Enter, I-B-VP-8-v2, 2026, pp. 1-38

I-B-VP-8 Friedrichs thermal states on the prescribed conical spacetime

The unstable-product wall, the thermal infrared estimate, and a compact-test quasifree KMS construction

Ngo Cheung

Version 1

Abstract

We construct the massless, zero-chemical-potential Friedrichs thermal state on the prescribed regular conical product of I-B, for every opening parameter $0 < \alpha < 1$, every flat-bundle character $0 \le \beta < 1$, and every inverse temperature $\beta_T > 0$. The extension, temperature, and Bose prescription remain inputs. The construction precedes, and does not repeat, the thermal one-point extension calculation of I-B-VP-1.

The key analytical distinction is infrared: compact-test ground-state finiteness involves $B_F^{-1/4}$, whereas positive-temperature field variance requires $B_F^{-1/2}$. We prove the latter estimate directly. More precisely, if $\rho = \min_{n \in \mathbb{Z}} |n + \beta|/\alpha$, the compactly tested Friedrichs heat semigroup decays as $s^{-3/2-\rho}$. Its leading coefficient is an explicit positive sum of squared lowest-angular-mode moments. This gives an optimal fractional inverse-power threshold for tests with a nonzero leading moment, and a quantitative thermal infrared-tail bound of order $\varepsilon^{1/2+\rho}$.

We construct a positive, regular, centered quasifree state, prove the full bounded-observable KMS strip condition, verify the extension-selected field equation and commutator, and prove continuity and finite second moments for all compact smooth regular spacetime tests. A weak Euclidean periodization identity identifies its prescribed two-point function without taking coincidence limits. At fixed massless operator, the states converge weakly to the spectral ground state as $T \mathrm{downarrow} 0$, with an explicit covariance estimate.

The contribution is a domain-explicit completion of the Friedrichs thermal-state gate and its stronger infrared

obligation; thermal cosmic-string Green functions and quasifree KMS methods are established subjects. The inherited no-go for unstable product extensions is retained with its compact-test proof. No massive thermal state, non-Friedrichs or non-product thermal state, excited-state classification, thermal wavefront theorem, new apex-contact classification, or backreaction is constructed.

Bibliographic record

Title	I-B-VP-8 Friedrichs thermal states on the prescribed conical spacetime
Author	Cheung, Ngo
Publication date	2026
Institution	Enter
Report number	I-B-VP-8-v2
Publisher	Cheung Ngo Medical Limited
First page	1
Last page	38
Language	en

Book edition note (t425u)

Standing Enter refusals, scope vocabulary (proved here/supplied input/conditional/ not constructed/not claimed), and Scholar bibliography policy are frozen in the companion chapter *I-B-VP Book Ledger* v1. O.1 remains NOT_CLOSED; XVI-N is not discharged; XV-B is not opened; no Host C, Page/island, or spacetime-emergence claim; wired=false; SOURCE_DATE_EPOCH=1792627200. This sheet is a companion manuscript record and is *not* claimed to be Google Scholar-indexed. The published Enter Calculus article is cited only for finite-instrument / operational interface discussions, never as a PDE premise.

Book expansion: IR before KMS

Keep the compact-test infrared estimate before the KMS construction. Key contrast: $B_F^{-1/4}$ for the ground covariance versus $B_F^{-1/2}$ for thermal field variance. Euclidean periodization is proved weakly first; differentiability/coincidence is a separate bridge.

Scope, background, and the corrected target

One prescribed host

Work in units $c = \hbar = k_{\mathrm{B}} = 1$. The spacetime is

$$\begin{aligned} M &= \mathbb{R}_t \times N, \\ N &= (0,\infty)_r \times (\mathbb{R}/2\pi\mathbb{Z})_\varphi \times \mathbb{R}_z, \\ g &= -\mathrm{d}t^2 + \mathrm{d}r^2 + \alpha^2 r^2 \mathrm{d}\varphi^2 + \mathrm{d}z^2, \qquad 0 < \alpha < 1. \end{aligned}$$

Its declared completion adds the worldsheet

$$\mathscr{W} = \mathbb{R}_t \times \{o\} \times \mathbb{R}_z.$$

The cone, deficit $2\pi(1-\alpha)$, worldsheet, time orientation, and static time coordinate are prescribed I-B data [9].

Fix a flat Hermitian line bundle L_β by the angular character

$$u(r, \varphi + 2\pi, z) = e^{2\pi \mathrm{i}\beta} u(r, \varphi, z), \qquad 0 \le \beta < 1.$$

The symbol β always denotes holonomy. Inverse temperature is denoted separately by $\beta_{\mathrm{T}} = 1/T > 0$.

The spatial measure and complex Hilbert space are

$$\mathrm{d}V = \alpha r \, \mathrm{d}r \, \mathrm{d}\varphi \, \mathrm{d}z, \qquad \mathscr{H}_\beta = L^2(N, L_\beta; \mathrm{d}V).$$

In a lifted parallel representation the flat covariant derivative is ordinary differentiation.

Let B_F be the Friedrichs realization of

$$-\Delta_\beta = -\partial_r^2 - \frac{1}{r}\partial_r - \frac{1}{\alpha^2 r^2}\partial_\varphi^2 - \partial_z^2$$

initially defined on compact smooth sections in the regular space N. Thus the operator is fixed before the state is constructed.

The first-stage release

The first manuscript suggested in the plan is adopted:

1. retain the unstable-product no-go;

2. construct the massless Friedrichs KMS state;

3. prove its compact-test infrared estimate. The sharp heat asymptotic, weak Euclidean identification, and fixed-operator Tdownarrow 0 limit are auxiliary results proved within this stage.

The broader V8 row is not declared complete. In particular, no state construction for the longitudinally non-product operators of the supplied I-B-VP-6-v2.tex is attempted [13].

State construction is not one-point extension

The supplied I-B-VP-1 thermal theorem starts with a Friedrichs KMS state and studies the extension of its one-point quantities to the apex [12]. The present task is logically earlier: establish that a specific positive-temperature covariance defines a state on all compact regular tests.

The state prescription here is the centered Bose covariance with zero chemical potential. Proving its existence does not select a physical temperature or establish uniqueness among all equilibrium states.

Established methods and the contribution claimed

Quasifree thermal representations and KMS strip methods are standard [1, 2, 3]. Finite-temperature Green functions around magnetic-flux cosmic strings have a substantial prior literature; see, for example, Guimarães [4]. Neither thermal fields on cones nor Bose factors originate in the Enter framework.

The contribution claimed here is narrower:

• a proof of the stronger inverse-power estimate required by the thermal covariance, on the precise Friedrichs cone

domain and for all holonomies;

• its optimal fractional threshold when the leading angular moment is nonzero;

• a state, algebra, test class, and KMS proof whose domains are written explicitly;

• a separation from assumed-state one-point formulas and from non-product extension questions. No exhaustive historical priority claim is made.

Enter and the philosophy of non-identification

Enter is destructive finite-register readout: the measured quantum names are consumed and a classical code is returned. Its operational characterization is relative to an explicitly declared strategy class [7]. The philosophical companion separates reporting, quantum recovery, thermodynamic implementation, and physical identification [8].

A finite report can name an installed inverse temperature and a proved estimate. It does not prepare, identify, or acquire an unknown continuum thermal state merely by reporting those words. The finite reporting channel in Section is independent of the following state-existence proofs.

The regular spacetime is not globally hyperbolic. For example, a sufficiently large causal diamond contains a sequence tending to $r = 0$, with no convergent subsequence in M. The operator prescription supplies the evolution used below. It does not change that geometric fact or make the regular slices geometric Cauchy surfaces [10].

Real fields, compact tests, and the unstable-product wall

Realification and field normalization

For a charged scalar, use the underlying real Hilbert space $\mathscr{H}_{\mathrm{R}}$ of $\mathscr{H}_{\beta}$, with inner product

$$(u, v)_{\mathrm{R}} = \mathrm{Re}\,\langle u, v\rangle_{\mathscr{H}_{\beta}}.$$

The associated real bundle is denoted E_{β}. For a neutral scalar at $\beta = 0$, one may instead use the ordinary real-valued subspace.

All complex Hilbert-space inner products in this manuscript are antilinear in the first argument. Realification is not replacement of a charged two-point kernel by its pointwise real part in the same complex sector. The charged field is reconstructed from the real components; reversed charged ordering involves the conjugate bundle.

The regular spacetime test class is

$$\mathcal{T} = C_c^\infty(M, E_\beta).$$

Its topology is the usual test-section topology, with fixed-compact-support stages.

The extension-selected compact-test CCR

For this subsection only, let

$$B = T_\Lambda \otimes I + I \otimes (-\partial_z^2)$$

be a massless product extension, and assume B is bounded below. The transverse cone extensions under discussion have this property; the finite-dimensional radial boundary ledger has finitely many negative transverse eigenvalues [11, 5].

Define the bounded operator

$$E_B(t) = \{ \sin(t\sqrt{B})/\sqrt{B}, \text{ by spectral calculus,}$$

where the scalar value at $\lambda = 0$ is t, and for $\lambda = -a^2 < 0$ it is $\sinh(at)/a$. For fixed t, boundedness follows from the lower bound on B.

On $\mathcal{T}$, set

$$\sigma_B(f, g) = -\int_{\mathbb{R}^2} (f(t), E_B(t - s)g(s))_{\mathrm{R}}\, \mathrm{d}t\, \mathrm{d}s.$$

The Weyl algebra is the CCR algebra of this presymplectic space, with its radical removed if necessary:

$$W(f)W(g) = e^{-\mathrm{i}\sigma_B(f,g)/2} W(f + g), \qquad W(f)^* = W(-f).$$

Time translation acts by

$$f_a(t) = f(t - a), \qquad \tau_a(W(f)) = W(f_a).$$

Definition (Finite second moments). A state has the compact-test finite-second-moment property if it is regular in every one-parameter Weyl subgroup and its GNS vector belongs to the domain of the self-adjoint field generator $\Phi(f)$ for every $f \in \mathcal{T}$. Equivalently, its field variance $\|\Phi(f)\Omega\|^2$ is finite for every such test.

The commutator normalization is

$$= \mathrm{i}\sigma_B(f, g)I.$$

At the unsmeared time level this is $-\mathrm{i}E_B(t - s)$, in agreement with the convention of the supplied product-state sheets.

Theorem (Retained unstable-product no-go). *Suppose T_Λ has a negative eigenvalue $-\kappa^2$, $\kappa > 0$. There is no time-translation-invariant state on the extension-selected compact-test algebra with finite second moments for every test in $\mathcal{T}$.*

In particular, there is no regular KMS state with that finite-moment property on this unstable massless product extension.

Proof. The transverse bound channel gives the full dispersion $k_z^2 - \kappa^2$, so the negative spectral projection of B is nonzero. Choose

$$[-b, -a] \subset (-\infty, 0), \qquad a > 0,$$

with nonzero spectral projection. Compact smooth spatial sections are dense in $\mathscr{H}_{\mathrm{R}}$, so some such section v has spectral measure

$$\mu_v(S) = (v, E_B^{\mathrm{sp}}(S)v)_{\mathrm{R}}$$

with $\mu_v([-b, -a]) > 0$. Here E_B^{sp} denotes the spectral measure, not the propagator .

Choose nonnegative, nonzero $\chi \in C_c^\infty([-A, A])$, and put $f(t) = \chi(t)v$. For $t_0 > 2A$, the contribution of a spectral value

$\lambda = -\omega^2 < 0$ to the absolute-sign-fixed commutator pairing is

$$\frac{\sinh(\omega t_0)}{\omega}\Big(\int \chi(s) e^{\omega s}\,\mathrm{d}s\Big)\Big(\int \chi(s) e^{-\omega s}\,\mathrm{d}s\Big).$$

Indeed, expand $\sinh(\omega(t_0 + s - s'))$; the two exponential integrals multiply symmetrically.

Every negative spectral contribution has the same sign. On $[-b, -a]$, the coefficient in has a uniform positive lower bound, apart from its exponential factor. This part therefore grows at least as $Ce^{\sqrt{a}\,t_0}$ for large t_0.

For $\lambda \geq 0$, the inequality

$$\left|\frac{\sin(u\sqrt{\lambda})}{\sqrt{\lambda}}\right| \leq |u|$$

bounds the entire nonnegative contribution by

$$C_\chi(t_0 + 2A)\|v\|^2.$$

Consequently $|\sigma_B(f_{t_0}, f)|$ is unbounded.

In an invariant state with the stated second moments, Cauchy–Schwarz and the CCR give

$$|\sigma_B(f_{t_0}, f)| = |\omega([\Phi(f_{t_0}), \Phi(f)])| \leq 2\|\Phi(f_{t_0})\Omega\|\|\Phi(f)\Omega\| = 2\|\Phi(f)\Omega\|^2.$$

This contradicts the unbounded commutator.

This is the compact-test argument of I-B-VP thm:nogo, retained here with its hypotheses explicit [11]. It does not assume that a generally noncompact unstable phase-space vector is already a compact field test.

Corollary (The excluded massless product rows). *The theorem excludes every logarithmic untwisted* L_L*, every diagonal product extension with a negative power-channel parameter* λ_j*, and the positive-order relation charts with* $H_W \not\geq 0$*, in the normalizations of the supplied product positivity ledger.*

Proof. Each listed case has a negative transverse eigenvalue by that ledger [11]. Apply Theorem .

Remark (No transfer to non-product extensions). A negative transverse eigenvalue is not, by itself, a negative

full spectral band for a momentum-dependent boundary law. The supplied I-B-VP-6 examples make this distinction explicit [13]. Neither existence nor absence of a KMS state for those operators is asserted here.

The Friedrichs heat kernel and its sharp infrared exponent

From now on $B = B_F$, with zero mass. Set

$$q_n = n + \beta, \qquad \nu_n = \frac{|n+\beta|}{\alpha}, \qquad \rho = \min_{n \in \mathbb{Z}} \nu_n, \qquad a_\rho = \frac{3}{2} + \rho.$$

Let

$$J_{\min} = \{n \in \mathbb{Z} : \nu_n = \rho\}.$$

There is one lowest mode, except that there are two when $\beta = 1/2$. The number ρ is not a deficiency count.

The full heat kernel

The Friedrichs radial spectral transform uses $J_{\nu_n}(pr)$ and measure $p\,\mathrm{d}p$. The angular factors are

$$\psi_n(\varphi) = \frac{e^{\mathrm{i}(n+\beta)\varphi}}{\sqrt{2\pi\alpha}}.$$

The standard Hankel diagonalization is compatible with the Friedrichs condition at zero [5].

Lemma (Heat kernel). *The integral kernel of e^{-sB_F}, relative to* $\mathrm{d}V$*, is*

$$K_s(x,x') = \frac{e^{-(z-z')^2/(4s)}}{\sqrt{4\pi s}} \frac{e^{-(r^2+r'^2)/(4s)}}{4\pi\alpha s} \cdot \sum_{n \in \mathbb{Z}} e^{\mathrm{i}(n+\beta)(\varphi-\varphi')} I_{\nu_n}\left(\frac{rr'}{2s}\right), \qquad s > 0.$$

The angular series converges locally uniformly for $s > 0$ and regular spatial arguments.

Proof. The longitudinal Gaussian is the Fourier inverse of e^{-sk^2}. In one angular mode, the radial heat kernel is

$$\int_0^\infty e^{-sp^2} J_\nu(pr) J_\nu(pr')\, p\,\mathrm{d}p = \frac{1}{2s} e^{-(r^2+r'^2)/(4s)} I_\nu\left(\frac{rr'}{2s}\right).$$

This is the Bessel Gaussian integral [6, Eq. 10.22.67]. Multiplying by $\psi_n(\varphi)\overline{\psi_n(\varphi')}$ and summing gives .

For y in a bounded nonnegative interval, the series

$$I_{\nu}(y) = \sum_{j=0}^{\infty} \frac{(y/2)^{2j+\nu}}{j!\,\Gamma(j+\nu+1)}$$

gives a bound

$$I_{\nu}(y) \le e^{y^2/4} \frac{(y/2)^{\nu}}{\Gamma(\nu+1)}, \qquad \nu \ge 0.$$

Here $\Gamma(j+\nu+1) \ge \Gamma(\nu+1)j!$ would already suffice with an adjusted bound; the weaker inequality $\Gamma(j+\nu+1) \ge \Gamma(\nu+1)$ also gives the displayed exponential estimate after keeping the $j!$ in . Since ν_{n} grows linearly with $|n|$, the resulting angular majorant is summable, uniformly on the stated compact sets. The series is [6, Eq. 10.25.2].

Remark (A harmless inequality can be sharpened). In the preceding proof one may use directly

$$\frac{\Gamma(j+\nu+1)}{\Gamma(\nu+1)} = \prod_{\ell=1}^{j} (\nu+\ell) \ge 1,$$

which is valid for every integer $j \ge 0$ and $\nu \ge 0$. No monotonicity assertion for the Gamma function on the entire positive axis is being used.

Large heat time and lowest angular moments

For a compactly supported smooth section v, define

$$m_{n}(v) = \int_{N} r^{\rho} e^{-\mathrm{i}(n+\beta)\varphi} v(x)\, \mathrm{d}V(x), \qquad n \in J_{\min}.$$

The integrand is single-valued because its two angular characters cancel.

Theorem (Compact heat bound and leading coefficient). *For every compact* $K \subset N$*, there exist constants* $C_{K}, s_{K} > 0$ *such that*

$$0 \le \langle v, e^{-sB_{F}} v \rangle \le C_{K} s^{-a_{\rho}} \|v\|_{L^{1}}^{2}, \qquad s \ge s_{K}, \quad \operatorname{supp} v \subset K.$$

For each such v*,*

$$\lim_{s\to\infty} s^{a_\rho}\langle v, e^{-sB_F} v\rangle = A_\rho \sum_{n\in J_{\min}} |m_n(v)|^2,$$

where

$$A_\rho = \frac{1}{8\pi^{3/2}\alpha\, 4^\rho \Gamma(1+\rho)} > 0.$$

In particular B_F has no L^2 zero mode.

Proof. For $x, x' \in K$, the Bessel argument in tends uniformly to zero as $s \to \infty$. Choose s_K so that $rr'/(4s) \le 1$ on $K \times K$.

The estimate from gives a summable majorant for

$$s^\rho I_{\nu_n}\left(\frac{rr'}{2s}\right).$$

Indeed,

$$s^\rho \left(\frac{rr'}{4s}\right)^{\nu_n} = \left(\frac{rr'}{4}\right)^\rho \left(\frac{rr'}{4s}\right)^{\nu_n - \rho},$$

and $\nu_n \ge \rho$. Together with the longitudinal and radial prefactors, this proves

$$|K_s(x, x')| \le C_K s^{-3/2-\rho}.$$

Integration against $\overline{v(x)} v(x')$ proves the upper bound. Positivity is operator positivity.

For a lowest mode,

$$s^\rho I_\rho\left(\frac{rr'}{2s}\right) \to \frac{(rr'/4)^\rho}{\Gamma(1+\rho)}.$$

For every other mode the limit is zero. The Gaussian factors tend to one. Dominated convergence in the angular sum and then in $K \times K$ yields with coefficient . The double integral for each lowest mode is $|m_n(v)|^2$.

Finally, e^{-sB_F} tends strongly as $s \to \infty$ to the zero-eigenvalue projection. Equation makes that projection vanish on the dense compact-test subspace, hence everywhere.

Remark (Untwisted normalization check). For $\beta = 0$, $\rho = 0$, $J_{\min} = \{0\}$, and

$$\langle v, e^{-sB_F} v \rangle \sim \frac{\left| \int_N v \, dV \right|^2}{8\pi^{3/2} \alpha \, s^{3/2}}$$

when the integral is nonzero. At $\alpha = 1$, the coefficient becomes the usual three-dimensional Euclidean heat coefficient. The result itself was proved for $0 < \alpha < 1$.

Theorem (Fractional infrared threshold). *For every q with $0 < q < a_\rho$ and every compact $K \mathrm{Subset} N$,*

$$\left\| B_F^{-q/2} v \right\|^2 \le C_{K,q} (\|v\|_{L^2}^2 + \|v\|_{L^1}^2), \qquad \operatorname{supp} v \subset K.$$

If at least one lowest moment $m_n(v)$ is nonzero, then

$$v \notin \operatorname{dom} B_F^{-q/2} \qquad \text{for every} q \ge a_\rho.$$

At the endpoint $q = a_\rho$, the large-heat-time integral diverges logarithmically.

Proof. For $q > 0$, the spectral theorem and Tonelli give the extended nonnegative identity

$$\left\| B_F^{-q/2} v \right\|^2 = \frac{1}{\Gamma(q)} \int_0^\infty s^{q-1} \langle v, e^{-sB_F} v \rangle \, \mathrm{d}s.$$

There is no zero-mode atom by Theorem .

For $0 < s \le s_K$, use contraction of the heat semigroup. For $s \ge s_K$, use . The latter integral converges exactly under the sufficient condition $q < a_\rho$, proving .

If a lowest moment is nonzero, supplies a positive lower bound $c_v s^{-a_\rho}$ for sufficiently large s. Then diverges for $q \ge a_\rho$, logarithmically when equality holds.

Corollary (The thermal compact-test estimate). *For every compact smooth spatial section,*

$$\mathrm{boxed}\ B_F^{-1/2} v \in \mathscr{H}_\beta .$$

Moreover, if

$$\mu_v(S) = \langle v, 1_S(B_F) v \rangle,$$

then, for sufficiently small $\varepsilon > 0$,

$$\operatorname{boxed} \int_{(0,\varepsilon]} \lambda^{-1}\, \mathrm{d}\mu_v(\lambda) \le C_K \|v\|_{L^1}^2 \varepsilon^{1/2+\rho}, \qquad \operatorname{supp} v \subset K.$$

The constants may depend on α, β, K, *but not on* v *or* ε *in the stated ranges.*

Proof. Take $q = 1 < a_\rho$ in Theorem .

For the tail estimate, when $\lambda^{-1} \ge s_K$,

$$\mu_v([0,\lambda]) \le e \left\langle v, e^{-B_F/\lambda} v \right\rangle \le C_K \|v\|_{L^1}^2 \lambda^{a_\rho}.$$

Integration by parts, with the zero-endpoint term vanishing, gives

$$\begin{aligned} \int_{(0,\varepsilon]} \lambda^{-1} \mathrm{d}\mu_v(\lambda) &= \varepsilon^{-1} \mu_v([0,\varepsilon]) + \int_0^\varepsilon \lambda^{-2} \mu_v([0,\lambda])\, \mathrm{d}\lambda \\ &\le C_K \|v\|_{L^1}^2 \frac{a_\rho}{a_\rho - 1} \varepsilon^{a_\rho - 1}. \end{aligned}$$

Absorb the fixed factor into the constant.

This proves a new obligation relative to the supplied ground-state estimate: $q = 1$, rather than $q = 1/2$, is required below. The proof does not substitute I-B-VP lem:infrared for a thermal bound.

The Bose covariance and its exact form domain

Let

$$h = \sqrt{B_F}$$

on the real Hilbert space and on its complexification $\mathscr{K} = \mathscr{H}_{\mathrm{R}}^{\mathbb{C}}$. The same notation is used for these compatible realizations. Let

$$n_{\beta_{\mathrm{T}}}(\omega) = \frac{1}{e^{\beta_{\mathrm{T}}\omega} - 1}, \qquad c_{\beta_{\mathrm{T}}}(\omega) = 1 + 2n_{\beta_{\mathrm{T}}}(\omega) = \coth\left(\frac{\beta_{\mathrm{T}}\omega}{2}\right).$$

No chemical potential is introduced.

Lemma (Ground and thermal infrared weights). *For* $\omega > 0$,

$$\frac{1}{\beta_{\mathrm{T}}\omega^{2}} \le \frac{1}{2\omega}\coth\left(\frac{\beta_{\mathrm{T}}\omega}{2}\right) \le \frac{1}{2\omega} + \frac{1}{\beta_{\mathrm{T}}\omega^{2}}.$$

Consequently, for $v \in \mathscr{H}_{\mathrm{R}}$*, the thermal configuration variance*

$$\frac{1}{2}(v, h^{-1}c_{\beta_{\mathrm{T}}}(h)v)_{\mathrm{R}}$$

is finite if and only if $v \in \operatorname{dom} h^{-1}$*. By contrast, the ground configuration variance is finite if and only if* $v \in \operatorname{dom} h^{-1/2}$.

For $w \in \operatorname{dom} h^{1/2}$,

$$\frac{1}{2}(w, hc_{\beta_{\mathrm{T}}}(h)w)_{\mathrm{R}} \le \frac{1}{2}\|h^{1/2}w\|^{2} + \beta_{\mathrm{T}}^{-1}\|w\|^{2}.$$

All expressions with an unbounded operator are quadratic-form expressions.

Proof. For $x > 0$, $\coth x \ge 1/x$, because $\tanh x \le x$. Also

$$\coth x = 1 + \frac{2}{e^{2x} - 1} \le 1 + \frac{1}{x}.$$

Substitute $x = \beta_{\mathrm{T}}\omega/2$.

The lower bound proves necessity of $\|h^{-1}v\| < \infty$. For sufficiency, the upper bound applies and

$$\omega^{-1} \le 1 + \omega^{-2}$$

controls the remaining ground term. The ground assertion is its spectral form-domain definition. Multiplying the upper bound for coth by $\omega/2$ gives .

Thus the precise contrast is

$c\|c\|c$	configuration variance multiplier	required inverse-power domain
hline $T = 0$	$(2\sqrt{B_F})^{-1}$	$\operatorname{dom} B_F^{-1/4}$
$T > 0$	$(2\sqrt{B_F})^{-1}\coth(\beta_{\mathrm{T}}\sqrt{B_F}/2)$	$\operatorname{dom} B_F^{-1/2}$.

Nonnegativity alone would not establish the second row.

A time-invariant phase domain

Define

$$\mathcal{D}_* = \operatorname{dom} h^{-1} \cap \bigcap_{j=0}^{\infty} \operatorname{dom} h^j, \qquad \mathscr{P} = \mathcal{D}_* \oplus \mathcal{D}_*.$$

Use the real domains for phase vectors. Every compact smooth regular spatial section belongs to $\mathcal{D}_*$: positive powers follow because it is in the domain of every differential power of B_F, and the negative power follows from Corollary . Spectral cutoffs also show that $\mathcal{D}_*$ is dense.

For $F = (v, w)$, define

$$S_t F = \big(\cos(th)v - h\sin(th)w, \quad h^{-1}\sin(th)v + \cos(th)w\big).$$

The multiplier $h^{-1}\sin(th)$ is bounded, with norm at most $|t|$.

Lemma (Phase dynamics). *The maps S_t preserve $\mathscr{P}$, form a group, and preserve the nondegenerate symplectic form*

$$\sigma(F, G) = (v, w')_{\mathrm{R}} - (w, v')_{\mathrm{R}}, \qquad G = (v', w').$$

They are continuous in the graph seminorms defining $\mathcal{D}_*$.

The real-linear map

$$k_0(v, w) = \frac{1}{\sqrt{2}}\big(h^{-1/2}v + \mathrm{i}h^{1/2}w\big) \in \mathscr{K}$$

satisfies

$$k_0 S_t = e^{\mathrm{i}th} k_0, \qquad 2\mathrm{Im}\,\langle k_0 F, k_0 G\rangle = \sigma(F, G).$$

Proof. Positive-power graph bounds follow directly from spectral calculus and the availability of all positive powers. For the inverse domain, use

$$h^{-1}h\sin(th)w = \sin(th)w,$$

and

$$\|h^{-2}\sin(th)v\| \le |t|\|h^{-1}v\|.$$

These give preservation and continuity of the inverse graph seminorm as well.

The group and symplectic identities follow either by the trigonometric formulas in the spectral representation or direct multiplication of the two-by-two matrix in . Nondegeneracy follows by testing against compact smooth v' and w'.

Finally,

$$h^{-1/2}(S_tF)_1 + \mathrm{i}h^{1/2}(S_tF)_2 = e^{\mathrm{i}th}(h^{-1/2}v + \mathrm{i}h^{1/2}w).$$

Expanding the inner product gives the second identity in .

The covariance form

For $F = (v, w) \in \mathscr{P}$, set

$$\mu_{\beta_{\mathrm{T}}}(F,F) = \tfrac{1}{2}\left\| c_{\beta_{\mathrm{T}}}(h)^{1/2}h^{-1/2}v\right\|^2 + \tfrac{1}{2}\left\| c_{\beta_{\mathrm{T}}}(h)^{1/2}h^{1/2}w\right\|^2.$$

Its real symmetric polarization is denoted $\mu_{\beta_{\mathrm{T}}}(F,G)$. All these quantities are finite by Lemma .

For compact v supported in K, that lemma and Theorem give

$$\mu_{\beta_{\mathrm{T}}}((v,0),(v,0)) \le C_K(1+\beta_{\mathrm{T}}^{-1})(\|v\|_{L^2}^2 + \|v\|_{L^1}^2).$$

Construction of the quasifree KMS state

The algebra and the KMS convention

Let $\mathfrak{A}(\mathscr{P},\sigma)$ be the Weyl C^*-algebra with

$$W(F)W(G) = e^{-\mathrm{i}\sigma(F,G)/2}W(F+G), \qquad W(F)^* = W(-F),$$

and dynamics

$$\tau_t(W(F)) = W(S_t F).$$

Definition (KMS condition used here). A β_{T}-KMS state for τ is an invariant state ω such that, for every A, B in the algebra, there is a bounded continuous function on

$$\{z : 0 \le \operatorname{Im} z \le \beta_{\mathrm{T}}\},$$

holomorphic in its interior, satisfying

$$F_{A,B}(t) = \omega(A\tau_t(B)), \qquad F_{A,B}(t + \mathrm{i}\beta_{\mathrm{T}}) = \omega(\tau_t(B)A).$$

For the state constructed below, time translations are strongly continuously implemented in its GNS representation.

Time evolution on the universal Weyl algebra need not be point-norm continuous. The strip formulation and the representation-level continuity are specified rather than silently assuming that missing property; compare [3, Sections 2.1–2.2].

Positivity by a doubled one-particle construction

Let $\overline{\mathscr{K}}$ be the conjugate Hilbert space. For $u = k_0 F$, define

$$k_{\beta_{\mathrm{T}}} F = \left((1 + n_{\beta_{\mathrm{T}}}(h))^{1/2} u, \quad \overline{n_{\beta_{\mathrm{T}}}(h)^{1/2} u} \right) \in \mathscr{K} \oplus \overline{\mathscr{K}}.$$

Lemma (Thermal one-particle identities). *For $F, G \in \mathscr{P}$,*

$$\left\| k_{\beta_{\mathrm{T}}} F \right\|^2 = \mu_{\beta_{\mathrm{T}}}(F, F), \qquad 2\operatorname{Im} \left\langle k_{\beta_{\mathrm{T}}} F, k_{\beta_{\mathrm{T}}} G \right\rangle = \sigma(F, G).$$

Moreover,

$$k_{\beta_{\mathrm{T}}} S_t = U_{\beta_{\mathrm{T}}}(t) k_{\beta_{\mathrm{T}}}, \qquad U_{\beta_{\mathrm{T}}}(t) = e^{\mathrm{i}th} \oplus e^{-\mathrm{i}t\overline{h}}.$$

Proof. The squared norm is

$$\left\langle u, (1 + 2n_{\beta_{\mathrm{T}}}(h))u \right\rangle.$$

Since v, w are real vectors in the complexification and the spectral functions of h preserve the real space, the mixed term between $h^{-1/2}v$ and $\mathrm{i}h^{1/2}w$ has zero real part. This gives .

The inner product on the conjugate space reverses the imaginary part. Hence the two Bose contributions cancel in the imaginary part, leaving $2\mathrm{Im}\left\langle k_0 F, k_0 G \right\rangle = \sigma(F, G)$. The intertwining identity follows from Lemma and commutation of spectral functions.

Proposition (A regular centered quasifree state). *The formula*

$$\text{boxed}\, \omega_{\beta_{\mathrm{T}}}(W(F)) = \exp\left[-\frac{1}{2}\mu_{\beta_{\mathrm{T}}}(F, F)\right]$$

defines a positive normalized centered quasifree state on $\mathfrak{A}(\mathscr{P}, \sigma)$. It is invariant under τ_t, is regular, and has finite polynomial field moments on $\mathscr{P}$.

Proof. Take the symmetric Fock space over $\mathscr{K} \oplus \overline{\mathscr{K}}$. Use Fock Weyl operators normalized by

$$W_{\mathrm{F}}(\xi)W_{\mathrm{F}}(\eta) = e^{-\mathrm{i}\mathrm{Im}\,\langle \xi, \eta \rangle} W_{\mathrm{F}}(\xi + \eta), \qquad \left\langle \Omega, W_{\mathrm{F}}(\xi)\Omega \right\rangle = e^{-\|\xi\|^2/2}.$$

Lemma makes $W(F) \mapsto W_{\mathrm{F}}(k_{\beta_{\mathrm{T}}} F)$ a representation of the declared CCR. Its vacuum expectation is . This proves positivity and normalization.

Equation proves invariance. For each F, the characteristic function on $s \mapsto W(sF)$ is a finite Gaussian, proving regularity. Products of finitely many corresponding Fock fields have finite vacuum moments, or equivalently their moments are given by the finite Wick sums of the finite two-point forms.

The doubled construction is the Araki–Woods mechanism [1, 2]. It is not a Gibbs density-matrix construction in a finite spatial box, and no finite partition function is assumed.

The strip proof, including bounded observables

Theorem (Friedrichs KMS state). *For every $0 < \alpha < 1$, $0 \le \beta < 1$, and $\beta_{\mathrm{T}} > 0$, the state satisfies Definition on*

$\mathfrak{A}(\mathscr{P},\sigma)$. *Its GNS dynamics is strongly continuous.*

In particular, restriction to any invariant compact-test subalgebra constructed in Section is a regular KMS state with finite compact-test second moments.

Proof. Let $F, G \in \mathscr{P}$, $u = k_0 F$, $v = k_0 G$. For z in the closed KMS strip define

$$H_{F,G}(z) = \left\langle u, (1 + n_{\beta_T}(h))e^{izh}v \right\rangle + \left\langle v, n_{\beta_T}(h)e^{-izh}u \right\rangle.$$

These are weighted spectral pairings.

If $0 \le y \le \beta_T$, their nonnegative scalar weights obey

$$(1 + n_{\beta_T}(\omega))e^{-y\omega} \le 1 + n_{\beta_T}(\omega), \qquad n_{\beta_T}(\omega)e^{y\omega} \le 1 + n_{\beta_T}(\omega).$$

Weighted Cauchy–Schwarz therefore bounds uniformly on the closed strip. Dominated spectral convergence proves continuity. Inside the strip, exponential damping at large ω and the already integrable Bose weight at zero justify holomorphy, locally uniformly in z.

At real t,

$$H_{F,G}(t) = \left\langle k_{\beta_T} F, U_{\beta_T}(t) k_{\beta_T} G \right\rangle.$$

The scalar identities

$$(1 + n_{\beta_T}(\omega))e^{-\beta_T \omega} = n_{\beta_T}(\omega), \qquad n_{\beta_T}(\omega)e^{\beta_T \omega} = 1 + n_{\beta_T}(\omega)$$

give

$$H_{F,G}(t + i\beta_T) = H_{G,F}(-t).$$

For a pair of Weyl generators set

$$\mathscr{F}_{F,G}(z) = \exp\left[-\frac{1}{2}\mu_{\beta_T}(F,F) - \frac{1}{2}\mu_{\beta_T}(G,G) - H_{F,G}(z) \right].$$

It is bounded and continuous on the strip, and holomorphic inside. The Weyl relations and the Fock expectation

yield

$$\mathscr{F}_{F,G}(t) = \omega_{\beta_T}(W(F)\tau_t(W(G))).$$

Equation gives precisely the reversed upper boundary in .

Finite linear combinations give the strip condition for Weyl polynomials. For such polynomials A, B, both boundary functions have absolute value at most $\|A\|\|B\|$. The maximum principle for bounded holomorphic functions on a strip gives the same bound throughout the strip.

Now approximate arbitrary A, B in norm by Weyl polynomials. The strip functions form a uniformly Cauchy sequence, by the same bound applied to differences in either argument. Their uniform limit has the required continuity, holomorphy, and boundary values. This proves the condition for every bounded observable in the algebra.

Finally, $U_{\beta_T}(t)$ is strongly continuous. Its second quantization strongly continuously implements the dynamics in the Fock representation and fixes the vacuum. Restricting to the cyclic subspace generated by the represented Weyl algebra gives the strongly continuous GNS implementation.

Remark (Existence, not global uniqueness). No uniqueness among all regular KMS states is asserted. In particular, the absence of an L^2 zero eigenvector does not by itself classify all distributional stationary additions or all possible equilibrium sectors. The displayed Bose prescription defines the state proved here.

Compact spacetime fields, continuity, and the commutator

For $f \in \mathcal{T}$, define a phase label by

$$F_f = (v_f, w_f) = \left(\int_{\mathbb{R}} \cos(th) f(t)\, \mathrm{d}t, \quad \int_{\mathbb{R}} h^{-1} \sin(th) f(t)\, \mathrm{d}t \right).$$

The integrals converge in every needed graph seminorm: all spatial supports lie in one compact set, positive powers are uniformly controlled by test seminorms, and

$$\|h^{-1} f(t)\| \le C_K(\|f(t)\|_{L^2} + \|f(t)\|_{L^1}).$$

Thus $F_f \in \mathscr{P}$.

Define

$$\Phi(f) = \varphi(F_f),$$

where $\varphi(F)$ is the regular Weyl field. Let $\mathfrak{A}_c$ be the C^*-subalgebra generated by the $W(F_f)$, $f \in \mathcal{T}$. It is invariant, since

$$F_{f_a} = S_a F_f.$$

Theorem (Compact-test thermal field). *The restriction of ω_{β_T} to $\mathfrak{A}_c$ has the following properties.*

1. *Its two-point form*

$$\omega_{\beta_T,2}(f,g) = \left\langle k_{\beta_T} F_f, k_{\beta_T} F_g \right\rangle$$

is continuous in the compact-test topology and has finite diagonal values.

2. *It satisfies the field equation*

$$\Phi((\partial_t^2 + B_F)f) = 0.$$

3. *Its commutator is extension selected:*

$$= -\mathrm{i} \int_{\mathbb{R}^2} (f(t), E_F(t-s)g(s))_{\mathrm{R}} \mathrm{d}t\, \mathrm{d}s\, I, \qquad E_F(u) = \frac{\sin(uh)}{h}.$$

4. *It is a centered quasifree β_T-KMS state.*

Proof. Write $F_f = \int S_t(f(t), 0)\mathrm{d}t$. The thermal one-particle map and its intertwining property give

$$k_{\beta_T} F_f = \int U_{\beta_T}(t) k_{\beta_T}(f(t), 0)\, \mathrm{d}t.$$

Minkowski's inequality and imply

$$\left\| k_{\beta_T} F_f \right\| \le C_K (1 + \beta_T^{-1})^{1/2} \int_{\mathbb{R}} (\|f(t)\|_{L^1} + \|f(t)\|_{L^2}) \mathrm{d}t.$$

On every fixed compact spacetime support, the right side is bounded by a test-section seminorm. Cauchy–Schwarz

proves continuity of the two-point form. The kernel theorem therefore also gives its distributional two-point kernel, without any wavefront assertion.

For the equation, integrate twice by parts in t in . There are no temporal boundary terms, and

$$(\partial_t^2 + h^2)\cos(th) = 0, \qquad (\partial_t^2 + h^2)h^{-1}\sin(th) = 0.$$

All operator operations are allowed by the graph-domain control. Hence $F_{(\partial_t^2 + B_F)f} = 0$.

For the commutator, use the symplectic form:

$$\begin{aligned}\sigma(F_f, F_g) &= \int_{\mathbb{R}^2} (f(t), h^{-1}[\cos(th)\sin(sh) - \sin(th)\cos(sh)]g(s))_{\mathrm{R}}\,\mathrm{d}t\,\mathrm{d}s \\ &= -\int_{\mathbb{R}^2} (f(t), h^{-1}\sin((t-s)h)g(s))_{\mathrm{R}}\,\mathrm{d}t\,\mathrm{d}s.\end{aligned}$$

This is .

The final assertion follows from restriction of Theorem to the invariant subalgebra.

The two-point spectral expression

In weak compact-test notation, the configuration two-point function is

$$\text{boxed}\, C^+_{\beta_{\mathrm{T}}}(t-s) = \frac{(1 + n_{\beta_{\mathrm{T}}}(h))e^{-\mathrm{i}(t-s)h} + n_{\beta_{\mathrm{T}}}(h)e^{\mathrm{i}(t-s)h}}{2h}.$$

This equation denotes the pairing on compact spatial tests and its subsequent compact time smearing. It does not claim that its multiplier is a bounded operator on all of L^2.

The antisymmetric part is temperature independent:

$$C^+_{\beta_{\mathrm{T}}}(u) - C^+_{\beta_{\mathrm{T}}}(-u) = -\mathrm{i}\frac{\sin(uh)}{h}$$

in the real-field ordering, with the corresponding conjugate ordering for the charged field.

Remark (Why the proof is not just a formal Bose substitution). The construction has supplied all of:

form domain, positive state, KMS strip, compact-test continuity, field equation, CCR.

In particular, controls the factor $n_{\beta_T}(h)/h \sim (\beta_T h^2)^{-1}$. A ground-state estimate involving only $h^{-1/2}$ would not justify these same thermal integrals.

Weak Euclidean identification and the fixed-operator limit

Periodization without a coincidence limit

Proposition (Euclidean two-point periodization). *For* $0 < \tau < \beta_T$, *the thermal Euclidean covariance has the weak compact-test expression*

$$C^E_{\beta_T}(\tau) = \frac{e^{-\tau h} + e^{-(\beta_T - \tau)h}}{2h(1 - e^{-\beta_T h})}.$$

On compact spatial tests it equals the absolutely convergent weak periodization

$$\text{boxed}\, C^E_{\beta_T}(\tau) = \sum_{n \in \mathbb{Z}} \frac{e^{-|\tau + n\beta_T|h}}{2h}.$$

The equality is locally uniform for τ *in compact subintervals of* $(0, \beta_T)$.

Proof. Analytically continue to $t - s = -i\tau$. This gives . For each scalar $\omega > 0$, summing the two geometric series gives

$$\sum_{n \in \mathbb{Z}} \frac{e^{-|\tau + n\beta_T|\omega}}{2\omega} = \frac{e^{-\tau\omega} + e^{-(\beta_T - \tau)\omega}}{2\omega(1 - e^{-\beta_T \omega})}.$$

Near zero the right side is $O(\omega^{-2})$; away from zero it is bounded on a closed interior τ-interval.

For a diagonal compact-test pairing, the summands are nonnegative spectral forms. Tonelli and Corollary prove convergence and the scalar-to-operator identity. For a mixed pairing, apply Cauchy–Schwarz to each positive summand and then to the sum. The same dominating form proves the local uniformity.

Thus the Euclidean Bose periodization used in a prescribed Friedrichs calculation is identified at the two-point compact-test level. This does not by itself justify a coincidence limit, differentiation at the apex, or a distributional extension of one-point quantities.

The Tdownarrow 0 limit

Let

$$\mu_\infty(F,F) = \frac{1}{2}\|h^{-1/2}v\|^2 + \frac{1}{2}\|h^{1/2}w\|^2, \qquad F = (v,w),$$

and

$$\omega_\infty(W(F)) = e^{-\mu_\infty(F,F)/2}.$$

This is the spectral ground-state covariance on the common phase domain.

Theorem (Zero-temperature convergence at fixed operator). *For every $F = (v,w) \in \mathscr{S}$,*

$$0 \le \mu_{\beta_T}(F,F) - \mu_\infty(F,F) \le \beta_T^{-1}(\|h^{-1}v\|^2 + \|w\|^2).$$

Consequently, as $\beta_T \to \infty$:

1. *$\omega_{\beta_T} \to \omega_\infty$ weakly as states on the common Weyl algebra;*

2. *the compact-spacetime two-point distributions converge to the spectral ground-state distribution, with an $O(\beta_T^{-1})$ bound on fixed test-support stages. No mass limit is asserted.*

Proof. The difference of the two quadratic forms is

$$\left\|n_{\beta_T}(h)^{1/2}h^{-1/2}v\right\|^2 + \left\|n_{\beta_T}(h)^{1/2}h^{1/2}w\right\|^2.$$

Use $n_{\beta_T}(\omega) \le (\beta_T\omega)^{-1}$ to obtain .

The characteristic functions therefore converge on every Weyl generator, hence on their finite linear span. States have norm one, so norm density extends convergence to every element of the common C^*-algebra.

For spacetime tests supported in a fixed bounded time interval,

$$\left\|h^{-1}v_f\right\| \le \int \|h^{-1}f(t)\|\mathrm{d}t, \qquad \|w_f\| \le \int |t|\|f(t)\|\mathrm{d}t.$$

These are bounded by fixed-support test seminorms using Corollary . The difference of the two-point forms is the positive symmetric form obtained by polarizing $\mu_{\beta_T} - \mu_\infty$; their commutators agree. Cauchy–Schwarz for this difference form yields the claimed $O(\beta_T^{-1})$ bilinear distribution estimate.

The limit keeps the massless operator B_F, the host, and holonomy fixed. It is not I-B-VP-3's $m$downarrow 0 limit of a massive ground state [14]. The two parameters, constructions, and comparison statements are different.

What remains downstream or outside the release

The recorded one-point theorem

For its own full one-point prescription, the supplied I-B-VP-1 thm:thermal records:

• a locally integrable thermal correction across the apex;

• the associated regular Ward identities and vacuum residue comparison;

• loss of longitudinal boost invariance;

• an enlarged contact class, including the longitudinal traceless anisotropic contact

$$\lambda_{\mathrm{th}}\,\mathrm{diag}\,(1,-1)\delta_{\mathscr{W}}$$

in the t, z block;

• the reported five/four contact counts in its stated classes. These are version-qualified attributions [12], not new derivations in this manuscript.

Theorems and now supply the canonical centered Friedrichs KMS state, and Proposition identifies its weak Euclidean Bose prescription. They do not independently audit every differentiation, coincidence limit, Ward extension, or contact count in the one-point sheet.

No apex limit such as the supplied untwisted $T^2/(12\alpha)$ limit is used to prove state existence.

The dilation obstruction is not repaired

The vacuum part and its nonzero residues are not removed by adding a thermal correction. The dilation no-go of the supplied vacuum contact class remains its own result [12].

The present state is not claimed to be dilation invariant. Its temperature is an additional scale. A thermal contact is neither an interacting counterterm nor an operator-domain parameter.

Other non-ground states: a terminology correction

No excited, coherent, or stationary non-KMS state is constructed in this first stage. Those are separate successor classes, requiring their own domains and infrared checks.

One sentence of the proposed plan needs qualification: a coherent displacement is Gaussian and is sometimes called a quasifree state with nonzero mean. It is not *centered* quasifree in the convention used here. Nor is every coherent displacement automatically non-KMS: a displacement commuting with the dynamics can preserve a KMS condition. Therefore ``coherent states are not quasifree and not KMS'' is not used as a universal theorem.

A coherent-state construction would have to specify the classical displacement functional and its continuity. A finite-particle excitation would have to specify its representation and excitation vectors. Neither is obtained merely by changing apex contacts.

Massive thermal states

No massive KMS state is constructed here. The massive product ground-state gap and threshold results of I-B-VP-3 concern a different state prescription [14]. Above-threshold thermal existence would require its own Bose form-domain argument; at threshold the infrared problem must be checked rather than cancelled by a contact term.

The elementary inequality

$$\frac{1}{2\omega}\coth\left(\beta_{\mathrm{T}}\omega/2\right) \geq \frac{1}{2\omega}$$

also shows that an already divergent positive ground covariance is not made finite by adding positive Bose occupation to that same spectral prescription. This observation constructs no threshold state and does not classify every possible alternative prescription.

Non-Friedrichs and non-product thermal states

A nonnegative non-Friedrichs product operator may have a constructed spectral ground state, but its compact-test membership in dom $B^{-1/2}$ is a new obligation. The ground-state Krein correction does not answer that question.

All longitudinally non-product operators remain outside the state construction. The supplied I-B-VP-6 release is an operator-extension and spectral result, not a state theorem [13].

Wavefronts, currents, and response

No thermal wavefront theorem is asserted. The construction of a distributional two-point function does not identify its ordinary global wavefront set with the ground-state result of I-B-VP-2 [15].

No thermal charged-current formula is inferred from the ground-state current of I-B-VP-0, and no thermal Maxwell response is inferred from I-B-VP-4 [16, 17]. The latter's frozen-source calculation is not a self-consistent scalar–Maxwell state problem.

I-B-VP-5's operator-selection result is not a state-limit theorem and does not normalize thermal one-point contacts [18].

No geometric or gravitational identification

Breaking boost symmetry in a thermal rest frame does not produce a new metric. A cone KMS state is not a de Sitter state or an expanding universe. A horizon detailed-balance statement on a fixed black-hole background is not a proof of this cone KMS condition.

No thermal stress tensor is constructed here as a source. In particular, it is not a cosmological vacuum-energy calculation or a swampland test. The host is unchanged, and I-B's no-source distinction remains in force [9].

Finite Enter reporting

A finite installed dictionary may describe

$$\text{model identifier}, \quad \alpha, \quad \beta, \quad \beta_{\mathrm{T}}, \quad \text{Friedrichs prescription},$$
$$\text{test-support class}, \quad \text{infrared bound}, \quad \text{scope}, \quad \text{evidence and failure labels}.$$

Exact real numbers and arbitrary test functions require a declared representation convention; they are not automatically finite words.

Let $C \cong \mathbb{C}^{2^q}$, $q \geq 1$, have calibrated orthonormal basis $\{|\tau_j\rangle\}$. Let A contain all surviving finite registers, and install finite report words w_j, including unused and failure outcomes. Put

$$D_j = \langle \tau_j|_C \otimes I_A, \qquad \mathscr{M}(X) = \sum_j |j, w_j\rangle\langle j, w_j|_R \otimes D_j X D_j^\dagger.$$

Proposition (Complete supplied-temperature report). *The map is completely positive and trace preserving, and*

$$\mathrm{Tr}_R \mathscr{M}(X) = \mathrm{Tr}_C X.$$

The identities hold after adjoining every untouched finite reference. The returned quantum interface contains A, not C. A positive-probability branch retains its actual conditional residual; no normalized state is assigned to a zero branch.

Proof. The recorded Kraus maps are $|j, w_j\rangle_R \otimes D_j$, and

$$\sum_j D_j^\dagger D_j = \sum_j |\tau_j\rangle\langle\tau_j|_C \otimes I_A = I_{CA}.$$

Expanding X in clock matrix units identifies the output blocks with its diagonal clock blocks. Their sum is the partial trace. Tensoring the same calculation with a reference identity proves the complete assertion.

This is the established I-B/En09-A supplied-report interface, not a continuum-state acquisition theorem [9, 19]. It does not select a temperature, prepare the thermal field, or establish its own analytical estimate by measurement. A finite dictionary state is not a continuum field state.

Finite testing could certify an identity on an independently proved finite-dimensional residual class if the probe map were injective there. A finite table alone establishes neither that class membership nor injectivity, as emphasized in the supplied coverage audit [20]. No thermometry apparatus or statistical identification protocol is constructed.

Release and unchanged boundaries

Theorem (I-B-VP-8 first-stage release). *For the prescribed massless regular conical product, fixed holonomy, Friedrichs extension, and centered zero-chemical-potential Bose prescription:*

1. *The compact-test heat estimate and its leading lowest-angular-mode coefficient are –.*

2. *Compact tests belong to* $\operatorname{dom} B_F^{-1/2}$*, with the thermal infrared tail . For nonzero leading moments, the fractional threshold in Theorem is optimal.*

3. *For every* $\beta_{\mathrm{T}} > 0$*, formula constructs a positive regular centered quasifree KMS state. Its compact-spacetime restriction is continuous, has finite second moments, and satisfies the field equation and the extension-selected CCR.*

4. *Its weak Euclidean covariance is the periodization . At fixed massless operator the states have the* $T\mathrm{downarrow}\,0$ *limit and bound of Theorem .*

5. *The inherited no-go remains valid on unstable massless product extensions in the compact-test finite-second-moment class.*

6. *No massive, non-Friedrichs, non-product, excited, or coherent state is released by these results. No thermal wavefront theorem, new one-point contact theorem, backreaction law, or continuum acquisition interface is constructed.*

Proof. Theorem proves (i). Theorem and Corollary prove (ii). Proposition , Theorem , and Theorem prove (iii). Section proves (iv). Theorem proves (v). Section and the declared construction domains give the exclusions in (vi).

begingroup 1.12 toprule Gate Verdict Exact scope

midrule endfirsthead toprule Gate Verdict Exact scope

midrule endhead bottomrule endfoot

V8.0: host and interface Frozen Prescribed regular cone, holonomy, dynamics, temperature, and finite report.

V8.1: unstable product wall Retained, proof given No invariant compact-test finite-second-moment state when the massless product has a negative transverse bound channel.

V8.2: Friedrichs KMS Constructed Every $\beta_{\mathrm{T}} > 0$, zero chemical potential, all compact smooth regular tests, stronger infrared estimate, bounded-observable strip condition.

V8.3: one-point corrections Cited downstream No recomputation or independent contact-count audit.

V8.4: other non-ground states Deferred No excitation or displacement construction. Coherent-state terminology is qualified.

V8.5: massive thermal Deferred No transfer from the massive spectral ground state or its threshold theorem.

V8.6: other extensions Deferred No non-Friedrichs or non-product KMS conclusion.

V8.7: further observables Deferred No thermal wavefront, charged current, Maxwell response, or new apex-contact classification.

endgroup

The unrelated characteristic programme

The supplied XIV–XV characteristic-composition material is not the problem solved by this state construction. No full characteristic graph map, joint section, transverse graph estimate, contact compatibility theorem, output wavefront theorem, or admitted interacting coefficient is supplied here.

XIV-I's version-specific I2 internal-amplitude advance is not withdrawn. Its existence does not discharge the full I3/J2 composition obligations. XIV-H's historical H2 ending and those later stops retain their version-qualified

meanings. The downstream XV-B quantum-admission exit is not opened.

Standing statuses

No result here discharges XVI-N or Route B. The standing XVI-O status remains

$$\text{nolinkurl}\, O.1 = NOT_C LOSED, \qquad \text{nolinkurl}\, bulk_s ymmetry_r estored = false.$$

These are scope statements, not a new audit of that programme. They do not erase the separately scoped two-point claim recorded in the supplied XVI-Q Q11 passages; that claim does not update XVI-O or become a premise of the present KMS construction.

No OpenAI catalogue entry, family number, abstract, or formalization report is used as a theorem premise. The philosophical essay is interpretive background, not a registered analytical premise. I-B-VP-AB and I-B-VP-SAE are not used as registered premises.

There is no emergence, Einstein dynamics, Maxwell or Einstein backreaction, Page curve, de Sitter state, cosmological bounce, or admitted coefficient of the inherited characteristic quantum graph.

In the requested programme reading, Problems 1, 2, and 8 remain unfinished; Problems 3–7 and 9–11 are not reached; Problem 6 is explicitly not the implication attempted here; and Problem 12 remains unresolved.

appendix

Debugging and domain audit

begingroup 1.12 toprule Potential error Required correction

midrule endfirsthead toprule Potential error Required correction

midrule endhead bottomrule endfoot

Using β for holonomy and inverse temperature Use β for the bundle character and β_{T} for inverse temperature.

Calling nonnegativity an infrared proof Prove compact-test membership in $\operatorname{dom} B_F^{-1/2}$.

Reusing the ground-state $B_F^{-1/4}$ estimate as the thermal estimate The Bose configuration multiplier is of order B_F^{-1} at zero; Lemma states the exact form domain.

Assuming a one-point thermal formula constructs a state Supply positivity, CCR, continuity, regularity, and the KMS condition before coincidence limits.

Using a globally hyperbolic existence theorem on the entire punctured cone Construct the algebra and dynamics from the fixed self-adjoint operator.

Assuming point-norm continuous dynamics on the universal Weyl algebra Use the stated strip formulation and prove strong GNS continuity.

Proving only field detailed balance and calling it the full KMS theorem Extend the Weyl strip functions to every bounded algebra element by norm approximation and the strip maximum principle.

Replacing the charged covariance by its pointwise real part Use realification and reconstruct the charge and conjugate sectors consistently.

Using noncompact unstable data as though they were compact field tests Theorem starts from a compact spatial test with nonzero negative spectral measure.

Ignoring the nonnegative spectral contribution in the no-go Bound it by $O(t_0)$; the negative part grows exponentially with a fixed sign.

Transferring product instability to a momentum-dependent boundary law No such state or no-go conclusion is made for I-B-VP-6.

Identifying absence of an L^2 zero mode with uniqueness of every KMS state Only the prescribed centered Bose state is constructed; distributional stationary additions are not classified.

Claiming every coherent state is non-Gaussian or non-KMS Distinguish centered quasifree from displaced Gaussian, and check whether the displacement commutes with dynamics.

Equating $T\downarrow 0$ with $m\downarrow 0$ The proved limit fixes the massless operator. No mass limit is included.

Promoting weak periodization to an apex coincidence theorem Proposition concerns compact-test two-point forms only.

Calling a thermal contact an extension parameter It changes a one-point distributional extension, not the operator domain or the Bose state prescription.

Calling a supplied temperature code continuum thermometry A preparation or acquisition interface and its calibration would be separate constructions.

endgroup

Source and verification discipline

The online I-B and philosophy pages were consulted for background and interface scope. The publisher record of the Enter article and the primary sources listed below were consulted for the attributed frameworks. The consulted publisher record reports acceptance of the Enter article on September 28, 2026 and says that its final formatted version is forthcoming.

The supplied version-qualified I-B-VP passages identify the open thermal-state row, the unstable-product no-go, and the separate ground-state estimate. Their use is explicit. The needed Friedrichs heat, inverse-power, state, and KMS arguments are given in this manuscript.

The supplied I-B-VP-6-v2.tex is used to identify the non-product boundary and the absence of a state construction there. None of its operator theorems is needed as a premise for the Friedrichs KMS proof.

The supplied I-B-VP-1 description of thm:thermal is cited as an assumed-state one-point result. The complete online manuscript was not independently inspected here. No missing metadata or unavailable proof is reconstructed from a label.

The large supplied retrieval inventory is not treated as a corpus-wide audit, a source dependency theorem, or

permission to import unrelated results.

Declarations

Authorship. The author field is left blank for submission preparation. The bibliography attributes the companion records to N. Cheung without assigning authorship of this draft.

Novelty scope. The claimed contribution is the explicit thermal infrared gate, its sharp lowest-mode heat analysis, and the domain-complete first-stage KMS construction on the fixed cone. Quasifree KMS theory, Araki–Woods doubling, Bessel heat kernels, and finite-temperature cosmic-string Green functions are established prior subjects.

Physical scope. The metric, cone, holonomy, extension, time flow, temperature, and Bose prescription remain inputs. No physical preference among temperatures or states is claimed.

Quantum scope. A free scalar thermal state is constructed. No interacting characteristic quantum coefficient, Ward admission, interacting algebra, or quantum-gravitational completion is claimed.

Verification. No successful pdflatex execution, numerical computation, proof-assistant verification, independent rebuild, independent review, registry installation, or journal acceptance of this manuscript is asserted. The results claimed are the written mathematical theorems on their stated domains.

References

[1] H. Araki and E. J. Woods, Representations of the canonical commutation relations describing a nonrelativistic infinite free Bose gas, *Journal of Mathematical Physics* **4** (1963), 637–662. DOI: 10.1063/1.1704002. The doubled quasifree thermal representation is established prior work.

[2] J. Dereziński, Introduction to representations of the canonical commutation and anticommutation relations, *Lecture Notes in Physics* **695** (2006), 63–143. DOI: 10.1007/3-540-32579-4_3. https://arxiv.org/abs/math-ph/0511030. Especially the KMS convention and Section 9.3 on Araki–Woods representations.

[3] K. Sanders, Thermal equilibrium states of a linear scalar quantum field in stationary spacetimes, *International Journal of Modern Physics A* **28** (2013), 1330010. DOI: 10.1142/S0217751X1330010X. https://arxiv.org/abs/1209.6068. Algebraic KMS and quasifree-state framework. No globally hyperbolic existence theorem is applied to the entire regular cone.

[4] M. E. X. Guimarães, *Vacuum Polarization at Finite Temperature around a Magnetic Flux Cosmic String*. Primary preprint, arXiv:gr-qc/9502003 (1995). https://arxiv.org/abs/gr-qc/9502003. Historical thermal Green-function and one-point context; not a substitute for the compact-test KMS proof here.

[5] J. Dereziński and S. Richard, On Schrödinger operators with inverse square potentials on the half-line, *Annales Henri Poincaré* **18** (2017), 869–928. DOI: 10.1007/s00023-016-0520-7. https://arxiv.org/abs/1604.03340. Radial realizations and generalized Hankel spectral analysis.

[6] F. W. J. Olver et al., editors, *NIST Digital Library of Mathematical Functions*. Chapter 10, especially equations 10.22.67 and 10.25.2. https://dlmf.nist.gov/10.22. https://dlmf.nist.gov/10.25. Bessel Gaussian integral and modified-Bessel power series.

[7] N. Cheung, ``The Enter Calculus: A Linear Quantum Lambda Calculus with an Operational Characterization of Measurement," *Frontiers in Physics* **14** (2026), article 1954943. DOI: 10.3389/fphy.2026.1954943. https://doi.org/10.3389/fphy.2026.1954943. Interface/operational background only; not a PDE premise.

[8] N. Cheung, *The Philosophy of Enter: Destructive Readout, Quantum Recovery, and the Refusal of Identification*, Cheung Ngo Medical Limited (2026). Framing essay; not a theorem premise. Programme note: https://enter.grok.me/enter.

[9] N. Cheung, *I-B-v1: A Finite Enter Spacetime: Calibrated Destructive Time Readout on a Prescribed Conical Product*, I-B-v1.tex, version 1. Enter Calculus Companion. https://enter.grok.me/browse/I-B. Prescribed host, thm:instrument, and prop:not-source. Companion record; not a Scholar-indexed journal article.

[10] N. Cheung, *I-B-G*, I-B-G-v2.tex, version 2. Enter Calculus Companion. Supplied version-qualified passages:

Friedrichs positivity, product dynamics, deficiency count, and regular-host non-global-hyperbolicity. No thermal state is imported from its Green operators. https://enter.grok.me/browse/I-B-G. Companion record; not a Scholar-indexed journal article.

[11] N. Cheung, *I-B-VP*, I-B-VP-v4.tex, version 3. Enter Calculus Companion. Supplied lem:infrared, thm:nogo, thm:friedrichs, thm:krein, and open thermal row. The compact-test unstable-product argument is retained; the thermal infrared estimate is proved separately. https://enter.grok.me/browse/I-B-VP. Companion record; not a Scholar-indexed journal article.

[12] N. Cheung, *I-B-VP-1*, I-B-VP-1-v4.tex, version 3. Enter Calculus Companion. Supplied thermal one-point description and thm:nogo dilation passages. Its thm:thermal assumes the state and concerns one-point extensions and contacts. Those calculations are not repeated or independently certified here. https://enter.grok.me/browse/I-B-VP-1. Companion record; not a Scholar-indexed journal article.

[13] N. Cheung, *I-B-VP-6: Longitudinally non-product self-adjoint extensions on the prescribed conical spacetime*, I-B-VP-6-v2.tex, version 1. Full source supplied with the present problem. Measurable relation fields, non-product spectral examples, and thm:release. It constructs no thermal state. https://enter.grok.me/browse/I-B-VP-6. Companion record; not a Scholar-indexed journal article.

[14] N. Cheung, *I-B-VP-3*, I-B-VP-3-v2.tex, version 1. Enter Calculus Companion. Supplied thm:gapped, thm:threshold, and thm:masslesslimit. Massive spectral ground-state and threshold results, not a massive KMS construction. https://enter.grok.me/browse/I-B-VP-3. Companion record; not a Scholar-indexed journal article.

[15] N. Cheung, *I-B-VP-2*, I-B-VP-2-v2.tex, version 1. Enter Calculus Companion. Supplied thm:main. Ordinary global wavefront of the massless Friedrichs ground covariance; no thermal wavefront is transferred. https://enter.grok.me/browse/I-B-VP-2. Companion record; not a Scholar-indexed journal article.

[16] N. Cheung, *I-B-VP-0*, I-B-VP-0-v2.tex, version 1. Enter Calculus Companion. Supplied Friedrichs charged-current record. No thermal current is inferred. https://enter.grok.me/browse/I-B-VP-0. Companion record; not a Scholar-indexed journal article.

[17] N. Cheung, *I-B-VP-4*, I-B-VP-4-v2.tex, version 1. Enter Calculus Companion. Supplied frozen-current Maxwell response and prop:nofeedback. Not a thermal scalar–Maxwell state construction. https://enter.grok.me/browse/I-B-VP-4. Companion record; not a Scholar-indexed journal article.

[18] N. Cheung, *I-B-VP-5*, I-B-VP-5-v2.tex, version 1. Enter Calculus Companion. Supplied thm:selection, prop:contactseparation, and finite supplied-report instrument. Operator selection is not thermal-state convergence or contact normalization. https://enter.grok.me/browse/I-B-VP-5. Companion record; not a Scholar-indexed journal article.

[19] N. Cheung, *En09-A*, version 1. Enter Calculus Companion. Supplied complete relational Enter theorem. https://enter.grok.me/browse/En09-A. Finite destructive extraction and the distinction between a reporting channel and additional dynamical hypotheses. Companion record; not a Scholar-indexed journal article.

[20] N. Cheung, *Oai282-A*, version 1. Enter Calculus Companion. Supplied prop:coverage and prop:invisible. https://enter.grok.me/browse/Oai282-A. Finite-probe coverage discipline only; no continuum thermometry theorem is imported. Companion record; not a Scholar-indexed journal article.

[21] N. Cheung, *Black-Hole Information as an Access Problem: Enter, Quantum Recovery, and Complete Residuals*, figshare (2026). DOI: 10.6084/m9.figshare.34335024.v1. https://doi.org/10.6084/m9.figshare.34335024.v1.

[22] N. Cheung, *Operational Wormhole Certificates—Existence, Dimension-Dependent Scaling, Operational Functions, and Composition*, figshare (2026). DOI: 10.6084/m9.figshare.34549575.v1. https://doi.org/10.6084/m9.figshare.34549575.v1.

[23] N. Cheung, *Enter, Quantum Magic, and Quantum Gravity: Readout, Recovery, and the Limits of Geometric Response*, figshare (2026). DOI: 10.6084/m9.figshare.34335582.v1. https://doi.org/10.6084/m9.figshare.34335582.v1.

[24] N. Cheung, *Operational Consistency Does Not Select Lorentzian Geometry*, figshare (2026). DOI:

10.6084/m9.figshare.34441758.v1. https://doi.org/10.6084/m9.figshare.34441758.v1.

[25] N. Cheung, *Realization of Relational Time by Destructive Enter on a Clock Register*, figshare (2026). DOI: 10.6084/m9.figshare.34442907.v1. https://doi.org/10.6084/m9.figshare.34442907.v1.