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I-B-VP-8-Ex Regular Friedrichs thermal one-point functions Wick square, stress tensor, and charged current before any apex extension

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Abstract

For the prescribed four-dimensional ultrastatic cone, an arbitrary fixed flat character, and the massless Friedrichs realization, we construct the regular-host Wick square, stress tensor, and charged current of the centered zero-chemical-potential Bose KMS state at every inverse temperature $\mathrm{B}_T > 0$. The principal contribution is the missing *compact-test-to-coincidence bridge*: weak Euclidean periodization is upgraded to normally convergent differentiated image sums near every regular diagonal point, after an independent check of the local Hadamard subtraction. The current is calculated from the imaginary angular first jet, not inferred from its vacuum value.

The resulting formulas give an explicit positive zeta-function coefficient for the leading thermal Wick correction,

$$\delta F(r) \sim \frac{d_* \zeta(2 + 2\nu_*)}{2\pi^2 \alpha\, \mathrm{B}_T^{2+2\nu_*}} r^{2\nu_*},$$

including the untwisted value $\delta F(0+) = 1/(12\alpha \mathrm{B}_T^2)$. They also give the corresponding signed lowest-mode coefficient of the thermal current, a strictly nonzero longitudinal boost-breaking stress difference at every positive temperature, and convergence of all three regular one-points to their vacuum values in the fixed-operator zero-temperature limit. These estimates provide a controlled local starting point for subsequent observable calculations without presupposing an apex renormalization.

The contribution is a domain-explicit regular-host package, not a claim that thermal cone Green functions, Bose KMS methods, or vacuum-polarization phenomena are new. A separate finite-register argument explains why a supplied report of these coefficients is neither continuum thermometry nor quantum recovery, and why a heat

kernel is not reservoir heat. No apex-contact classification, global thermal wavefront theorem, massive or non-product state construction, backreaction, spacetime-emergence theorem, or inherited gravitational quantum-coefficient admission is asserted.

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Book expansion: observable-calculation companion

This sheet is an **observable-calculation companion**, not another state-construction paper. Local Hadamard subtraction and the rule ``differentiate the complex kernel before taking coincidence" precede the formulae. Reservoir/thermometry remarks are scope framing only (see Book Ledger).

Purpose, background, and proof discipline

The precise extension

The parent I-B-VP-8 constructs the centered massless Friedrichs KMS state on compact regular tests, proves its stronger thermal infrared estimate, identifies weak Euclidean periodization, and establishes a fixed-operator zero-temperature limit [1]. Its release explicitly stops before an independent one-point theorem. The present manuscript fills that regular-host gap.

The observables are

$$F_{\mathrm{B}_T} = \langle \Phi^\dagger \Phi \rangle_{\mathrm{ren},\mathrm{B}_T}, \qquad T^\mu{}_{\nu,\mathrm{B}_T} = \langle T^\mu{}_\nu \rangle_{\mathrm{ren},\mathrm{B}_T}, \qquad J_{\varphi,\mathrm{B}_T} = \langle J_\varphi \rangle_{\mathrm{ren},\mathrm{B}_T},$$

on $r > 0$. The field is one charged complex scalar. The curvature coupling $\xi \in \mathbb{R}$ occurs in the declared stress improvement; it does not alter the regular massless operator, because the regular metric is flat.

The vacuum formulas recorded in I-B-VP and I-B-VP-0, and the thermal formulas recorded in I-B-VP-1, are documentary comparisons [8, 9, 10]. They are not analytical premises. In particular, no apex extension, contact count, or distributional Ward identity from I-B-VP-1 is used below.

We re-establish the compact-test infrared gate, the unstable-product obstruction, and the field-equation and commutator statements needed to identify the state. The additional work is then local: prove differentiated image-sum convergence, verify the subtraction, and take coincidence limits.

Standing input sentence. The cone, the extension, the character, the centered Bose prescription, and the temperature are inputs. I-B, I-B-G, I-B-G-S, I-B-VP, I-B-VP-0, I-B-VP-1, and the OpenAI catalogue are not

premises. The Philosophy of Enter is an introductory map and is not a registered premise.

Established literature and the scope of novelty

Finite-temperature scalar Green functions and vacuum polarization on cosmic-string geometries have an established literature. Guimarães treats thermal vacuum polarization in the presence of a magnetic-flux cosmic string; Frolov, Pinzul, and Zelnikov calculate finite-temperature stress on a cone [24, 25]. Quasifree equilibrium states and Wick rotation are likewise standard subjects [26]. The local composite-field framework belongs to the established theory of Wick polynomials [27].

The contribution claimed here is narrower than historical priority for any of those subjects. It is a self-contained compatibility package for the exact operator, test class, charged normalization, and subtraction specified below. Its useful findings are the proved passage from weak periodization to regular coincidence jets, the explicit lowest-mode thermal coefficients, and a stress formula that requires no integration constant selected at the removed apex. These are the mathematical improvements over merely quoting a closed thermal kernel.

No exhaustive priority claim is made for the resulting zeta-function asymptotics or current formula. Their significance here is that they are derived from the specified KMS covariance, with the necessary interchange of limits justified.

Enter, spacetime Enter, and the shredder reading

Cheung's Enter calculus distinguishes destructive finite-register readout from a quantum-return interface. Its operational characterization is relative to a stated strategy class, not a derivation of measurement semantics without hypotheses [2, 4]. A measured quantum name is consumed at that interface and a classical code is returned. This is a statement about access and output type; it is not a claim that all physical carriers of information are annihilated.

The prescribed-spacetime use of Enter adds a finite interface to a given geometric host [5]. Consuming a finite clock encoding does not consume the geometric time axis, the conical worldsheet, or the future cone. Nothing in a

finite readout derives the metric used in this paper.

The Philosophy of Enter organizes these distinctions as a refusal to identify unlike constructions [3]. The shredder reading of I-C makes the corresponding separation between loss of accessible quantum input, persistence of a classical record, and thermodynamic erasure [16, 17, 18]. Section proves the finite statements actually used here. They do not supply an acquisition instrument for the continuum field.

This background motivates a proof discipline rather than a field equation. A report is not a state. A state is not a detector. A heat semigroup is not heat delivered to a bath. A regular stress expectation is not a sourced metric.

Corrections to tempting shortcuts

Four repairs are essential.

First, weak convergence of covariances does not alone permit differentiation and restriction to the diagonal. The local normal-convergence estimates below supply that missing step, including for the zero-temperature limit.

Second, realifying a charged field is not permission to replace its complex-sector kernel by its pointwise real part. Such replacement destroys the angular first jet and therefore the current.

Third, the thermal stress is calculated directly from local second jets. It is not reconstructed by integrating radial conservation from $r = 0$, which could conceal an unproved boundary normalization.

Fourth, the classical-record obstruction has a fidelity ceiling $1/d$ and, in half-diamond distance, an error lower bound $1 - 1/d$. Those two numbers must not be interchanged.

Host, Friedrichs operator, and the infrared gate

Prescribed geometry and bundle

Work in units $c = \hbar = k_{\mathrm{B}} = 1$. Let

$$M = \mathbb{R}_t \times N, \qquad N = (0,\infty)_r \times (\mathbb{R}/2\pi\mathbb{Z})_\varphi \times \mathbb{R}_z, \qquad g = -\mathrm{d}t^2 + \mathrm{d}r^2 + \alpha^2 r^2 \mathrm{d}\varphi^2 + \mathrm{d}z^2,$$

where $0 < \alpha < 1$. The removed worldsheet would be $\mathscr{W} = \mathbb{R}_t \times \{o\} \times \mathbb{R}_z$, but it is not part of the domain of the observables constructed here.

The regular spacetime is not globally hyperbolic. For example, choose two events on a common radial ray at radius $r_0 > 0$, with time separation greater than $2r_0$. Their causal diamond contains events with radius tending to zero. That sequence has no convergent subsequence in M. The operator prescription below supplies an evolution; it does not turn the regular slices into geometric Cauchy surfaces. This is the same geometric distinction documented in I-B-G [6].

Fix $0 \le \beta < 1$ and the flat Hermitian line bundle L_β represented on the angular cover by

$$u(r, \varphi + 2\pi, z) = e^{2\pi i\beta} u(r, \varphi, z).$$

In a lifted parallel frame the covariant derivative is ordinary differentiation. The Hilbert space and measure are

$$\mathscr{H}_\beta = L^2(N, L_\beta; \mathrm{d}V), \qquad \mathrm{d}V = \alpha r \, \mathrm{d}r \, \mathrm{d}\varphi \, \mathrm{d}z.$$

Inner products are antilinear in their first argument.

Let B_F be the Friedrichs realization of

$$-\Delta_\beta = -\partial_r^2 - r^{-1}\partial_r - (\alpha^2 r^2)^{-1}\partial_\varphi^2 - \partial_z^2$$

initially defined on $C_c^\infty(N, L_\beta)$. Thus $B_F \ge 0$. Write $h = B_F^{1/2}$.

Set

$$q_n = n + \beta, \qquad \nu_n = \frac{|q_n|}{\alpha}, \qquad \rho = \nu_* = \min_{n \in \mathbb{Z}} \nu_n, \qquad \mathscr{I}_* = \{n : \nu_n = \rho\}, \qquad d_* = |\mathscr{I}_*|.$$

There is one lowest mode, except at $\beta = 1/2$, where $d_* = 2$. At $\beta = 0$, $\rho = 0$.

Heat kernel and compact-test estimate

Lemma (Friedrichs heat kernel). *Relative to* $\mathrm{d}V$*, the heat kernel is*

$$K_s(x,x') = \frac{e^{-(z-z')^2/(4s)}}{\sqrt{4\pi s}} \frac{e^{-(r^2+r'^2)/(4s)}}{4\pi\alpha s} \sum_{n\in\mathbb{Z}} e^{iq_n(\varphi-\varphi')} I_{\nu_n}\left(\frac{rr'}{2s}\right).$$

The series converges locally normally for $s>0$ and regular spatial arguments.

Proof. The angular functions $(2\pi\alpha)^{-1/2}e^{iq_n\varphi}$, the longitudinal Fourier transform, and the radial Hankel transform with $J_{\nu_n}(pr)$ diagonalize the Friedrichs form. The radial Gaussian identity is

$$\int_0^\infty e^{-sp^2} J_\nu(pr) J_\nu(pr') p\,dp = \frac{1}{2s} e^{-(r^2+r'^2)/(4s)} I_\nu\left(\frac{rr'}{2s}\right).$$

This standard Bessel identity and the defining series for I_ν are recorded in the DLMF [28]. They give with the displayed normalization.

For completeness, the relevant convergence estimate follows directly from

$$I_\nu(y) = \sum_{j=0}^{\infty} \frac{(y/2)^{2j+\nu}}{j!\,\Gamma(j+\nu+1)}.$$

For $\nu \ge 0$, $\Gamma(j+\nu+1)/\Gamma(\nu+1) = \prod_{\ell=1}^{j}(\nu+\ell) \ge 1$. Consequently

$$I_\nu(y) \le \frac{(y/2)^\nu}{\Gamma(\nu+1)} e^{y^2/4}$$

on bounded nonnegative y-intervals. Since ν_n grows linearly in $|n|$, this is summable after multiplication by any fixed polynomial in ν_n. The same argument applies to derivatives on compact regular sets.

Proposition (Compact thermal infrared gate). *Let $a_\rho = 3/2+\rho$. For every compact $K\subset N$, there are constants $C_K, s_K>0$ such that*

$$0 \le \langle v, e^{-sB_F} v\rangle \le C_K s^{-a_\rho} \|v\|_{L^1}^2, \qquad s \ge s_K, \quad \operatorname{supp} v \subset K.$$

For $n\in\mathcal{I}_*$, put

$$m_n(v) = \int_N r^\rho e^{-iq_n\varphi} v(x)\,dV(x).$$

Then

$$\lim_{s\to\infty} s^{a_\rho}\langle v, e^{-sB_F}v\rangle = \frac{\sum_{n\in\mathcal{I}_*}|m_n(v)|^2}{8\pi^{3/2}\alpha\,4^\rho\Gamma(1+\rho)}.$$

In particular, B_F has no L^2 zero mode, compact smooth tests belong to $\operatorname{Dom} B_F^{-1/2}$*, and*

$$\int_{(0,\varepsilon]} \lambda^{-1}\,\mathrm{d}\mu_v(\lambda) \le C_K \mathrm{lVertv}\|_{L^1}^2 \varepsilon^{1/2+\rho}$$

for sufficiently small $\varepsilon > 0$, where $\mu_v(S) = \langle v, 1_S(B_F)v\rangle$.

Proof. On $K \times K$, the argument $rr'/(2s)$ tends uniformly to zero. The preceding majorant yields

$$|K_s(x,x')| \le C_K s^{-3/2-\rho}.$$

Integration gives . For a lowest mode,

$$s^\rho I_\rho(rr'/(2s)) \to \frac{(rr'/4)^\rho}{\Gamma(1+\rho)}.$$

All higher-mode contributions vanish after that rescaling. Dominated convergence in the angular sum and then in the two spatial variables proves .

The strong limit of e^{-sB_F} as $s \to \infty$ is the zero-eigenvalue projection. The heat bound makes it zero on a dense test subspace, hence everywhere.

For $q > 0$, the spectral theorem and Tonelli give the extended identity

$$\mathrm{lVertB_}F^{-q/2}v\|^2 = \frac{1}{\Gamma(q)}\int_0^\infty s^{q-1}\langle v, e^{-sB_F}v\rangle\,\mathrm{d}s.$$

Small heat times are controlled by contraction, and large heat times by . Thus the right side is finite for $q < a_\rho$. Taking $q = 1$ proves the thermal domain statement. If a lowest moment is nonzero, also shows that the threshold $q < a_\rho$ is optimal, with logarithmic divergence at equality.

Finally,

$$\mu_v([0,\lambda]) \le e\,\langle v, e^{-B_F/\lambda}v\rangle \le C_K\|v\|_{L^1}^2\lambda^{a_\rho}.$$

Stieltjes integration by parts gives

$$\int_{(0,\varepsilon]} \lambda^{-1} \mathrm{d}\mu_{v} = \frac{\mu_{v}([0,\varepsilon])}{\varepsilon} + \int_{0}^{\varepsilon} \frac{\mu_{v}([0,\lambda])}{\lambda^{2}}\,\mathrm{d}\lambda,$$

where the lower boundary vanishes because $a_{\rho} > 1$. This proves .

The ground covariance requires only the weaker inverse-power gate $\mathrm{Dom}\, B_{F}^{-1/4}$. At positive temperature, the factor $n_{\mathrm{B}_{T}}(\omega) = (e^{\mathrm{B}_{T}\omega} - 1)^{-1}$ behaves as $(\mathrm{B}_{T}\omega)^{-1}$, so the field variance has an $h^{-2} = B_{F}^{-1}$ infrared weight. The stronger gate just proved is therefore an actual thermal obligation, not optional notation.

State identification and the unstable-product wall

Realification and the centered Bose state

Use the underlying real Hilbert space

$$\mathscr{H}_{\mathbb{R}} = (\mathscr{H}_{\beta}, \mathrm{Re}\,\langle \cdot, \cdot \rangle).$$

This is the real rank-two bundle associated with L_{β}. The complex field is reconstructed locally as $\Phi = (\varphi_{1} + \mathrm{i}\varphi_{2})/\sqrt{2}$. Consequently $F = \langle \Phi^{\dagger} \Phi \rangle$ has the one-real-component covariance normalization, whereas the complex stress is twice the corresponding real-scalar stress.

For real phase-space vectors $F = (q, p)$, use

$$\sigma(F, G) = (q, p')_{\mathbb{R}} - (p, q')_{\mathbb{R}},$$

and

$$\mu_{\mathrm{B}_{T}}(F, F) = \frac{1}{2}(q, h^{-1} \coth(\mathrm{B}_{T} h/2) q)_{\mathbb{R}} + \frac{1}{2}(p, h \coth(\mathrm{B}_{T} h/2) p)_{\mathbb{R}}.$$

Take the invariant linear space generated by smooth compact phase data, their time translates, and compact-time smooth integrals of those translates. All displayed forms are finite there. Indeed, at zero frequency the q-term requires $h^{-1} q \in \mathscr{H}_{\mathbb{R}}$, proved in Proposition , and the p-multiplier is bounded. At high frequency, compact smooth

tests belong to every positive power domain of B_F.

The evolution on test phase data is

$$S_t(q,p) = (\cos(ht)q - h\sin(ht)p, h^{-1}\sin(ht)q + \cos(ht)p).$$

The apparent inverse in the sine quotient is harmless at zero.

Proposition (The specified KMS state). *The functional*

$$\omega_{\mathrm{B}_T}(W(F)) = \exp\left[-\frac{1}{2}\mu_{\mathrm{B}_T}(F,F)\right]$$

defines a positive regular centered quasifree state. It is invariant under S_t and satisfies the bounded-observable KMS strip condition at inverse temperature B_T.

In the complex-sector kernel notation its two-point function is

$$W_{\mathrm{B}_T}(t-s) = \frac{(1+n_{\mathrm{B}_T}(h))e^{-\mathrm{i}h(t-s)} + n_{\mathrm{B}_T}(h)e^{\mathrm{i}h(t-s)}}{2h}.$$

This is the centered zero-chemical-potential Friedrichs state specified in I-B-VP-8.

Proof. Complexify $\mathscr{H}_{\mathbb{R}}$, and set

$$k(q,p) = \frac{h^{-1/2}q + \mathrm{i}h^{1/2}p}{\sqrt{2}}.$$

Then $2\mathrm{Im}\,\langle kF, kG\rangle = \sigma(F,G)$. With $n = n_{\mathrm{B}_T}(h)$, define the doubled map

$$k_{\mathrm{B}_T}F = ((1+n)^{1/2}kF, n^{1/2}\overline{kF}).$$

Its squared norm is $\mu_{\mathrm{B}_T}(F,F)$, and its symplectic form is still σ, because the second conjugate summand subtracts the n-contribution from the imaginary part. Pulling back the Fock Weyl functional along this map gives , proving positivity and regularity. It also proves finite second moments.

Equation gives $k(S_tF) = e^{\mathrm{i}ht}kF$. Hence μ_{B_T} and σ are invariant. The two-point formula follows by polarization.

Here are the strip details. First cut the spectral support to $\varepsilon \le h \le R$. The scalar identity

$$n_{\mathrm{B}_T}(\omega)e^{\mathrm{B}_T\omega} = 1 + n_{\mathrm{B}_T}(\omega)$$

interchanges the two orderings at the upper boundary of the strip $0 \le \operatorname{Im} z \le \mathrm{B}_T$. The Gaussian Weyl-product formula therefore gives the KMS boundary identity for Weyl generators and their finite products. The spectral integrals are holomorphic inside the strip. They are bounded in the horizontal direction; their boundary values are expectations of products of unitaries. The maximum principle gives the corresponding strip bound.

Remove the cutoffs in the μ_{B_T}-norm. The infrared estimate and the positive-power bounds make the cutoff vectors converge. Cauchy–Schwarz for the positive state gives convergence of bounded Weyl correlations on both boundaries. The strip bounds and uniqueness of the holomorphic limit pass the boundary identity to the limit. Finite linear combinations and norm approximation then give the bounded-observable strip condition.

The universal Weyl time action need not be norm continuous. The assertion is the bounded-correlation KMS condition, or equivalently the KMS condition in the regular GNS von Neumann realization, where the implemented time group is strongly continuous. No norm-continuity claim for universal Weyl generators is needed.

The construction is the standard doubled quasifree mechanism; the domain-explicit argument above, rather than a general globally hyperbolic existence theorem, supplies its application to this host. Compare the general equilibrium discussion in [26].

Compact spacetime tests

Let $\mathcal{T} = C_c^\infty(M, E_\beta)$, with its usual fixed-support test-section topology. For $f \in \mathcal{T}$, define

$$q_f = \int_{\mathbb{R}} \cos(ht) f(t)\,\mathrm{d}t, \qquad p_f = \int_{\mathbb{R}} h^{-1} \sin(ht) f(t)\,\mathrm{d}t.$$

These are the phase data of the smeared field.

Proposition (Field equation and selected commutator). *The pullback of ω_{B_T} to compact spacetime tests is continuous, regular, and has finite second moments. For $P = \partial_t^2 + B_F$, it satisfies the field equation and*

$$\sigma_{B_F}(f,g) = -\int_{\mathbb{R}^2} (f(t), h^{-1}\sin(h(t-s))g(s))_{\mathbb{R}}\, \mathrm{d}t\, \mathrm{d}s.$$

Thus the field commutator is $[\varphi(f), \varphi(g)] = \mathrm{i}\sigma_{B_F}(f,g)I$*, in the extension-selected sense.*

Proof. On a fixed compact spacetime support, Proposition bounds $h^{-1}f(t)$ by fixed-support test seminorms. The inequalities

$$\frac{1}{2h}\coth(\mathrm{B}_T h/2) \le C_{\mathrm{B}_T}(h^{-1} + h^{-2}), \qquad \frac{h}{2}\coth(\mathrm{B}_T h/2) \le C_{\mathrm{B}_T}(1+h)$$

control the phase covariance. The bounded multiplier $h^{-1}\sin(ht)$ has absolute value at most $|t|$. These bounds, with integration in t, prove continuity and finite second moments.

Integrating twice by parts in time gives $q_{Pf} = p_{Pf} = 0$; compact time support removes the endpoint terms. The spatial differential expression preserves compact regular tests and agrees there with B_F. This proves the field equation.

Finally,

$$\cos(ht)\frac{\sin(hs)}{h} - \frac{\sin(ht)}{h}\cos(hs) = -\frac{\sin(h(t-s))}{h}.$$

Substitution into the phase symplectic form gives . This proof makes no assertion that an ordinary compact-spacetime Green exact sequence has been established.

The no-go is retained locally, not imported

Theorem (Unstable massless product obstruction). *Let*

$$B = T_\Lambda \otimes I + I \otimes (-\partial_z^2)$$

be a semibounded massless product extension. If T_Λ *has an eigenvalue* $-\kappa^2 < 0$*, there is no time-translation-invariant state on its compact regular test CCR algebra having finite second moments for every compact test. In particular, no KMS state with that property exists.*

Proof. The bound channel has dispersion $k_z^2 - \kappa^2$. Thus some interval $[-b, -a]\mathrm{Subset}(-\infty, 0)$, $a > 0$, has nonzero

full spectral projection. By density, a compact smooth spatial section v has positive spectral measure on that interval.

Choose nonnegative nonzero $\chi \in C_c^\infty([-A,A])$ and put $f(t) = \chi(t)v$. The selected propagator is the spectral multiplier

$$w_u(\lambda) = \begin{cases} \sin(u\sqrt{\lambda})/\sqrt{\lambda}, & \lambda > 0, \\ u, & \lambda = 0, \\ \sinh(u\sqrt{-\lambda})/\sqrt{-\lambda}, & \lambda < 0. \end{cases}$$

For a large translation $u > 2A$, a negative spectral value $\lambda = -\omega^2$ contributes, with one common sign, the quantity

$$\frac{\sinh(\omega u)}{\omega}\Big(\int \chi(s)e^{\omega s}\mathrm{d}s\Big)\Big(\int \chi(s)e^{-\omega s}\mathrm{d}s\Big)$$

to the commutator pairing. On $[-b,-a]$, this grows at least as $ce^{\sqrt{a}\,u}$. All negative spectral contributions have the same sign. The nonnegative contribution has magnitude at most $C(u + 2A)\|v\|^2$, since $|\sin(\tau\sqrt{\lambda})/\sqrt{\lambda}| \le |\tau|$. Therefore $|\sigma_B(f_u, f)|$ is unbounded.

In an invariant state with the stated finite moments, the weak field CCR and Cauchy–Schwarz instead give

$$|\sigma_B(f_u, f)| \le 2\|\varphi(f_u)\Omega\|\|\varphi(f)\Omega\| = 2\|\varphi(f)\Omega\|^2.$$

The weak CCR here follows by differentiating Weyl relations; it needs the field vectors, not a common domain for products of two unbounded fields. This is a contradiction.

The product hypothesis is substantive. A momentum-dependent boundary law can change the relation between a transverse bound value and the full spectrum. No no-go or existence result for the non-product operators of I-B-VP-6 is inferred from this proof [15].

Euclidean kernel and the coincidence bridge

A kernel derived from the chosen operator

Let $u > 0$. The vacuum Euclidean covariance is

$$G_E(u) = \frac{e^{-uh}}{2h}.$$

Its heat representation is

$$G_E(u;x,x') = \int_0^\infty \frac{e^{-u^2/(4s)}}{\sqrt{4\pi s}} K_s(x,x')\,\mathrm{d}s.$$

The scalar identity follows by an elementary Gaussian integral, and the operator identity follows spectrally.

Write $\theta = \varphi - \varphi'$, $v = z - z'$, and define

$$\cosh\eta = \frac{r^2 + r'\hat{}2 + u^2 + v^2}{2rr'}.$$

Inserting , or taking the Laplace transform of its Bessel series, gives

$$G_E(u;x,x') = \frac{1}{8\pi^2 \alpha rr' \sinh\eta} \sum_{n\in\mathbb{Z}} e^{\mathrm{i}q_n\theta} e^{-\nu_n\eta}.$$

For example, after the change $y = rr'/(2s)$, the required Laplace identity is

$$\int_0^\infty e^{-y\cosh\eta} I_\nu(y)\,\mathrm{d}y = \frac{e^{-\nu\eta}}{\sinh\eta}.$$

It can also be checked by termwise integration of the I_ν-series for $\cosh\eta > 1$. All interchanges are absolutely convergent for $u > 0$ on the regular compact sets in question.

The angular sum has the elementary form

$$\mathscr{S}_\beta(a,\theta) = \frac{e^{-\beta a + \mathrm{i}\beta\theta}}{1 - e^{-a+\mathrm{i}\theta}} + \frac{e^{-(1-\beta)a - \mathrm{i}(1-\beta)\theta}}{1 - e^{-a-\mathrm{i}\theta}}, \qquad a > 0.$$

This includes $\beta = 0$. At $\theta = 0$,

$$S_\beta(a) := \sum_n e^{-|q_n|a} = \frac{\cosh((1/2-\beta)a)}{\sinh(a/2)}.$$

Thus, at equal radii and equal angle,

$$g(w;r) = \frac{S_\beta(\eta/\alpha)}{8\pi^2 \alpha r^2 \sinh\eta}, \qquad \cosh\eta = 1 + \frac{w}{2r^2}, \qquad w > 0.$$

This derives, rather than assumes, the documentary closed form called eq:Gtherm in I-B-VP-1 [10].

Weak periodization first

Proposition (Weak Euclidean periodization). *For* $0 < \tau < \mathrm{B}_T$*, on compact spatial tests,*

$$C^E_{\mathrm{B}_T}(\tau) = \frac{e^{-\tau h} + e^{-(\mathrm{B}_T - \tau)h}}{2h(1 - e^{-\mathrm{B}_T h})}$$

$$= \sum_{m \in \mathbb{Z}} \frac{e^{-|\tau + m\mathrm{B}_T|h}}{2h}.$$

The sum is absolutely convergent as a weak pairing, locally uniformly for τ *in compact subintervals of* $(0, \mathrm{B}_T)$.

Proof. Analytic continuation of gives . For each $\omega > 0$, two geometric series give

$$\sum_{m \in \mathbb{Z}} \frac{e^{-|\tau + m\mathrm{B}_T|\omega}}{2\omega} = \frac{e^{-\tau\omega} + e^{-(\mathrm{B}_T - \tau)\omega}}{2\omega(1 - e^{-\mathrm{B}_T\omega})}.$$

The right side is $O(\omega^{-2})$ at zero. Proposition therefore controls it on compact tests. For diagonal pairings, the summands are positive forms, so Tonelli applies. Mixed pairings follow by Cauchy–Schwarz for each positive summand and then for the sum. The same majorant gives local uniformity.

This is precisely a two-point, compact-test statement. No coincidence limit or apex estimate was used to construct the state or prove the identity.

The differentiated local upgrade

Lemma (Large-separation jet bound). *Fix compact regular spatial sets and an integer* $k \geq 0$*. For sufficiently large real* u*, every endpoint derivative of total order at most* k *of* $G_E(u; x, x')$ *satisfies*

$$|\partial^\gamma G_E(u; x, x')| \leq C_{K,k} u^{-2-2\rho}.$$

The same bound holds when u *is replaced by* $u + \zeta$*, with* ζ *in a fixed bounded complex neighborhood. Constants depend on the fixed parameters and compact sets. They are not asserted uniform as* α, β *vary.*

Proof. For large u, uniformly on the stated sets,

$$e^{-\eta} = O(rr' u^{-2}), \qquad \frac{1}{rr' \sinh \eta} = O(u^{-2}).$$

Consequently one angular term is bounded by $Cu^{-2}(C_K/u^2)^{\nu_n}$. A fixed number of radial or angular derivatives introduces only bounded inverse powers of r, r', polynomial factors in ν_n, q_n, and bounded derivatives of η. Longitudinal or u-derivatives do not worsen the displayed large-u power.

Factor out $u^{-2-2\rho}$. For sufficiently large u, the remaining angular series is majorized by

$$C\sum_n (1+\nu_n)^{N_k} c^{\nu_n-\rho}, \qquad 0<c<1.$$

It converges. The explicit formula for η, with the branch $\operatorname{Re}\eta>0$, gives the same estimates for bounded complex perturbations of u. Termwise differentiation is therefore justified and proves .

Theorem (Regular coincidence bridge). *Near every regular spacetime diagonal point, $W_{\mathrm{B}_T}-W_{\mathrm{vac}}$ has a smooth representative. Its Euclidean representative for $|\tau|<\mathrm{B}_T$, locally near $\tau=0$, is*

$$R^E_{\mathrm{B}_T}(\tau;x,x') = \sum_{m=1}^{\infty}\{G_E(m\mathrm{B}_T+\tau;x,x')+G_E(m\mathrm{B}_T-\tau;x,x')\}.$$

Every finite-order endpoint derivative is obtained termwise. The Lorentzian representative near equal times is

$$R_{\mathrm{B}_T}(t-t';x,x') = \sum_{m=1}^{\infty}\{G_E(m\mathrm{B}_T+\mathrm{i}(t-t');x,x')+G_E(m\mathrm{B}_T-\mathrm{i}(t-t');x,x')\}.$$

In particular,

$$R^E_{\mathrm{B}_T}(0;x,x') = 2\sum_{m=1}^{\infty} G_E(m\mathrm{B}_T;x,x').$$

Proof. Choose a neighborhood of the regular diagonal small enough that the finitely many nearest nonzero images stay at positive Euclidean separation. Those terms are smooth, with holomorphic continuation in a small complex time neighborhood.

For the remaining images, Lemma gives the summable derivative majorant

$$C_{K,k}\sum_{m\ge m_0}(m\mathrm{B}_T)^{-2-2\rho}.$$

The Weierstrass test proves local normal convergence of every differentiated series, both in and in .

It remains to identify this smooth kernel with the covariance difference rather than with an unrelated sum. On compact tests and initially inside the Euclidean strip, Proposition gives that identification. Alternatively, summing the positive-frequency operator series in gives

$$\sum_{m \geq 1} \frac{e^{-m\mathrm{B}_T h}}{h} \cos\left(h(t - t')\right) = \frac{n_{\mathrm{B}_T}(h)}{h} \cos\left(h(t - t')\right),$$

which is exactly minus the vacuum term. The infrared gate justifies this operator identity weakly. Normal convergence then identifies its local smooth representative and all the required jets.

The compact-test infrared estimate controls the low-frequency meaning of the periodization. The jet estimate is the additional local input that permits coincidence. Neither statement replaces the other.

The local Hadamard subtraction is checked again

Choose a simply connected regular coordinate neighborhood and parallelize the flat bundle there. Its local flat Euclidean singular kernel is

$$H_E^{\mathrm{sing}} = \frac{1}{4\pi^2(u^2 + v^2 + r^2 + r'\wedge 2 - 2rr'\cos(\alpha\theta))} = \frac{1}{8\pi^2 rr'(\cosh\eta - \cos(\alpha\theta))}.$$

The corresponding Lorentzian boundary value is obtained by $u = \varepsilon + \mathrm{i}(t - t')$. No logarithmic curvature term occurs: both the regular metric curvature and the flat connection curvature vanish.

Lemma (Analytic regular vacuum remainder). *The difference $G_E - H_E^{\mathrm{sing}}$ is real analytic in regular local coordinates near the Euclidean diagonal. Its Lorentzian continuation is smooth near the regular spacetime diagonal. Thus the same local Hadamard subtraction works for W_{vac} and W_{B_T}.*

Proof. Use , with $a = \eta/\alpha$. Near $(\eta, \theta) = (0, 0)$, its only local pole divisors are $\eta = \pm\mathrm{i}\alpha\theta$. At either divisor, the numerator of the corresponding geometric term equals one. The residues of

$$\frac{\mathscr{S}_\beta(\eta/\alpha, \theta)}{\alpha\sinh\eta} \quad \text{and} \quad \frac{1}{\cosh\eta - \cos(\alpha\theta)}$$

therefore agree.

More explicitly, put both expressions over the local common denominator obtained from the factors $\eta - i\alpha\theta$ and $\eta + i\alpha\theta$. Residue cancellation makes the analytic numerator divisible by each factor, hence by their product. At their intersection, the apparent additional factor $1/\sinh\eta$ is removable: the geometric expression $\mathscr{S}_\beta(a,\theta)$, analytically continued away from its two divisors, is odd in a. The quotient by $\sinh\eta$, and the subtracted difference, are even in η. The remainder is consequently analytic in η^2, θ.

Finally, η^2 is analytic in

$$\cosh\eta - 1 = \frac{(r - r')^2 + u^2 + v^2}{2rr'}$$

near zero. This proves analyticity in ordinary local coordinates. Continuation in the small complex time neighborhood gives a smooth Lorentzian remainder.

Theorem adds a smooth thermal difference. Hence no temperature-dependent local subtraction is needed.

This verifies the subtraction itself rather than using a vacuum sibling as a premise. It is a local Hadamard statement. No global wavefront relation, including the treatment of diffractive fronts, is proved here.

Regular one-point functions

Point-split definitions and normalization

Let K be the smooth local remainder after subtracting the Lorentzian kernel corresponding to . Brackets denote coincidence in a local parallel frame, and parallel transport is understood when comparing endpoint indices.

Define

$$F[K] = [K],$$

$$J_\mu[K] = i[(D_\mu - D_{\mu'})K],$$

$$A_{\mu\nu}[K] = \mathrm{Re}\,[D_\mu D_{\nu'} K].$$

The Hermitian endpoint property makes F and J_μ real and $A_{\mu\nu}$ symmetric. For one complex scalar, set

$$T_{\mu\nu}[K] = 2A_{\mu\nu}[K] - g_{\mu\nu}A^{\lambda}{}_{\lambda}[K] + 2\xi(g_{\mu\nu}\mathrm{Box} - \nabla_{\mu}\nabla_{\nu})F[K].$$

The current is in unit-charge normalization. For electric charge e, the physical electric current is eJ_{μ}.

Because K is a smooth bisolution,

$$A^{\lambda}{}_{\lambda}[K] = \frac{1}{2}\mathrm{Box}\,F[K].$$

Thus

$$T^{\mu}{}_{\nu}[K] = 2A^{\mu}{}_{\nu}[K] + (2\xi - \tfrac{1}{2})\delta^{\mu}{}_{\nu}\mathrm{Box}\,F[K] - 2\xi\nabla^{\mu}\nabla_{\nu}F[K].$$

These definitions fix the finite normalization. In particular, no additional cosmological constant term is added, and the locally flat untwisted vacuum is normalized to zero.

Explicit thermal sums

Write $\delta X = X_{\mathrm{B}_T} - X_{\mathrm{vac}}$, and let

$$w_m = m^2\mathrm{B}_T^2, \qquad \eta_m = 2\operatorname{arsinh}\frac{m\mathrm{B}_T}{2r}, \qquad a_m = \frac{\eta_m}{\alpha}, \qquad \mathscr{L}_r = \frac{\mathrm{d}^2}{\mathrm{d}r^2} + \frac{1}{r}\frac{\mathrm{d}}{\mathrm{d}r}.$$

For the angular sums introduce

$$A_{\beta}(a) := \sum_{n\in\mathbb{Z}} q_n e^{-|q_n|a} = -\frac{\mathrm{d}}{\mathrm{d}a}\frac{\sinh((1/2-\beta)a)}{\sinh(a/2)},$$

$$Q_{\beta}(a) := \sum_{n\in\mathbb{Z}} q_n^2 e^{-|q_n|a} = S_{\beta}''(a).$$

All derivatives of g below hold r fixed.

Define

$$f(r) = 2\sum_{m\geq 1} g(w_m; r),$$

$$U(r) = 2\sum_{m\geq 1} (2g_w(w_m; r) + 4w_m g_{ww}(w_m; r)),$$

$$Z(r) = 4\sum_{m\geq 1} g_w(w_m; r),$$

$$V(r) = \frac{1}{4\pi^2\alpha^3 r^4}\sum_{m\ge 1}\frac{Q_\beta(a_m)}{\sinh\eta_m},$$

$$D(r) = \frac{1}{2}\mathscr{L}_r f + U + Z - V.$$

Theorem (Regular thermal Wick, stress, and current package). *For every fixed* $0 < \alpha < 1$, $0 \le \beta < 1$, $\mathrm{B}_T > 0$, *and* $\xi \in \mathbb{R}$, *the locally subtracted coincidence limits exist on* $r > 0$. *Their thermal corrections are*

$$\boxed{\delta F = f,}$$

$$\boxed{\delta T^{\mu}{}_{\nu} = 2\mathrm{diag}\,(-U, D, V, -Z) + (2\xi - \tfrac{1}{2})(\mathscr{L}_r f)\delta^{\mu}{}_{\nu} - 2\xi\,\mathrm{diag}\,(0, f'', f'/r, 0),}$$

in the order (t, r, φ, z), *and*

$$\boxed{\delta J_\varphi = -\frac{1}{2\pi^2\alpha r^2}\sum_{m\ge 1}\frac{A_\beta(a_m)}{\sinh\eta_m}, \qquad \delta J_t = \delta J_r = \delta J_z = 0.}$$

All displayed sums and their regular radial derivatives converge locally normally.

The complete one-points are obtained by adding the vacuum expressions in Proposition . They are stationary, longitudinally translation invariant, and rotation invariant. The stress is diagonal. The current is azimuthal, conserved, and independent of ξ.

Proof. Existence and termwise differentiation follow from Theorem and Lemma . Equation is the regular diagonal of .

For the time jet, the two Euclidean images have arguments $m\mathrm{B}_T \pm \tau$. Since g depends on their squares,

$$[\partial_\tau^2 R^E_{\mathrm{B}_T}] = 2\sum_{m\ge 1}(2g_w + 4w_m g_{ww}) = U.$$

For the longitudinal difference $v = z - z'$,

$$[\partial_v^2 R^E_{\mathrm{B}_T}] = 4\sum_{m\ge 1} g_w = Z.$$

Lorentzian continuation and differentiation in both endpoints give

$$A^t{}_t = -U, \qquad A^z{}_z = -Z.$$

The angular two-endpoint derivative gives $A^{\varphi}_{\varphi} = V$.

For clarity, the radial component is not fixed by integrating a conservation law from the apex. Let $D = [\partial_r \partial_{r'} R^E_{\mathrm{B}_T}]$.

Endpoint symmetry gives $[\partial_r R^E_{\mathrm{B}_T}] = \dot{f}/2$. The Euclidean field equation in the first endpoint implies

$$[\partial_r^2 R^E_{\mathrm{B}_T}] = -\frac{\dot{f}}{2r} + V - U - Z.$$

Differentiating the diagonal twice gives $\ddot{f} = 2[\partial_r^2 R^E_{\mathrm{B}_T}] + 2D$. Therefore

$$D = \tfrac{1}{2}\mathscr{L}_r f + U + Z - V,$$

which proves . For a radial scalar,

$$\nabla^{\mu}\nabla_{\nu} f = \mathrm{diag}\,(0, \ddot{f}, \dot{f}/r, 0).$$

Substitution into proves .

For the current, keep the complex kernel:

$$\partial_\theta G_E = \frac{\mathrm{i}}{8\pi^2 \alpha r^2 \sinh \eta} \sum_n q_n e^{-|q_n|\eta/\alpha}$$

at equal radii. The operation $\mathrm{i}(\partial_\varphi - \partial_{\varphi'}) = 2\mathrm{i}\partial_\theta$, and the two thermal images, give . Time and longitudinal first jets vanish by evenness. The radial difference jet vanishes by endpoint symmetry.

Off-diagonal stress components vanish by the same evenness properties; a radial-angular jet is purely imaginary and drops out of $A_{\mu\nu}$. The current has only an angular component independent of φ, so

$$\nabla_\mu J^\mu = \frac{1}{\alpha r}\partial_\varphi(\alpha r J^\varphi) = 0.$$

No occurrence of ξ entered its definition or evaluation.

Remark (Why the complex first jet is indispensable). Replacing $R^E_{\mathrm{B}_T}$ by its pointwise real part makes the angular dependence even at $\theta = 0$. Its first angular derivative is then zero. That operation would erase , not calculate it. Realification retains the two real fiber components and their cross-correlations; it is not this replacement. The

distinction is the same one emphasized for the vacuum current in I-B-VP-0 [9].

Vacuum terms, independently recovered

Define

$$P_\beta = \alpha^{-2}[1 - 6\beta(1-\beta)] - 1, \qquad C_\beta = \alpha^{-4}[1 - 30\beta^2(1-\beta)^2] - 1.$$

Proposition (Vacuum reference values). *In the normalization above,*

$$F_{\mathrm{vac}} = \frac{P_\beta}{48\pi^2 r^2},$$

$$T^{\mu}{}_{\nu,\mathrm{vac}} = \frac{C_\beta}{720\pi^2 r^4}\mathrm{diag}\,(1,1,-3,1) + 2(\xi - \tfrac{1}{6})\frac{P_\beta}{48\pi^2 r^4}\mathrm{diag}\,(4,-2,6,4),$$

$$J_{\varphi,\mathrm{vac}} = -\frac{\mathrm{B}_3(\beta)}{6\pi^2\alpha^2 r^2} = \frac{\beta(1-\beta)(2\beta-1)}{12\pi^2\alpha^2 r^2},$$

where $\mathrm{B}_3(\beta) = \beta^3 - \frac{3}{2}\beta^2 + \frac{1}{2}\beta$. *The other current components vanish.*

Proof. At equal radii and angle, make a Euclidean time split s. The geometric sum has expansion

$$S_\beta(a) = \frac{2}{a} + \mathrm{B}_2(\beta)a + \frac{\mathrm{B}_4(\beta)}{12}a^3 + O(a^5),$$

where

$$\mathrm{B}_2(\beta) = \beta^2 - \beta + \tfrac{1}{6}, \qquad \mathrm{B}_4(\beta) = \beta^4 - 2\beta^3 + \beta^2 - \tfrac{1}{30}.$$

Using $\eta = 2\mathrm{arsinh}\,(s/(2r))$ and subtracting $1/(4\pi^2 s^2)$, direct multiplication gives

$$H_E(s) = \frac{P_\beta}{48\pi^2 r^2} - \frac{C_\beta + 10P_\beta}{2880\pi^2 r^4}s^2 + O(s^4).$$

Lemma ensures that these are actual regular jets, not formal summation values. This proves .

The vacuum kernel is longitudinally boost invariant. Together with scaling, stationarity, and the real-part symmetries, this gives $T^{\mu}{}_{\nu} = r^{-4}\mathrm{diag}\,(a,b,c,a)$. Conservation gives $c = -3b$. The time component follows from

$$T^{t}_{\ t} = -2H_E''(0) + (2\xi - \tfrac{1}{2})\mathscr{L}_r F_{\mathrm{vac}}.$$

The trace identity proved below gives $T^{\mu}_{\ \mu} = (6\xi - 1)\mathscr{L}_r F_{\mathrm{vac}}$. These two equations determine a, b, c and yield .

For the current use a nonzero longitudinal spacelike split $s = z - z'$. Then

$$J_\varphi(s) = -\frac{A_\beta(\eta/\alpha)}{4\pi^2 \alpha r^2 \sinh\eta}, \qquad \cosh\eta = 1 + \frac{s^2}{2r^2}.$$

The direct flat subtraction has zero angular difference jet. From ,

$$A_\beta(a) = \frac{2}{3}\mathrm{B}_3(\beta)a + O(a^3).$$

Taking $s \to 0$ proves . Every differentiation was performed before coincidence on an absolutely convergent kernel. The remaining components vanish by the symmetries already used.

These agree with the documentary vacuum siblings [8, 9], but the proof did not assume their coefficients.

Regular Ward, trace, and improvement identities

Proposition (Identities on *M*). *The full one-points and their thermal differences satisfy*

$$\nabla_\mu T^{\mu}_{\ \nu} = 0,$$

$$T^{\mu}_{\ \mu} = (6\xi - 1)\mathrm{Box}\, F,$$

$$T^{\mu}_{\ \nu}(\xi') - T^{\mu}_{\ \nu}(\xi) = 2(\xi' - \xi)(\delta^{\mu}_{\ \nu}\mathrm{Box} - \nabla^\mu\nabla_\nu)F.$$

For the thermal correction, $\mathrm{Box}\, f = \mathscr{L}_r f$. *These are identities on* $r > 0$, *not distributional identities through the removed worldsheet.*

Proof. It suffices to use an arbitrary smooth Hermitian bisolution K in a flat local parallel frame. Differentiating the diagonal gives $\mathrm{Box}\, F = 2A^{\lambda}_{\ \lambda}$. Endpoint symmetry and the two field equations give

$$\nabla^\mu A_{\mu\nu} = \tfrac{1}{2}\nabla_\nu A^{\lambda}_{\ \lambda}.$$

Therefore the divergence of $2A_{\mu\nu} - g_{\mu\nu}A^{\lambda}_{\ \lambda}$ vanishes. The improvement divergence vanishes because the regular

Ricci tensor is zero. This proves conservation.

Taking the trace in gives

$$2 \cdot \tfrac{1}{2}\mathrm{Box}\, F + 4(2\xi - \tfrac{1}{2})\mathrm{Box}\, F - 2\xi \mathrm{Box}\, F = (6\xi - 1)\mathrm{Box}\, F.$$

The improvement formula follows by subtraction. The same proof applies to the analytic vacuum remainder and to the smooth thermal difference. No boundary term at $r = 0$ has been evaluated.

At $\xi = 1/6$, the regular trace vanishes. This is not a statement about possible supported trace terms at the apex, which are outside the present construction.

Lowest-mode limits, boost breaking, and the vacuum limit

Quantitative regular-host approach to the removed apex

Let v_+ be the smallest angular order strictly larger than ρ, and define

$$\delta_* = \min\{2, 2(v_+ - \rho)\} > 0, \qquad Q_* = \sum_{n \in \mathscr{I}_*} q_n.$$

These are fixed-sector parameters.

Theorem (Leading thermal Wick and current coefficients). *As* $r/\mathrm{B}_T \mathrm{downarrow}\, 0$, *through regular points,*

$$\delta F(r) = \frac{1}{\mathrm{B}_T^2}\left[\frac{d_* \zeta(2+2\rho)}{2\pi^2 \alpha}\left(\frac{r}{\mathrm{B}_T}\right)^{2\rho} + O\left(\left(\frac{r}{\mathrm{B}_T}\right)^{2\rho+\delta_*}\right)\right],$$

$$\delta J_\varphi(r) = \frac{1}{\mathrm{B}_T^2}\left[-\frac{Q_* \zeta(2+2\rho)}{\pi^2 \alpha}\left(\frac{r}{\mathrm{B}_T}\right)^{2\rho} + O\left(\left(\frac{r}{\mathrm{B}_T}\right)^{2\rho+\delta_*}\right)\right].$$

At $\beta = 0$ *and* $\beta = 1/2$, *the current correction is identically zero.*

In particular,

$$\beta = 0: \qquad \lim_{r\,\mathrm{downarrow}\,0} \delta F(r) = \frac{1}{12\alpha \mathrm{B}_T^2},$$

whereas for $0 < \beta < 1$, $\delta F = O(r^{2v_*})$, *with a strictly positive leading coefficient. For twisted sectors, the physical current component obeys*

$$\delta J^{\wedge}_{\varphi} = \frac{\delta J_{\varphi}}{\alpha r} = O(r^{2\rho - 1}).$$

No value or distribution at the apex is defined by this theorem.

Proof. Put $s = m\mathrm{B}_T$. At equal radii,

$$\frac{1}{r^2 \sinh \eta} = \frac{2}{s^2}\left(1 + \frac{4r^2}{s^2}\right)^{-1/2}, \qquad e^{-\eta} = \frac{r^2}{s^2}(1 + O(r^2/s^2)).$$

Thus the n-th angular term of $g(s^2; r)$ is

$$\frac{1}{4\pi^2 \alpha}\frac{r^{2\nu_n}}{s^{2+2\nu_n}}(1 + O((1+\nu_n) r^2/s^2)).$$

Separate the lowest modes. Their common leading coefficient is the displayed one with $\nu_n = \rho$. Higher modes gain at least $(r/s)^{2(\nu_+ - \rho)}$; the geometric correction gains $(r/s)^2$. The angular tail is bounded by a convergent geometric series with polynomial weights, uniformly for r/B_T in a sufficiently small fixed interval.

Consequently the remainder can be summed in m, including the zeta tails. The factor two in gives . The current operation multiplies an angular term by $-2q_n$; this gives . The same majorants justify the finite radial differentiations used below.

At $\beta = 0$, the terms n and $-n$ cancel in the signed sum. At $\beta = 1/2$, the terms n and $-n-1$ cancel. Thus the current vanishes identically in both sectors. Finally, $\rho = 0, d_* = 1$ at $\beta = 0$, and $\zeta(2) = \pi^2/6$, proving .

For $0 < \beta < 1/2$, $Q_* = \beta$. For $1/2 < \beta < 1$, $Q_* = \beta - 1$. At $\beta = 1/2$, $d_* = 2$ but $Q_* = 0$. The scalar enhancement and current cancellation at that character are therefore different lowest-mode statements.

The asymptotics are not claimed uniform as $\beta \to 0$. In particular, taking the holonomy limit and taking $r \to 0$ need not commute.

Corollary (Size and integrability of thermal corrections). *For* $0 < \beta < 1$, *the thermal stress components in a regular orthonormal frame satisfy*

$$\delta T^{\hat{\mu}}_{\hat{\nu}} = O(r^{2\rho - 2}) \quad (r\mathrm{downarrow}\, 0).$$

For $\beta = 0$, they remain bounded. They are smooth and hence locally integrable on M. Moreover, their absolute radial integrals with measure $r\,\mathrm{d}r$ are finite near zero. These statements concern the thermal correction, not the generally nonintegrable vacuum stress.

Proof. For a twisted sector, the differentiated expansion in the proof of Theorem gives

$$f^{(j)} = O(r^{2\rho - j}), \qquad j = 0, 1, 2.$$

The time and longitudinal jets are $O(r^{2\rho})$, while the physical angular second jet is $O(r^{2\rho - 2})$. Equations and give the stress bound.

In the untwisted sector, the zero angular mode is analytic in $r^2, r'\text{^}2$ at nonzero image separation. Its first two physical jets are bounded. Every nonzero angular order is at least $1/\alpha > 1$, so its second physical jets are $O(r^{2/\alpha - 2})$, also bounded. Normal convergence supplies the corresponding summed bounds.

For a twisted sector,

$$\int_0^R r\, r^{2\rho - 2}\, \mathrm{d}r = \frac{R^{2\rho}}{2\rho} < \infty.$$

For the untwisted sector, boundedness suffices. No extension of a derivative or Ward identity through zero is being made.

Strict longitudinal boost breaking

Proposition (A sign-definite boost-breaking difference). *At every $r > 0$, every $\mathrm{B}_T > 0$, and every fixed character,*

$$\text{boxed}\, \delta T^{t}{}_{t} - \delta T^{z}{}_{z} = -16 \sum_{m \geq 1} w_m g_{ww}(w_m; r) < 0.$$

It is independent of ξ. Thus the positive-temperature one-point stress is not invariant under longitudinal boosts.

Proof. The improvement terms in the two mixed components agree. Equations , , and give the displayed equality.

To prove strictness rather than merely a possible difference, use on the spatial diagonal:

$$g(w;r) = \int_0^\infty \frac{e^{-w/(4s)}}{\sqrt{4\pi s}} K_s(x,x)\,\mathrm{d}s.$$

The diagonal heat kernel is strictly positive: its Bessel-mode expression has positive summands. Differentiating twice in $w > 0$ is justified by the exponential factor at small s and the heat bound at large s. Hence

$$g_{ww}(w;r) = \int_0^\infty \frac{e^{-w/(4s)}}{16s^2\sqrt{4\pi s}} K_s(x,x)\,\mathrm{d}s > 0.$$

The convergent sum in is therefore strictly negative.

A diagonal longitudinally boost-invariant mixed tensor must have $T^t{}_t = T^z{}_z$. The vacuum already has that equality. The thermal difference violates it.

The preferred rest frame is the supplied static KMS dynamics. This loss of state symmetry neither changes the prescribed metric nor constructs a gravitational response.

Temperature conventions

Convention (Declared comparison of image periods). The defining Bose factor in this paper is

$$n_{\mathrm{B}_T}(\omega) = \frac{1}{e^{\mathrm{B}_T\omega} - 1}.$$

If a comparison calculation uses a symbol T_{img} through image shifts m/T_{img}, and uses the *same normalized time coordinate and the same units*, then the conventions agree precisely on declaring

$$T_{\mathrm{img}} = \mathrm{B}_T^{-1}.$$

Indeed, their image spacings then coincide, and Proposition identifies the same weak Euclidean covariance. Theorem identifies its regular jets. Without that declared coordinate and unit identification, no conversion is inferred from the letter T.

Under this declared conversion, becomes $T_{\mathrm{img}}^2/(12\alpha)$, matching the downstream record in I-B-VP-1 [10]. This is a convention comparison, not continuum thermometry.

Fixed-operator zero-temperature limit

Theorem (Vacuum limit of regular one-points). *Keep* B_F, α, β, ξ *and the subtraction fixed. As* $\mathrm{B}_T \to \infty$,

$$F_{\mathrm{B}_T} \to F_{\mathrm{vac}}, \qquad T^{\mu}{}_{\nu,\mathrm{B}_T} \to T^{\mu}{}_{\nu,\mathrm{vac}}, \qquad J_{\mu,\mathrm{B}_T} \to J_{\mu,\mathrm{vac}}$$

locally smoothly on $r > 0$. *For each compact regular set and each fixed finite derivative order, the differences are*

$$O_K(\mathrm{B}_T^{-2-2\rho}).$$

This bound need not be optimal for a particular component or character.

Proof. For large B_T, all nonzero image separations lie in the large-separation regime of Lemma . Thus every prescribed finite-order local derivative of the thermal remainder is bounded by

$$C_{K,k} \sum_{m \geq 1} (m\mathrm{B}_T)^{-2-2\rho} = C_{K,k}\zeta(2+2\rho)\mathrm{B}_T^{-2-2\rho}.$$

Apply the coincidence differential operators defining F, T, J and their regular derivatives.

This is the local-jet strengthening of the parent's fixed-operator zero-temperature covariance limit. A weak covariance limit alone would not justify this conclusion; the present proof supplies the required derivative control.

No mass is varied in this theorem. It is neither a massless-limit theorem nor a formulation of Nernst's law. It does not establish the zeroth law or select a physical temperature.

A normalization check

Although the construction assumes $0 < \alpha < 1$, the formulas have the ordinary untwisted flat specialization $\alpha = 1, \beta = 0$. There $g(w; r) = 1/(4\pi^2 w)$, and direct summation gives

$$\delta F = \frac{1}{12\mathrm{B}_T^2}, \qquad \delta J_{\varphi} = 0, \qquad \delta T^{\mu}{}_{\nu} = \frac{\pi^2}{45\mathrm{B}_T^4}\mathrm{diag}\,(-3, 1, 1, 1).$$

The energy density is therefore $-\delta T^{t}{}_{t} = \pi^2/(15\mathrm{B}_T^4)$, as appropriate to two real scalar components. This checks both the complex multiplicity and the Lorentzian mixed-component sign. It is an analytical substitution, not a reported numerical execution.

Finite reports and the shredder ledger

A supplied report, not a continuum instrument

A finite report may contain a manuscript identifier, encoded parameter values or intervals, formula identifiers, a list of regular observation radii, and certified finite approximations to the coefficients. A finite word does not encode arbitrary exact real numbers without an encoding prescription.

Let C be a finite clock or label register with orthonormal basis $\{|\tau_j\rangle\}$, and let A contain every returned quantum register. For preinstalled finite report words w_j, put

$$D_j = \langle \tau_j|_C \otimes I_A, \qquad \mathscr{M}(X) = \sum_j \mathrm{lvert}j, w_j\rangle\mathrm{langle}j, w_j|_R \otimes D_j X D_j^\dagger.$$

Proposition (Complete supplied-report identity). *The map $\mathscr{M}$ is CPTP and, for every untouched finite reference E,*

$$\mathrm{Tr}_R\,[(\mathscr{M} \otimes \mathrm{id}_E)(X)] = \mathrm{Tr}_C X.$$

The output contains no quantum C-factor. This is an instrument for reporting a supplied finite description, not for acquiring an unknown continuum state or temperature.

Proof. The recorded Kraus maps are $\mathrm{lvert}j, w_j\rangle_R \otimes D_j$, and their squared moduli sum to I_{CA}. Expanding $X = \sum_{j,k} |\tau_j\rangle\langle\tau_k| \otimes X_{jk}$ shows that the recorded blocks are X_{jj}. Their sum is $\mathrm{Tr}_C X$. The same calculation includes E.

The formula has no field input port, no field–probe coupling, and no likelihood depending on an unknown B_T. It therefore does not contain the data needed for a continuum thermometry theorem.

This is the local channel identity underlying the report interfaces in I-B-VP-8, I-C, and related Enter manuscripts [1, 16, 19]. The identity is proved here independently of any continuum interpretation.

Classical records cannot return an unknown code

Theorem (Record-only recovery obstruction). *Let $Q \cong \mathbb{C}^d$, $d \geq 2$. If a channel from Q has only a classical*

accessible output, then every deterministic decoding from that record back to $Q' \cong \mathbb{C}^d$ *has Bell-input entanglement fidelity at most* $1/d$*. Its distance from the identity channel satisfies*

$$\frac{1}{2}\| \mathscr{D}\mathscr{C} - \mathrm{id}_Q \|_\diamond \geq 1 - \frac{1}{d}.$$

The same statement applies to a classical report of the one-point coefficients when that report is the complete accessible output.

Proof. A finite classical-output channel has the form

$$\mathscr{C}(\rho) = \sum_x \mathrm{Tr}\,(E_x \rho)\mathrm{proj}\, x, \qquad E_x \geq 0, \quad \sum_x E_x = I.$$

After a decoder,

$$\mathscr{D}\mathscr{C}(\rho) = \sum_x \mathrm{Tr}\,(E_x \rho)\sigma_x.$$

For a referenced input, this output is separable across reference versus decoded output. This is the measure-and-prepare, or entanglement-breaking, mechanism [29].

Let $|\Omega_d\rangle = d^{-1/2}\sum_{j=1}^{d} |j\rangle|j\rangle$. For normalized product vectors,

$$|\langle \Omega_d, a \otimes b \rangle|^2 \leq \frac{1}{d}$$

by Cauchy–Schwarz. Convexity gives the same overlap bound for every separable state. Thus the decoded Bell output has fidelity at most $1/d$.

Testing the effect $|\Omega_d\rangle\langle\Omega_d|$ distinguishes this output from the ideal Bell state with probability difference at least $1 - 1/d$. The trace distance is at least that difference, and the half-diamond distance is at least the Bell-input trace distance. This proves .

The $1/d$ number is a fidelity ceiling. The error lower bound in the displayed normalization is $1 - 1/d$. The conclusion does not apply to an interface that additionally returns a useful quantum residual to the decoder. Accessible output roles must remain fixed. This is the finite distinction documented by I-C thm:A and En07-A

[16, 18].

Append-only records and endpoint nonselection

Proposition (Record monotonicity and prefix nonselection). *For a finite append-only classical history* $R_1, \ldots, R_n$,

$$H(R_1, \ldots, R_{n+1}) \geq H(R_1, \ldots, R_n).$$

This does not determine a radiation entropy or a future physical endpoint. In particular, a finite supplied prefix of coefficient reports does not choose an endpoint.

Proof. The Shannon chain rule gives

$$H(R_1, \ldots, R_{n+1}) = H(R_1, \ldots, R_n) + H(R_{n+1} \mid R_1, \ldots, R_n),$$

and classical conditional entropy is nonnegative.

For endpoint nonselection, append an independent qudit Q and an inaccessible qudit B in a fixed pure state. Two continuations can have exactly the same existing record: one leaves Q untouched, while another swaps Q with B. If only Q is later accessible, the first continuation returns the code and the second gives a replacement channel. The prefix does not distinguish those continuations. Neither a radiation system nor its time-dependent state was specified by the Shannon calculation.

These are the local contents relevant to I-C thm:B and thm:E [16]. Monotonicity of a classical ledger is not a Page curve. Program termination is likewise not a physical evaporation endpoint.

Reservoir heat requires an added thermal model

Use $b > 0$ for a *finite bath* inverse temperature. This symbol is distinct from holonomy β and from the continuum inverse temperature B_T. Let M_0 be a finite memory, B_0 a finite bath with Hamiltonian H_{B_0}, and

$$\tau_{B_0} = \frac{e^{-bH_{B_0}}}{Z}.$$

Assume the initial product $\rho_{M_0} \otimes \tau_{B_0}$, and let a unitary U produce

$$\omega_{M_0B_0} = U(\rho_{M_0} \otimes \tau_{B_0})U^{\dagger}.$$

Define heat into the bath and memory entropy decrease by

$$Q_{B_0} = \operatorname{Tr} H_{B_0}(\omega_{B_0} - \tau_{B_0}), \qquad \Delta S_{M_0} = S(\rho_{M_0}) - S(\omega_{M_0}).$$

The sign convention for ΔS_{M_0} is essential.

Theorem (Finite Landauer identity and its one-way implication). *Under these finite Gibbs-product and unitary hypotheses,*

$$\text{boxed}\, bQ_{B_0} = \Delta S_{M_0} + I(M_0 : B_0)_{\omega} + D(\omega_{B_0} \| \tau_{B_0}).$$

Therefore strict memory entropy decrease implies $Q_{B_0} > 0$. The converse is false. Neither this identity nor a positive heat cost follows from a reduced channel name alone.

Proof. Since $\log \tau_{B_0} = -bH_{B_0} - \log Z$,

$$D(\omega_{B_0} \| \tau_{B_0}) = -S(\omega_{B_0}) + b\operatorname{Tr}(H_{B_0}\omega_{B_0}) + \log Z.$$

Subtract the corresponding Gibbs entropy identity to obtain

$$bQ_{B_0} = S(\omega_{B_0}) - S(\tau_{B_0}) + D(\omega_{B_0} \| \tau_{B_0}).$$

Unitary invariance and the initial product give

$$S(\omega_{M_0B_0}) = S(\rho_{M_0}) + S(\tau_{B_0}).$$

The definition of mutual information now gives

$$S(\omega_{B_0}) - S(\tau_{B_0}) = \Delta S_{M_0} + I(M_0 : B_0)_{\omega}.$$

This proves . Nonnegativity of mutual information and relative entropy gives the one-way implication. This is the equality form of the finite Landauer account [30], also recorded in I-C and I-C-Th [16, 17].

For failure of the converse, leave the memory unchanged and take a bath qubit with $H_{B_0} = \varepsilon \mathrm{proj}\, 1$, $\varepsilon > 0$. Apply a bit flip to the bath alone. Then

$$Q_{B_0} = \varepsilon \tanh(b\varepsilon/2) > 0, \qquad \Delta S_{M_0} = 0.$$

Thus positive heat does not imply memory entropy decrease.

Finally, a reduced channel specifies neither H_{B_0}, the initial Gibbs state, a complete dilation, nor the control and reset ledger. Those are additional hypotheses, not consequences of an input–output equality.

Example (Lost access with zero heat). Take $H_{B_0} = 0$, so that $\tau_{B_0} = I/d$, and let U swap a d-dimensional memory with the bath. The accessible memory output is I/d, independent of its input. The input has moved to the bath, while $Q_{B_0} = 0$. With an external reference, the reference correlations move with it. This is loss of the designated access, not annihilation of the complete information.

There is no contradiction with Theorem : the swap does not strictly decrease the memory entropy for an initially pure memory. It illustrates why an operational shredder and a positive-heat eraser are different interfaces.

For a driven finite implementation with fixed uncoupled initial and final Hamiltonians, the elementary energy balance reads

$$W = \Delta E_{M_0} + Q_{B_0},$$

where W is the energy supplied by the drive. An energy-conserving unitary has $W = 0$. This finite first-law ledger must likewise be specified; a count of consumed quantum names is not an energy. The non-determination by a channel is the separation recorded in En07-A [18].

In particular, the operator e^{-sB_F} in Section is a heat semigroup. The parameter s has the role of a spectral smoothing parameter. Neither it nor its kernel is Q_{B_0}. No finite bath was present in the cone-state construction.

Finite Gibbs KMS and zero temperature are separate facts

Proposition (Finite Gibbs identities do not construct the field state). *For a finite Hamiltonian H, the Gibbs*

state $\tau_b = e^{-bH}/\mathrm{Tr}\,(e^{-bH})$ *satisfies the KMS boundary identity for* $\alpha_t(A) = e^{itH}Ae^{-itH}$. *As* $b \to \infty$, *it converges to the normalized ground-space projection. These statements are independent of the continuum state construction above.*

Proof. The finite matrix function

$$F_{A,B}(z) = \mathrm{Tr}\,(\tau_b A e^{izH} B e^{-izH})$$

is entire. Cyclicity of trace and commutation of H with e^{-bH} give $F_{A,B}(t + ib) = \mathrm{Tr}\,(\tau_b \alpha_t(B)A)$. This is the KMS boundary identity.

Diagonalize H. After subtracting its lowest eigenvalue, the Gibbs weights on strictly positive energies vanish as $b \to \infty$, while the weights on the ground eigenspace become equal. If that eigenspace has dimension g and a positive gap Δ separates it from the remaining $D - g$ levels, the trace-norm error is at most $2(D - g)g^{-1}e^{-b\Delta}$. The assertion is immediate also when every level is a ground level.

This proof has neither B_F nor a continuum test space. Conversely, the continuum estimates do not prove the finite trace identities by substitution.

This is the appropriate separation from the finite KMS and zero-temperature propositions of I-C-Th [17]. Likewise, the surface-gravity Bose spectrum recorded in Em03-A is a fixed-background emission calculation, not a temperature table for the present cone [22].

Six distinct time-related objects

The following six objects remain separate:

1. the prescribed geometric time coordinate T, written t in ;

2. a finite destructive clock code and its calibration;

3. a harmonic relational scalar with its own equation and Cauchy data;

4. an incidence register and its installed update rule;

5. an X-A density ramp, which is a prescribed non-clock;

6. the inverse temperature B_T, a parameter in a Bose occupation law. The clock separations documented in XIX-A and the relational readout construction of En09-A do not identify these objects [20, 19, 21]. A finite report can print a value assigned to B_T. That does not make temperature a clock code, a lapse, an incidence parameter, or a reconstructed geometry.

Release, non-release, and debugging audit

Exact mathematical release

Theorem (I-B-VP-8-Ex regular-host release). *For the prescribed host , fixed flat character, massless Friedrichs operator, centered zero-chemical-potential Bose prescription, and every* $B_T > 0$*, the following statements are established within this manuscript:*

1. *Compact tests satisfy the thermal inverse-power gate and infrared tail ; the compact-test unstable-product obstruction is retained under its stated product hypotheses.*

2. *The specified positive regular quasifree KMS state satisfies the field equation and extension-selected commutator on compact regular tests.*

3. *Weak Euclidean periodization has the locally normally convergent differentiated representatives –. The local vacuum Hadamard subtraction is checked independently.*

4. *The regular Wick square, stress, and current are the vacuum expressions – plus the explicit thermal sums –.*

5. *The leading lowest-mode coefficients, regular conservation, trace, improvement, integrability estimates, strict longitudinal boost-breaking difference, and fixed-operator one-point vacuum limit hold with the scopes stated above.*

6. *A finite supplied report of these coefficients obeys the complete destructive-report identity, record-only recovery obstruction, append-only monotonicity, and endpoint nonselection. Reservoir heat enters only after the additional finite Gibbs-product and unitary model has been supplied. No assertion in this theorem extends an observable or a Ward identity through the apex.*

Proof. Proposition and Theorem prove the first item. Propositions and prove the second. Proposition , Theorem , and Lemma prove the third. Theorem and Proposition prove the fourth. Proposition , Theorem , Corollary , Proposition , and Theorem prove the fifth. The independently finite proofs in Section prove the last item. Each proof retains its own domain and does not invoke a documentary sibling as an analytical premise.

Gates not closed by this release

Here ``not constructed'' means not constructed by this manuscript. It is not a universal nonexistence claim or a claim that no result exists elsewhere.

toprule Gate Status here Exact qualification

midrule endfirsthead toprule Gate Status here Exact qualification

midrule endhead bottomrule endfoot Regular Wick, stress, current Constructed Local Hadamard subtraction and regular coincidence jets.

Apex extension and contacts Not redone I-B-VP-1 is a separate downstream record; no supported $\delta_{\mathscr{W}}$ term or contact dimension is released.

Global thermal wavefront Not proved here Local smooth subtraction is not a treatment of every geometric and diffractive front. I-B-VP-2 records the ground-state result.

Massive thermal state Not constructed No massive threshold analysis is performed. A divergent positive ground covariance cannot be cured by adding nonnegative Bose occupation.

Non-Friedrichs product thermal state Not constructed Positivity, the actual inverse-half-power domain, and local covariance estimates must be checked for that operator.

Non-product thermal state Not constructed The I-B-VP-6 operator ledger is not a state theorem. The product no-go is not transferred to it.

Excited or coherent states Not classified A coherent displacement is not automatically non-KMS; its admitted domain and covariance with time translations require separate analysis.

Frozen Maxwell response Not constructed I-B-VP-4 is a separate frozen-source calculation, not a self-consistent field response here.

Smooth-core state limit Not constructed Operator selection in I-B-VP-5 does not itself prove renormalized one-point convergence.

Backreaction Not constructed No Einstein–matter or self-consistent Maxwell equation is solved.

The ground-state wavefront, massive threshold, frozen-response, core-selection, and non-product distinctions are documented in the corresponding sheets [11, 12, 13, 14, 15]. They are separations, not proof transfers. Similarly, the free spin-one and spin-two operator constructions of I-B-G-S are not this scalar stress tensor, and a Green operator is not a positive state [7, 6].

For the coherent-state qualification, the elementary mechanism is worth stating. If an admitted Weyl displacement is defined by a real linear functional ℓ, and $\ell(S_t F) = \ell(F)$, its Weyl automorphism commutes with time evolution. Composing a KMS state with that automorphism preserves the KMS condition. This observation does not construct such a displacement in the present test domain; it only rules out the blanket assertion that every coherent displacement must fail to be KMS.

A compact debugging ledger

toprule Potential error Correction used in the proof

midrule endfirsthead toprule Potential error Correction used in the proof

midrule endhead bottomrule endfoot

Use the ground infrared gate at positive temperature Prove $C_c^\infty(N, L_\beta) \subset \operatorname{Dom} B_F^{-1/2}$, not only $\operatorname{Dom} B_F^{-1/4}$.

Take coincidence directly in a weak periodization identity Prove the large-image jet estimate and normal convergence before restriction to the diagonal.

Replace the charged kernel by its real part everywhere Use the real part only in the stress bilinear; retain the

complex angular first jet for the current.

Infer the thermal current from its vacuum polynomial Differentiate and sum the nonzero thermal images independently.

Copy a stress normalization without complex multiplicity Use $\Phi = (\varphi_1 + \mathrm{i}\varphi_2)/\sqrt{2}$: one-component Wick covariance and two-component stress.

Choose radial stress by an apex integration constant Compute $D = [\partial_r \partial_r R^E_{\mathrm{B}_T}]$ locally and eliminate it with the endpoint equation.

Call endpoint integrability a Ward extension theorem State only absolute-size estimates and regular identities; no apex distribution is normalized.

Infer one-point convergence from weak state convergence alone Prove convergence of the required local jets uniformly on compact regular sets.

Identify the symbols T, β, and B_T Reserve β for holonomy, B_T for continuum inverse temperature, and b for a separately supplied finite bath.

Use $1/d$ as the half-diamond error bound State Bell fidelity $\le 1/d$ and error $\ge 1 - 1/d$.

Read ΔS with an unstated sign Define it as memory entropy decrease in .

Treat a supplied report as thermometry Retain a finite register input only; no unknown field acquisition port is present.

Equate heat semigroup, KMS covariance, and reservoir heat Keep the spectral, equilibrium-state, and implementation ledgers separate.

Standing non-identifications

The construction does not derive a metric from records or entanglement. It does not establish Einstein dynamics, a Page curve, de Sitter physics, quantum-gravitational evaporation, or an ultraviolet completion. The cone remains

prescribed throughout.

No catalogue family is a temperature table or an analytical premise. The OpenAI catalogue's rem:not-premise is used only as a documentary non-transfer statement [23]. Families 282, 362, and 376 are not verified or used. No family abstract or reported proof artifact supplies a missing covariance, boundary condition, or thermal machine.

The inherited status sentences are unchanged:

O.1 remains NOTCLOSED,
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H2, I3, and J2 are not resolved by this manuscript.

These are scoped documentary boundaries, not newly proved universal obstruction theorems. No inherited gravitational quantum-response coefficient is admitted by the free-field coefficients evaluated here.

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review before actual submission.

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