is included to make this conclusion independent of a transferred scalar boundary classification. The scalar background is consistent with the inverse-square analysis in [5].

Remark (The elementary $\lambda = 0$ case). At $\lambda = 0$, $d_0 = -\mathrm{i}\sigma_1\partial_r$ on the half-line. For the $b = 0$ domain,

$$\operatorname{dom}(d_0^{(b)})^2 = \{u \in H^2 : u_1'(0) = 0,\ u_2(0) = 0\}.$$

Thus its square has a Neumann upper component and a Dirichlet lower component. For the $a = 0$ domain the roles are reversed. Componentwise Friedrichs would impose Dirichlet on both, which is not either first-order square.

Remark (Not a choice between the V1 one-point branches). The two axes constructed here are not identified with I-B-VP-1's branches A and B. Such an identification would require a comparison of first-order mode

domains, the chosen state, and the flux convention. Coincidence of formal squared angular orders is insufficient.

The longitudinal product realization

Return to the two-component spatial Hilbert space and use the unitary transform

$$(\mathscr{F}_z u)(k) = (2\pi)^{-1/2} \int_{\mathbb{R}} e^{-ikz} u(z) \mathrm{d}z.$$

Let $D = D_{\perp}^{(b)}$, in either the original transverse coordinates or their unitarily equivalent radial direct sum.

For each $k \in \mathbb{R}$, define

$$H(k) = D + k\Gamma_{\perp}, \qquad \operatorname{dom} H(k) = \operatorname{dom} D.$$

Lemma (Fibre self-adjointness and graph identity). *Each $H(k)$ is self-adjoint, depends continuously on k in norm resolvent sense at nonreal spectral parameters, and satisfies*

$$\|H(k)v\|^2 = \|Dv\|^2 + k^2\|v\|^2, \qquad v \in \operatorname{dom} D.$$

Moreover,

$$\operatorname{dom} H(k)^2 = \operatorname{dom} D^2, \qquad H(k)^2 = D^2 + k^2 I.$$

Proof. The perturbation $k\Gamma_{\perp}$ is bounded and self-adjoint. For example, for $|\operatorname{Im} \zeta| > |k|$, a Neumann-series resolvent argument for

$$H(k) - \zeta = \{I + k\Gamma_{\perp}(D - \zeta)^{-1}\}(D - \zeta)$$

gives surjectivity in both half-planes and hence self-adjointness on $\operatorname{dom} D$.

The resolvent identity gives

$$\|(H(k) - \zeta)^{-1} - (H(k') - \zeta)^{-1}\| \le \frac{|k - k'|}{|\operatorname{Im} \zeta|^2}.$$

For , the cross term has zero real part by self-adjointness of D, invariance of its domain under $\Gamma_{\perp}$, and .

Finally, for $v \in \operatorname{dom} D$, the condition $H(k)v \in \operatorname{dom} D$ is equivalent to $Dv \in \operatorname{dom} D$, since $\Gamma_\perp v \in \operatorname{dom} D$.

Anticommutation then gives the square formula.

Theorem (An explicit self-adjoint spatial Hamiltonian). *The operator*

$$H_W = \mathscr{F}_z^{-1}\left(\int_{\mathbb{R}}^{\oplus} H(k)\mathrm{d}k\right)\mathscr{F}_z$$

is a self-adjoint realization of . Its domain is exactly

$$\operatorname{dom} H_W = \{\Psi : \quad \widehat{\Psi}(k) \in \operatorname{dom} D \text{a.e.},\ \int_{\mathbb{R}} \left(\|D\widehat{\Psi}(k)\|^2 + k^2\|\widehat{\Psi}(k)\|^2\right)\mathrm{d}k < \infty\}.$$

Equivalently, this is the intersection of the transverse operator domain and the longitudinal H^1 domain.

A graph core is the algebraic tensor product of the transverse graph core described after with $C_c^\infty(\mathbb{R}_z)$.

The four-component massless Dirac Hamiltonian

$$H_D = \begin{pmatrix} H_W & 0 \\ 0 & -H_W \end{pmatrix}, \qquad \operatorname{dom} H_D = \operatorname{dom} H_W \oplus \operatorname{dom} H_W,$$

is self-adjoint. It gives unitary Hilbert-space evolution $e^{-\mathrm{i}tH_D}$ for the supplied domain.

Proof. Lemma gives a measurable self-adjoint fibre field. At every nonreal ζ, its fibre resolvents have the uniform bound $1/|\operatorname{Im}\zeta|$. Their direct integral is a resolvent of a self-adjoint operator.

The direct-integral domain requires square integrability of $H(k)\widehat{\Psi}(k)$. Identity makes this exactly , rather than merely a sufficient intersection domain.

To verify the core, first truncate momentum to a compact interval. There the fibre graph norms are uniformly equivalent to the graph norm of D. Finite transverse graph-core sums and smooth momentum coefficients approximate every truncated graph section. Their inverse Fourier transforms are Schwartz in z. Cut them off in z; the Schwartz tail and its first derivative tend to zero in the graph norm. Momentum truncation then removes the remaining tails. This proves the core assertion.

On the initial smooth domain the operator is the expression derived in Proposition , so the closure extends that operator. The block direct sum is self-adjoint. The spectral theorem supplies its unitary group.

Remark (Why the graph-domain proof matters). The exact intersection in follows from the first-order anticommutation identity and the grading-compatible domain. It is not copied from a scalar tensor sum.

For a different radial boundary line, $D + k\Gamma_{\perp}$ is still a bounded self-adjoint perturbation for fixed k, but the grading-domain identity used here need not hold. We make no corresponding global graph-norm claim for such a choice.

The unrestricted spatial minimal operator

Let $H_{W,\min}$ be the closure of the two-component expression on compactly supported smooth sections of Σ, without imposing the new critical boundary vector.

Theorem (Essential self-adjointness versus longitudinal boundary space). *If $m_D = 0$, $H_{W,\min}$ is self-adjoint and equals .*

If $m_D = 1$, its deficiency indices are (∞, ∞). Each deficiency space is unitarily equivalent to $L^2(\mathbb{R}_k)$. The four-component minimal Hamiltonian likewise has indices $(0, 0)$ in the first case and (∞, ∞) in the second.

Thus is a finite transverse boundary prescription for a chosen product realization. It is not a finite-dimensional classification of every spatial Dirac extension.

Proof. Fourier transformation in z gives the fibre minimal operators

$$D_{\perp,\min} + k\Gamma_{\perp}.$$

The identification with the closure follows by momentum truncation and the tensor-core argument used in Theorem . The minimal transverse domain is grading invariant, and the graph identity holds there as well. Testing against that tensor core gives the corresponding maximal direct-integral adjoint.

When $m_D = 0$, the transverse closure is self-adjoint. Theorem and the same initial-domain core argument then give

the first assertion.

For a critical channel, solve at $z_0 = \pm i\mu$. Put $Q = \sqrt{k^2 + \mu^2} > 0$. A decaying fibre solution is

$$v_{z_0,k}(r) = \sqrt{r}\begin{pmatrix} K_{\lambda - 1/2}(Qr) \\ \frac{iQ}{k + z_0} K_{\lambda + 1/2}(Qr) \end{pmatrix}.$$

The derivative identities from Lemma verify

$$(d_\lambda + k\sigma_3)v_{z_0,k} = z_0 v_{z_0,k}.$$

Both coefficients are nonzero; their relative modulus is one. The integrability criterion at zero is unchanged and the decaying solution at infinity is unique. All noncritical channels contribute zero.

The normalized vectors depend measurably on k. Multiplying them by arbitrary $L^2(\mathbb{R}_k)$ profiles identifies each full deficiency space with $L^2(\mathbb{R}_k)$. The block direct sum proves the four-component statement.

No raw boundary trace is asserted to belong to unweighted $L^2(\mathbb{R}_z)$ on the entire spatial adjoint domain. The theorem identifies the deficiency space through normalized defect profiles.

Explicit sectors and the scalar non-transfer

Corollary (A concrete declared critical realization). *Fix any* $0 < \alpha < 1$, *the bounding spin structure* $s = 1/2$, *and the character* $\beta = 1/2$. *There is exactly one critical angular mode,* $n = -1$, *and its radial expression is* $d_0 = -i\sigma_1\partial_r$.

Declaring $b = 0$ *in this mode and taking the unique closures in all other modes gives the self-adjoint four-component Hamiltonian , with domain . No state or observable is selected.*

Proof. Here $q_n = n + 1$; only $q_{-1} = 0$ satisfies the critical inequality. Apply Theorems and .

Corollary (Scalar emptiness and spinor emptiness differ). *The scalar count*

$$d_\beta^{sc} = \#\{n : |n + \beta| < \alpha\}$$

does not determine the Dirac count $m_D(s, \beta)$.

For example, with $\alpha = 1/3$ *and the bounding spin structure:*

$c\|c\|c\beta$	d_β^{sc}	$m_D(1/2,\beta)$
hline 0	1	0
1/2	0	1

Proof. For $\beta = 0$, the scalar mode $n = 0$ is counted. The spinor angular lattice is half-integral, so its distance from zero is $1/2 > \alpha/2$.

For $\beta = 1/2$, the scalar lattice has distance $1/2 > \alpha$, whereas the spinor combined lattice is integral and has the zero mode $n = -1$.

The scalar count and the non-product scalar ledger belong to I-B-G and I-B-VP-6 [12, 19]. Equation is a direct calculation, not an application of their scalar essential self-adjointness theorems to spinors.

We do not transfer the V6 scalar boost-rigidity result, flat-band example, or momentum-compensation mechanism. Nor does the first-order identity $H(k)^2 = D^2 + k^2$ identify a Dirac state threshold with I-B-VP-3's scalar product threshold [16].

One-point data, Maxwell obstructions, and deferred systems

The V1 Dirac one-point calculation is a different problem

I-B-VP-1 version 3 records numerical massless Dirac stress and azimuthal-current formulas, their distributional residues, and an allowed generic contact pair (τ, w) [14]. Its charge-conjugation-invariant untwisted subcase excludes w, leaving τ. This qualification corrects an unqualified assertion that the contact space always has two parameters.

Those contacts change extensions of c-number one-point distributions. They are not the trace parameter t in , a spin structure, or the line character.

The source manuscript reports comparisons with Linet and Bezerra de Mello–Saharian [6, 7], and discloses an unresolved comparison with Spinelly–Bezerra de Mello [8]. We neither repeat its numerical checks nor decide that discrepancy. The present domain proof is independent of those comparisons.

The one-point dilation obstruction recorded there concerns its stated observable-normalization class. It is not contradicted by homogeneous first-order domains in Proposition : the objects being dilated and the extension problems are different. No state comparison identifies our $b = 0$ realization with either one-point branch.

Maxwell: what is retained and what is not constructed

I-B-VP-4 concerns a restricted classical magnetic response to a frozen scalar current, with specified distributional completion rules [17]. It does not supply a photon Hamiltonian, Gauss-law-preserving Cauchy domain, physical Hilbert space, or quantum gauge algebra.

Two boundary facts can be separated without borrowing a Maxwell domain.

Proposition (A source-independent winding obstruction). *For $m \in \mathbb{Z} \setminus \{0\}$, the regular gauge map*

$$g_m(r, \varphi) = e^{\mathrm{i}m\varphi}$$

does not extend to a continuous $U(1)$ map on the transverse disk. This obstruction is independent of whether a source is frozen, dynamical, or zero.

Proof. Its restriction to a meridian has degree m. A map on a disk restricts to a null-homotopic map on the boundary and therefore has degree zero.

This is the topological content behind V4's distinction between regular gauges and smooth completion gauges. It does not define a completion gauge group or its physical phase space.

Proposition (The integrability obstruction is conditional). *Suppose a magnetic field in a neighborhood of the apex has orthonormal component*

$$F_{\hat{r}\overset{\wedge}{\varphi}} = Cr^{-2} + O(1), \qquad C \neq 0,$$

uniformly on a nonempty compact tangential patch. Then it has no locally integrable extension agreeing with that field off the worldsheet.

Nonvanishing of a source alone does not imply this hypothesis, and does not give a universal

locally-integrable-extension obstruction for Maxwell fields.

Proof. For sufficiently small r, the absolute value is bounded below by a positive multiple of r^{-2}. The transverse volume measure is $\alpha r\,\mathrm{d}r\,\mathrm{d}\varphi$, so its integral diverges as

$$\int_0^{\varepsilon} r^{-1}\mathrm{d}r.$$

The same conclusion holds in smooth Cartesian completion coordinates, whose norms and density are comparable here.

For the final assertion, choose a smooth one-form A supported in a compact set disjoint from the apex and put $F = \mathrm{d}A$, $J^{\nu} = \nabla_{\mu} F^{\mu\nu}$. One can choose A so that $J \neq 0$, for example by taking a nonconstant compact spatial coefficient for A_t on a regular flat chart. Both F and J extend smoothly by zero near the apex. Thus a nonzero source need not have the displayed singular asymptotic behavior.

V4's obstruction therefore survives a later dynamical-source construction only if that construction proves the relevant nonzero singular asymptotics in the same response sector. It cannot be generalized to every nonzero Maxwell source.

The linked field/current contact in V4 depends on its restricted distributional completion. No analogous contact classification is imported for a general constrained system. Substituting a response field into a previously frozen source does not by itself construct a self-consistent coupled state [17].

The unentered Maxwell constraint interface

A later Maxwell stage must specify an actual domain on which, in its chosen sign convention, the constraint includes

$$\mathrm{div}_q E = \rho.$$

In coordinates this divergence is

$$\mathrm{div}_q E = \frac{1}{\alpha r}\partial_i(\alpha r E^i).$$

It must also specify the allowed gauge maps, the residual gauge after any gauge fixing, the boundary flux conditions, and the quotient or reduced phase space.

Writing this equation is not a proof that a self-adjoint evolution preserves its constraint domain. No such domain or quotient is constructed here. In particular, the gauge-fixed Noether identities of XI-A-ST on its declared completion are not this cone's Gauss law [20].

Proca and twisted tensors

Proca is not obtained here by adding a mass term to V4's response or by taking a Maxwell mass limit. Its constraint structure, domain, and initial-data problem remain to be constructed.

Geometrically twisted tensor sectors remain unopened. The flat or supported graviton, ghost, and tensor blocks of later manuscripts are not conical tensor-bundle realizations supplied by the present spinor construction.

Evolution is not a state, and a report is not acquisition

State and observable obligations

Theorem constructs a one-particle self-adjoint Hamiltonian and its unitary evolution. It does not construct:

• a spectral vacuum or a distinguished fermionic state;

• a two-point distribution or a Hadamard condition;

• a global spinor wavefront theorem;

• renormalized current, stress, or coincidence limits;

• a coupled spinor–Maxwell state. Unbounded negative one-particle energies are not, by themselves, a nonexistence theorem for a quantum-field vacuum.

No scalar covariance, scalar current, scalar heat estimate, or scalar threshold theorem fills these obligations. Likewise, V1's one-point formulas cannot be assigned to the new domain without a state and domain comparison [13, 14, 15, 16].

We have not proved geometric finite propagation for the selected extension. Unitarity is the evolution claim. It does not turn Σ into a geometric Cauchy surface of the punctured host.

Enter background and philosophy

In the Enter calculus, destructive readout consumes the measured quantum names and produces a classical code. Its operational characterization is relative to an explicitly specified strategy class [9]. The philosophical companion distinguishes such reporting from quantum recovery and from identification with an independently given physical object [10].

Here those ideas fix a reporting boundary, not an analytical premise. A finite program can report a supplied description of

$$(\alpha, s, \beta, \ell), \quad \text{chosen radial law}, \quad \text{operator-domain scope}, \quad \text{proved or deferred obligations}.$$

It does not infer a spin lift or unknown continuum boundary condition merely by printing that description.

Complete finite report channel

Let $C \cong \mathbb{C}^D$, $D = 2^q$, have a calibrated orthonormal basis $\{|\tau_j\rangle\}$. Let A contain every surviving finite register. Install finite report words w_j, including unused and failure codes, and define

$$D_j = \langle \tau_j|_C \otimes I_A, \qquad \mathscr{M}(X) = \sum_j |j, w_j\rangle\langle j, w_j|_R \otimes D_j X D_j^\dagger.$$

Proposition (Complete supplied-description report). *The map is completely positive and trace preserving. For every untouched finite reference E,*

$$\mathrm{Tr}_R\,[(\mathscr{M} \otimes \mathrm{id}_E)(X)] = \mathrm{Tr}_C\, X.$$

Its returned quantum interface contains A, not C. A positive-probability branch has its actual conditional residual; a zero-probability branch has no assigned normalized conditional state.

The channel selects no spin structure, operator domain, gauge constraint, or metric.

Proof. The recorded Kraus maps are $|j, w_j\rangle_R \otimes D_j$, and

$$\sum_j D_j^\dagger D_j = I_C \otimes I_A.$$

Expansion in clock matrix units shows that the output blocks are precisely the diagonal clock blocks of X. Their sum is $\mathrm{Tr}_C X$. The calculation remains valid with A enlarged by E. Its codomain contains no quantum clock factor.

The installed words and finite register operations have no continuum input or geometry-update output. Thus the last sentence is a limitation of this declared interface, not an identification theorem.

This reproduces the finite I-B/En09-A interface on the supplied-description task [11, 21]. Additional clock stationarity or covariance assumptions are not imposed.

For example, the two clock states

$$(|\tau_0\rangle + |\tau_1\rangle)/\sqrt{2}, \qquad (|\tau_0\rangle - |\tau_1\rangle)/\sqrt{2}$$

with the same residual input give identical outputs. There is no universal clock recovery from this channel. The classical record also does not determine an arbitrary surviving A-state.

Finite tests could identify data within an independently proved identifiable model class. They do not prove their own coverage of an unrestricted continuum class [22]. No field-acquisition apparatus, statistical model, physical comparison theorem, or laboratory resource-fit certificate is constructed here.

Release, deferrals, and standing boundaries

Theorem (I-B-VP-7 first-stage release). *On the unchanged prescribed regular host:*

1. *There are exactly two spin structures, represented by . Their choice is separate from the flat line character.*

2. *The spin connection and first-order Hamiltonian expression are and . Their angular reduction is .*

3. *The transverse critical count is , with strict inequality. The radial maximal traces, Green form, and self-adjoint domains are those proved in Section .*

4. *The declared $b = 0$ product choice gives the self-adjoint four-component Hamiltonian , with the exact domain . The $a = 0$ axis gives a second such construction.*

5. *The unrestricted spatial minimal operator is essentially self-adjoint exactly when $m_D = 0$; otherwise its boundary freedom is infinite-dimensional.*

6. *In every critical radial channel, a componentwise Friedrichs square is not the operator square of any self-adjoint first-order radial realization.*

7. *No state, one-point observable, constrained Maxwell domain, Proca field, twisted tensor sector, or backreaction is released. The finite report is only a supplied-description interface.*

Proof. Theorem and Proposition prove (i). Proposition and the unitary radial reduction prove (ii). Theorems and , with Lemmas and , prove (iii). Theorem proves (iv). Theorem proves (v). Theorem proves (vi). Sections and state the construction boundary and prove the finite interface in (vii).

begingroup 1.12 toprule Stage Verdict Exact scope

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V7.0: host Retained Geometry and all spin/line/domain inputs remain prescribed.

V7.1: spin Constructed Two regular spin lifts, their polar signs, and the separate flat line datum.

V7.2: Dirac Constructed in the stated class First-order radial domains and a grading-compatible longitudinal product realization.

All spatial Dirac extensions Not classified The critical full boundary space is infinite-dimensional; no non-product spinor classification is attempted.

V7.3: Maxwell Deferred Topological and conditional integrability audits are not a Gauss-law-preserving evolution domain or physical phase space.

V7.4: Proca Deferred No massive-vector constraint construction or mass-limit argument.

V7.5: tensors Unopened No geometrically twisted conical tensor sector.

V7.6: states and observables Deferred No vacuum, Hadamard theorem, spinor current or stress expectation, or quantum gauge field.

endgroup

Documentary comparisons are not proof transfers

The scalar Friedrichs current and wavefront results, the massive scalar band, the scalar core-selection theorem, and the scalar non-product relation fields remain on their own operators [13, 15, 16, 18, 19]. Their use here is to locate a boundary, not to bypass a spinor proof.

XI-E, XI-F, and XI-G concern their declared flat multiplets and domains; their algebraic or microlocal conclusions are not conical spinor-domain theorems. The X supported-transfer sheets do not provide a new conical covariance. Em02-E's lapse and instantaneous kernel are not this spinor problem.

The supplied XIV-H, XIV-I, and XIV-J records concern an independent characteristic quantum composition. The historical H2 amplitude stop, the version-specific I2 internal-amplitude construction, and the unresolved I3/J2 joint-map obligations are not changed by this release. No joint characteristic map, quantum coefficient, or downstream quantum Ward admission is inferred. XV-B is not opened as a quantum-admission exit.

Standing refusals and version qualifications

No emergence, Einstein dynamics, Maxwell or Einstein backreaction, Page curve, de Sitter state, cosmological bounce, or inherited characteristic quantum coefficient is claimed. No physical preference among the two spin structures or the constructed domains is asserted.

XVI-N and Route B are not discharged. XVI-O retains its stated O.1 = NOT_CLOSED status, and bulk_symmetry_restored = false is not changed. The separately supplied scoped XVI-Q record is not erased; its distinct prescription and restricted claim are not a premise or a result of this manuscript. Outside independently

earned scopes, the series standing stop is unchanged.

I-B-VP-AB and I-B-VP-SAE are not analytical premises. The OpenAI catalogue, family numbers, abstracts, and formalization reports supply no theorem here. The philosophy essay is not a registered analytical premise.

In the requested programme reading, Problems 1, 2, and 8 remain unfinished. Problems 3–7 and 9–11 are not reached; Problem 6 is explicitly not this implication. Problem 12 remains unresolved.

appendix

Debugging and acceptance audit

begingroup 1.12 toprule Potential error Required correction

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Identifying the line character with a spin structure Keep the spin lift s and the flat character β separate. Their tensor product has combined monodromy.

Assuming the combined equation identifies both factors Proposition gives the simultaneous half-shift equivalence.

Calling geometric cone holonomy a spin lift The tangent connection and its chosen double-cover lift are different data.

Using the smooth-disk spin structure without declaring a core Only the regular host is used. A filling is additional and does not extend the singular metric automatically.

Counting the flux twice Use either the parallel-cover seam character or a constant angular connection in a spin-only seam gauge.

Removing the radial $1/(2r)$ term It follows from the spin connection and is removed only by the explicit unitary $u = r^{1/2}f$.

Using the scalar threshold $|n + \beta| < \alpha$ The first-order criterion is $|n + s + \beta| < \alpha/2$, including its strict endpoints.

Declaring both critical traces zero to obtain self-adjointness That is the critical minimal domain. A self-adjoint domain requires a maximal isotropic trace line.

Calling the Dirac realization Friedrichs The first-order operator is not semibounded. Use the explicitly declared axis domain.

Choosing Friedrichs independently for the squared components Theorem excludes this as the square of a self-adjoint critical first-order realization.

Promoting formal anticommutation to an operator identity Prove grading invariance of the chosen boundary domain. Only the two axes have it in the critical family.

Giving a finite-dimensional full spatial deficiency count The finite count is transverse. Longitudinal profiles make the critical spatial indices infinite.

Borrowing the scalar V6 domain or boost theorem Theorem is a new first-order domain proof. No spinor boost-rigidity theorem is imported.

Identifying a boundary parameter with (τ, w) The former changes a spinor operator domain; the latter normalizes c-number one-point distributions.

Saying that every V1 Dirac contact space is two-dimensional The supplied charge-conjugation-invariant untwisted subcase forbids w.

Resolving the Spinelly comparison by constructing a domain A state, flux, and formula comparison remains absent.

Generalizing V4's integrability obstruction to every nonzero source Its singular asymptotic hypothesis must survive. A smooth source supported away from the apex is a counterexample to an unrestricted generalization.

Calling the Gauss equation a constrained Maxwell construction A domain, preservation theorem, residual gauge group, and physical quotient are additional obligations.

Identifying unitary evolution with a quantum state or geometric Cauchy property Neither follows from the

self-adjoint-domain theorem.

Treating an installed finite report as continuum acquisition An apparatus and a separately proved identification or coverage contract are additional and absent.

endgroup

Source-use and verification statement

The supplied I-B-VP-6 version 1 is cited as the scalar non-product companion, not as a spinor premise. The supplied version-qualified V1 and V4 passages identify the one-point and classical-response boundaries. No assertion is made that every proof in those manuscripts has been independently verified.

The online I-B and philosophy pages were consulted for the prescribed-host and finite-interface background. The publisher record for the Enter article was consulted; it reports acceptance on September 28, 2026 and a forthcoming formatted version in that record.

Primary sources were consulted for spin structures, Aharonov–Bohm fermion extensions, cone Dirac analysis, and half-line Dirac and inverse-square realizations. The endpoint, trace, square-domain, and longitudinal-domain arguments used in the release have been written explicitly above.

No failed retrieval of an online companion page is evidence that a manuscript is absent from the registry. No corpus-wide scan, registry installation, or complete bibliographic priority audit is asserted.

Declarations

Authorship. The author field is left blank for submission preparation. The bibliography attributes the source manuscripts to N. Cheung; that attribution does not assign authorship of this draft.

Novelty scope. The contribution is a spin/line/domain interface and its explicit cone implementation, with the square-domain obstruction and scalar non-transfer examples. The general extension methods and underlying spin geometry are established prior work.

Physical scope. All geometric, spin, line, and nonunique operator data remain inputs. No core-selection principle, spinor vacuum, quantum gauge system, coupled state, or metric response is supplied.

Verification. No successful pdflatex execution, machine-checked proof, numerical computation, compiled circuit, independent rebuild, independent review, repository installation, or journal acceptance is asserted. The source is a single-file pdfLaTeX manuscript containing the stated written proofs.

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[11] N. Cheung, *I-B*, I-B-v1.tex, version 1, Enter Calculus Companion. https://enter.grok.me/browse/I-B. Prescribed regular conical host, geometric holonomy, thm:instrument, and prop:not-source. Geometric holonomy is not a chosen spin lift. Companion record; not a Scholar-indexed journal article.

[12] N. Cheung, *I-B-G*, I-B-G-v2.tex, version 2, Enter Calculus Companion. Supplied version-qualified passages: scalar lem:deficiency, ass:product, prop:nonGH, and open item O5. The scalar deficiency theorem is not used as a Dirac theorem. https://enter.grok.me/browse/I-B-G. Companion record; not a Scholar-indexed journal article.

[13] N. Cheung, *I-B-VP*, I-B-VP-v4.tex, version 3, Enter Calculus Companion. Supplied version-qualified open spinor/gauge row and scalar observable passages. The present release supplies only the stated spinor-domain stage of that row. https://enter.grok.me/browse/I-B-VP. Companion record; not a Scholar-indexed journal article.

[14] N. Cheung, *I-B-VP-1*, I-B-VP-1-v4.tex, version 3, Enter Calculus Companion. Supplied thm:diracclosed, thm:diracapex, and thm:nogo. One-point data and contacts, with a disclosed unresolved Spinelly comparison; not a first-order domain classification. https://enter.grok.me/browse/I-B-VP-1. Companion record; not a Scholar-indexed journal article.

[15] N. Cheung, *I-B-VP-2*, I-B-VP-2-v2.tex, version 1, Enter Calculus Companion. Supplied thm:main. Scalar Friedrichs covariance wavefront result; not a spinor or gauge result. https://enter.grok.me/browse/I-B-VP-2. Companion record; not a Scholar-indexed journal article.

[16] N. Cheung, *I-B-VP-3*, I-B-VP-3-v2.tex, version 1, Enter Calculus Companion. Supplied thm:spectrum and thm:threshold. Scalar product-class spectrum and state threshold; not a Dirac mass-shell theorem. https://enter.grok.me/browse/I-B-VP-3. Companion record; not a Scholar-indexed journal article.

[17] N. Cheung, *I-B-VP-4*, I-B-VP-4-v2.tex, version 1, Enter Calculus Companion. Supplied thm:L1, thm:potential, prop:largegauge, and prop:nofeedback. Restricted frozen-current classical response and its completion obstructions, not a constrained quantum Maxwell system. https://enter.grok.me/browse/I-B-VP-4. Companion record; not a Scholar-indexed journal article.

[18] N. Cheung, *I-B-VP-5*, I-B-VP-5-v2.tex, version 1, Enter Calculus Companion. Supplied thm:selection, prop:holonomy, and prop:contactseparation. Scalar core selection and flat-bundle filling distinction; not spin-structure selection. https://enter.grok.me/browse/I-B-VP-5. Companion record; not a Scholar-indexed journal article.

[19] N. Cheung, *I-B-VP-6: Longitudinally non-product self-adjoint extensions on the prescribed conical spacetime*, I-B-VP-6-v2.tex, version 1. Manuscript supplied with this problem. Scalar measurable-relation ledger, spectrum, and boost test. No scalar theorem from it is promoted to a spinor theorem.

https://enter.grok.me/browse/I-B-VP-6. Companion record; not a Scholar-indexed journal article.

[20] N. Cheung, *XI-A-ST*, version 1, supplied manuscript passages. Gauge-fixed identities and a finite report interface on its declared completion. Not the conical Maxwell constraint or a spinor domain. https://enter.grok.me/browse/XI-A-ST. Companion record; not a Scholar-indexed journal article.

[21] N. Cheung, *En09-A*, version 1, Enter Calculus Companion. Supplied complete destructive-clock and relational-extraction passages. https://enter.grok.me/browse/En09-A. The additional hypotheses for relational evolution are not assumed here. Companion record; not a Scholar-indexed journal article.

[22] N. Cheung, *Oai282-A*, version 1, Enter Calculus Companion. Supplied prop:coverage and prop:invisible. https://enter.grok.me/browse/Oai282-A. Finite-probe coverage discipline only. Companion record; not a Scholar-indexed journal article.

[23] N. Cheung, *Black-Hole Information as an Access Problem: Enter, Quantum Recovery, and Complete Residuals*, figshare (2026). DOI: 10.6084/m9.figshare.34335024.v1. https://doi.org/10.6084/m9.figshare.34335024.v1.

[24] N. Cheung, *Operational Wormhole Certificates—Existence, Dimension-Dependent Scaling, Operational Functions, and Composition*, figshare (2026). DOI: 10.6084/m9.figshare.34549575.v1. https://doi.org/10.6084/m9.figshare.34549575.v1.

[25] N. Cheung, *Enter, Quantum Magic, and Quantum Gravity: Readout, Recovery, and the Limits of Geometric Response*, figshare (2026). DOI: 10.6084/m9.figshare.34335582.v1. https://doi.org/10.6084/m9.figshare.34335582.v1.

[26] N. Cheung, *Operational Consistency Does Not Select Lorentzian Geometry*, figshare (2026). DOI: 10.6084/m9.figshare.34441758.v1. https://doi.org/10.6084/m9.figshare.34441758.v1.

[27] N. Cheung, *Realization of Relational Time by Destructive Enter on a Clock Register*, figshare (2026). DOI: 10.6084/m9.figshare.34442907.v1. https://doi.org/10.6084/m9.figshare.34442907.v1.