Enter, I-B-VP-9-Ex1-v2, 2026, pp. 1-40

I-B-VP-9-Ex Exceptional reductions, conditional apex contacts, and engineered smooth matrix cores

A self-contained extension on a prescribed conical spacetime

Ngo Cheung

Abstract

We study three questions left separate by the generic matrix-extension calculation: exceptional reductions of its radial special functions, the hypotheses needed for a finite apex-contact ledger, and realization of a declared boundary matrix by shrinking smooth cores. An important correction is necessary. An unrestricted assertion that Barnes reduction occurs only for equal orders, diagonal matrices, or scale-invariant relations is false. Mixed rank-one matrices reduce the Barnes denominator, and commensurate unequal orders admit finite univariate Barnes sums even for positive-definite mixed matrices. We prove instead a sharp structural classification: a constant decomposition into scalar Bessel channels exists precisely when the boundary relation respects the unequal-order decomposition, or when the orders coincide. We also classify all dilation-invariant relations and recover the equal-order projective line without extending it to unequal orders.

For contacts, we prove a conditional three-parameter classification in an explicitly specified homogeneous normal-jet convention. Its two-parameter charge-conjugation specialization requires invariance of the extension and prescription, not merely half holonomy. Rotation-breaking projectors and finite matrices are recorded in separate sectors.

The constructive result is an explicit nonnegative smooth-cap approximation of every fixed finite $H \geq 0$, including a declared positive-definite mixed example. A matrix ground-state factorization gives Mosco and strong-resolvent convergence to the product realization $Nb = Ha$. The cap interaction is smooth-coefficient and finite-rank in

angular variables, but angularly nonlocal; no local-curvature-only realization is claimed. This construction separates engineered operator realizability from physical core selection, state convergence, and contact normalization. We prove the limiting apex-arrival support bound and retain supported Friedrichs transfer windows. Finally, finite classical reports are separated from continuum observables, quantum return, reservoir heat, and endpoint dynamics. No numerical table, compilation, interacting gravitational coefficient, backreaction, or spacetime-emergence claim is made.

Bibliographic record

Title	I-B-VP-9-Ex Exceptional reductions, conditional apex contacts, and engineered smooth matrix cores
Author	Cheung, Ngo
Publication date	2026
Institution	Enter
Report number	I-B-VP-9-Ex1-v2
Publisher	Cheung Ngo Medical Limited
First page	1
Last page	40
Language	en

Book edition note (t425u)

Standing Enter refusals, scope vocabulary (proved here/supplied input/conditional/ not constructed/not claimed), and Scholar bibliography policy are frozen in the companion chapter *I-B-VP Book Ledger* v1. O.1 remains NOT_CLOSED; XVI-N is not discharged; XV-B is not opened; no Host C, Page/island, or spacetime-emergence claim; wired=false; SOURCE_DATE_EPOCH=1792627200. This sheet is a companion manuscript record and is *not* claimed to be Google Scholar-indexed. The published Enter Calculus article is cited only for finite-instrument

/ operational interface discussions, never as a PDE premise.

Book expansion: claim / counterexample / corrected theorem

Layout: (1) claim; (2) counterexample; (3) corrected theorem. Exceptional reductions, conditional contact classification, and the engineered core construction are separate contributions—not one broad universality claim. The engineered core is **angularly nonlocal**; that warning remains in the title/abstract/theorem reading.

Question, corrections, and philosophical orientation

The three mathematical tasks

The object of this paper is a prescribed massless scalar extension, not a derivation of its background. The generic calculation in Cheung's I-B-VP-9 provides the motivating matrix resolvent and three-variable Mellin–Barnes representation [1]. We do not repeat its nonrationality proof or replace its crossover functions by a Bernoulli polynomial. Instead, we address three logically distinct tasks:

1. identify structural reductions and distinguish them from accidental special-function reductions;

2. state exactly when a finite-dimensional contact space follows from symmetry, scaling, and a declared completion convention;

3. construct one fully declared shrinking-core family and prove convergence to its selected matrix relation. All mathematical ingredients used in the proofs are supplied below. Companion manuscripts are credited for the questions and comparisons, not used as unstated hypotheses.

The extension-theoretic background is classical. Boundary conditions at conical singularities, inverse-square radial operators, and finite-deficiency resolvent formulas are established subjects [16, 17, 18]. The contribution here is the particular structural classification, the correction of the proposed unrestricted exceptional list, the sector-by-sector contact audit, and the explicit factored cap construction for the declared matrix problem. No priority claim is made for self-adjoint extension theory, ground-state transformations, or Mosco convergence.

Two corrections that affect the release

The phrase ``the three-variable formula collapses'' is not a mathematical invariant until the allowed transformations are fixed. A constant unitary decomposition of radial channels is one precise notion. The number of contours in a chosen special-function representation is another. They are not equivalent.

For example, if H has rank one, its determinant term disappears, even when the orders are unequal and the off-diagonal entry is nonzero. More substantially, rationally related unequal orders permit partial fractions in one fractional momentum variable. Section proves this statement, including a positive-definite mixed example. Thus the development brief's proposed exceptional list is sharp only after restricting it to *structural channel reduction*. It cannot be submitted as a classification of every possible Barnes simplification.

A second correction concerns half holonomy. Charge conjugation is a symmetry of the half-holonomy differential expression, but need not preserve a selected boundary relation. A realization with one singular channel and one Friedrichs channel is an immediate counterexample. Consequently, the disappearance of an odd contact coefficient at $\beta = \frac{1}{2}$ requires a charge-conjugation-invariant realization and completion prescription. This distinction is essential in the projector sector.

Enter and spacetime Enter

The introductory *Philosophy of Enter* is useful as an interpretive map, not as a premise of the following analysis [2]. The finite Enter semantics and its characterization concern declared operations, names, records, and residuals [3]. Spacetime Enter places such finite interfaces on an already prescribed host [4]. It does not supply that host by solving the present scalar boundary problem.

The elementary interface can be written without importing a characterization theorem. For a finite measured register C, a surviving register B, and an orthonormal basis $\{|\tau_j\rangle\}$, set

$$D_j = \langle \tau_j|_C \otimes I_B, \qquad \mathscr{E}(X) = \sum_j |j\rangle\mathrm{langlej}\,|_R \otimes D_j X D_j^{\dagger}.$$

Then

$$\sum_j D_j^\dagger D_j = I_{CB}, \qquad \mathrm{Tr}_R \mathscr{E}(X) = \mathrm{Tr}_C X.$$

Both identities follow by expanding X in the clock basis. They also hold with an untouched reference.

Equation consumes a declared quantum access interface. It does not consume the geometric time axis, choose an apex extension, evaluate an unknown continuum expectation, or provide a field instrument. A finite report of a mathematical expression is not the physical acquisition of the corresponding field observable.

The Shredder Philosophy gives a related discipline: distinguish a classical report from coherent return, discarded access from physical annihilation, and a channel from its thermodynamic implementation [12, 14]. Section proves the finite-dimensional statements needed here. The analogy is not an identification of the cone apex with a horizon.

Standing inputs and exclusions

The cone, line holonomy, scalar normalization, extension, coupling ξ, and regular-host subtraction are inputs. In the core construction, the cap geometry, smooth magnetic connection, and engineered finite-angular-rank interaction are additional inputs.

Conservation on a prescribed host is not an Einstein source law. For instance, a constant null covector k_μ gives a nonzero conserved traceless tensor $k_\mu k_\nu$ in a flat chart, while its Einstein tensor is zero. This is the elementary separation relevant to I-B's source warning [4].

The present sheet does not construct a non-product relation field, a massive state, a KMS state, an interacting apex algebra, or a spin-2 core. The comparisons with I-B-VP-6, I-B-VP-8, and I-B-G-S remain comparisons of different problems [11, 6]. In particular, scalar conservation is not a transferred Maxwell or gravitational Ward identity.

The prescribed chart and its normalized resolvent

Assumption (Host and two deficient channels). Let

$$M = \mathbb{R}_t \times (0,\infty)_r \times S^1_\varphi \times \mathbb{R}_z, \qquad g = -\mathrm{mathop\,d}t^2 + \mathrm{mathop\,d}r^2 + \alpha^2 r^2 \mathrm{mathop\,d}\varphi^2 + \mathrm{mathop\,d}z^2,$$

with

$$\frac{1}{2} < \alpha < 1, \qquad 1 - \alpha < \beta < \alpha.$$

The scalar has angular equivariance

$$\Phi(\varphi + 2\pi) = e^{2\pi i \beta}\Phi(\varphi).$$

Write

$$q_0 = \beta, \quad q_1 = \beta - 1, \qquad u = \frac{\beta}{\alpha}, \quad v = \frac{1-\beta}{\alpha}, \quad N = \operatorname{diag}(u, v), \quad S = u + v = \alpha^{-1}.$$

Thus $0 < u, v < 1$ and $1 < S < 2$. All other angular channels are Friedrichs. The transverse realization is independent of longitudinal momentum.

The missing worldsheet is

$$\mathscr{W} = \mathbb{R}_t \times \{o\} \times \mathbb{R}_z.$$

The punctured spacetime is not globally hyperbolic: a causal curve can approach its missing timelike axis in finite coordinate time, and causal diamonds need not be compact. A global equation inverse therefore requires additional apex information. The distinction between local hyperbolicity and a selected Green operation is also central to I-B-G [5].

For $0 < v < 1$, the radial expression is

$$L_v = -\partial_r^2 - r^{-1}\partial_r + v^2 r^{-2} \quad \text{on} L^2((0,\infty), r\mathrm{mathop\,d}r).$$

Its maximal-domain traces are denoted by

$$f_j(r) = a_j r^{-v_j} + b_j r^{v_j} + \text{minimal-domain remainder}, \qquad (v_0, v_1) = (u, v).$$

These are boundary traces, not an assertion that every maximal-domain function has a convergent pointwise expansion to arbitrary order. Integration by parts gives the boundary form

$$\Omega(f,g) = 2(a_f^\dagger N b_g - b_f^\dagger N a_g).$$

Definition (Finite matrix chart). For a fixed Hermitian $H \geq 0$, the deficient-channel realization L_H is defined by

$$Nb = Ha.$$

Friedrichs means $a = 0$, not $H = 0$. The finite matrix $H = 0$ means $b = 0$, the singular homogeneous realization in both deficient channels.

The graph in is Lagrangian for ; it therefore defines a self-adjoint extension. We use the ordinary free longitudinal factor and write

$$A_H = L_H \otimes I + I \otimes (-\partial_z^2).$$

The factors α in the radial measure do not change the boundary relation.

For later dimensional bookkeeping,

$$[H_{jk}] = \text{length}^{-(\nu_j + \nu_k)}.$$

Whenever matrix norms combine different boundary coordinates, one fixed reference length is understood. In the core proof we work in the associated dimensionless units and restore the reference length by rescaling.

Lemma (Normalized resolvent correction). *Put*

$$e_\nu = 2^{\nu-1}\Gamma(\nu), \qquad s_\nu = \frac{\pi}{2\sin(\pi\nu)}, \qquad E(p) = \operatorname{diag}(e_u p^{-u}, e_v p^{-v}), \qquad S_0 = \operatorname{diag}(s_u, s_v).$$

For $p > 0$, the deficient-channel correction to the Friedrichs resolvent has matrix kernel

$$K_N(pr)\, C_H(p)\, K_N(pr'), \qquad C_H(p) = (S_0 + 2E(p)HE(p))^{-1},$$

where $K_N = \operatorname{diag}(K_u, K_v)$. Moreover,

$$0 \leq C_H(p) \leq S_0^{-1},$$

and $L_H \geq 0$.

Proof. The Friedrichs scalar Green kernel is $I_{\nu}(pr_{<})K_{\nu}(pr_{>})$, relative to $r^{'}\mathrm{mathop\,d}r^{'}$. The endpoint expansions are

$$K_{\nu}(pr) = e_{\nu}p^{-\nu}r^{-\nu} + d_{\nu}p^{\nu}r^{\nu} + \cdots, \qquad d_{\nu} = 2^{-\nu-1}\Gamma(-\nu),$$

and

$$I_{\nu}(pr) = \frac{p^{\nu}r^{\nu}}{2\nu e_{\nu}} + \cdots.$$

The gamma reflection identity gives $2\nu e_{\nu}d_{\nu} = -s_{\nu}$. Imposing $Nb = Ha$ on the sum of the Friedrichs kernel and the proposed correction gives

$$I - S_0C_H = 2EHEC_H,$$

which is . The Bessel Wronskian preserves the original Green jump. These Bessel identities are recorded in the NIST Digital Library of Mathematical Functions [19].

Positivity gives the matrix bound. A decaying negative-energy solution would require a null vector of $S_0 + 2EHE$, which is positive definite. The essential spectrum is unchanged by this finite-rank resolvent correction; hence the self-adjoint realization has no negative spectrum.

Set $\mathcal{N}_{\alpha} = (4\pi^2\alpha)^{-1}$ and $y = (\tau, z)$. The Euclidean covariance difference is

$$\begin{aligned}\delta W_{E,H}(x,x^{'}) = \quad & \mathcal{N}_{\alpha}\int_0^{\infty} p\,\mathrm{mathop\,d}p\,J_0(p|y-y^{'}|) \\ & \times\sum_{j,k=0}^{1} e^{\mathrm{i}q_j\varphi}K_{\nu_j}(pr)C_{H,jk}(p)K_{\nu_k}(pr^{'})e^{-\mathrm{i}q_k\varphi^{'}}.\end{aligned}$$

The angular normalization is $1/(2\pi\alpha)$; Fourier inversion in the two Euclidean longitudinal variables supplies the remaining $1/(2\pi)$.

Lemma (Regular-host differentiability). *On compact sets with $r, r^{'} > 0$, and all derivatives needed for stress and current converge absolutely. The difference is a smooth bisolution near every regular diagonal.*

Proof. At large p, differentiated Bessel factors are a polynomial in p times $e^{-p(r+r^{'})}$. At small p, a fixed radial derivative changes powers of r, but does not worsen the $p^{-\nu_j}$ behavior of $K_{\nu_j}(pr)$. The worst undifferentiated

momentum power is therefore $p^{1-\nu_j-\nu_k}$, integrable because $\nu_j + \nu_k < 2$. Longitudinal derivatives improve this power. The matrix bound in Lemma is uniform. Differentiation under the integral follows. Cancellation of the two resolvent Green jumps proves the bisolution statement.

For a complex scalar with the normalization of two canonical real scalars, define

$$F(x) = \delta W(x,x), \qquad A_{ab} = \mathrm{Re}\,[\nabla_a \nabla_{b'} \delta W(x,x')]_{x'=x}, \qquad K = g^{ab} A_{ab}.$$

Our regular differences are

$$\delta T_{ab} = 2A_{ab} - g_{ab}K + 2\xi(g_{ab}\mathrm{square}_g - \nabla_a\nabla_b)F,$$

$$\delta J_a = \mathrm{i}e[(\nabla_a - \nabla_{a'})\delta W(x,x')]_{x'=x}.$$

The differences need no additional regular-diagonal subtraction, because their kernels are smooth there. Direct differentiation gives

$$\nabla^a \delta T_{ab} = 0, \qquad \nabla^a \delta J_a = 0, \qquad \delta T^a{}_a = (6\xi - 1)\mathrm{square}_g F.$$

Indeed, the bisolution property gives $\mathrm{square}_g F = 2K$; the canonical divergence cancels by the field equations, and the improvement is divergence-free in regular flat charts.

Ledger 1: structural exceptions and genuine additional reductions

What is being classified

Definition (Structural scalar-channel reduction). A structural reduction is a constant orthogonal decomposition of the deficient angular space into one-dimensional subspaces that reduce both the radial differential expression and the boundary relation. For a finite graph this means simultaneous reduction of N and H. A homogeneous reduction means invariance of the complete boundary relation under radial dilations.

This definition does not classify every identity between special functions. It captures the precise operation often intended by ``diagonalize the boundary matrix and use two scalar channels."

Theorem (Sharp structural exceptional list). *Under Assumption :*

1. *If $u \neq v$, a finite graph has a structural scalar-channel reduction if and only if $H_{01} = 0$.*

2. *If $u = v$, every Hermitian finite graph has a structural scalar-channel reduction.*

3. *The only dilation-invariant finite graph is $H = 0$.*

4. *Among all self-adjoint boundary relations, the dilation-invariant relations at unequal orders are exactly the four coordinate choices of singular or Friedrichs branch.*

5. *At equal orders, every dilation-invariant relation is*

$$a \in \operatorname{ran} \Pi, \qquad Nb \in \ker \Pi$$

for an orthogonal projector Π. Rank-one projectors are parametrized by $\mathbb{CP}^1$. No general $\mathbb{CP}^1$ family of homogeneous relations exists at unequal orders.

Proof. A one-dimensional reducing projector for the radial expression must commute with N^2. When $u \neq v$, its eigenspaces are exactly the coordinate lines. These lines reduce the graph precisely when H is diagonal. Conversely, diagonal H gives two independent scalar boundary conditions.

When $u = v$, the differential expression is scalar in channel space. The spectral projectors of H then reduce both it and the boundary relation. This proves the first two assertions.

The unitary radial dilation $f(r) \mapsto \lambda f(\lambda r)$ acts on traces by

$$a \mapsto \lambda^{I-N} a, \qquad Nb \mapsto \lambda^{I+N} Nb.$$

A graph $Nb = Ha$ is preserved exactly when

$$\lambda^N H \lambda^N = H \quad \text{for every} \lambda > 0.$$

Entrywise, $\lambda^{v_j + v_k} H_{jk} = H_{jk}$. All exponents are positive, so $H = 0$.

For general relations, remove the common factor λ. The dilation generator on (a, Nb) has weights $-u, -v, u, v$. At

unequal orders these four weights are distinct. A subspace invariant under all dilations is consequently a sum of their eigenspaces. A two-dimensional Lagrangian subspace must choose exactly one member from each canonically paired $a_j, (Nb)_j$ pair. This gives the four coordinate relations.

At equal orders there are only two weights. A dilation-invariant Lagrangian is therefore $A \oplus B$, with A in the a -space and B in the Nb-space. Equation gives $B = A^{\perp}$. Writing $A = \operatorname{ran} \Pi$ proves the final assertion.

The same obstruction is visible in a normalized Weyl matrix:

$$H + \operatorname{diag}\left(\frac{s_u}{2e_u^2} p^{2u}, \frac{s_v}{2e_v^2} p^{2v} \right).$$

A constant diagonalizer of H need not diagonalize this family. At unequal orders, simultaneous reduction requires the coordinate decomposition.

The recovered scalar functions

For real η, χ, define

$$\mathscr{M}_{\eta\chi}(\lambda) = \int_0^{\infty} x^{\lambda - 1} K_{\eta}(x) K_{\chi}(x) \mathrm{mathop\,d}x = \frac{2^{\lambda - 3}}{\Gamma(\lambda)} \prod_{\varepsilon, \delta = \pm 1} \Gamma\left(\frac{\lambda + \varepsilon\eta + \delta\chi}{2} \right),$$

initially for $\operatorname{Re} \lambda > |\eta| + |\chi|$. One derivation inserts the two Schwinger integral representations, integrates x, and changes the remaining positive variables to their sum and ratio. The resulting integrals are respectively gamma and beta integrals. Absolute convergence justifies the changes of variables. This is the standard Bessel product moment [19, 17].

For $X > 0$, set

$$\mathscr{U}^{m}_{\eta\chi}(X; v) = \frac{1}{2\pi \mathrm{i}} \int_{c - \mathrm{i}\infty}^{c + \mathrm{i}\infty} \Gamma(z) \Gamma(1 - z) X^{-z} \mathscr{M}_{\eta\chi}(m + 2vz) \mathrm{mathop\,d}z,$$

where $0 < c < 1$ and $m + 2vc > |\eta| + |\chi|$. The elementary Mellin–Barnes formula for $(1 + t)^{-1}$ and Fubini give

$$\mathscr{U}^{m}_{\eta\chi}(X; v) = \int_0^{\infty} \frac{x^{m - 1} K_{\eta}(x) K_{\chi}(x)}{1 + X x^{-2v}} \mathrm{mathop\,d}x.$$

For diagonal H,

$$C_{jj}(p) = \frac{1}{s_{v_j}(1 + a_j p^{-2v_j})}, \qquad a_j = \frac{2e_{v_j}^2 H_{jj}}{s_{v_j}},$$

so all radial moments are univariate functions . At equal orders, the spectral decomposition $H = \sum_a \lambda_a \Pi_a$ gives

$$C_H(p) = \sum_a \frac{\Pi_a}{s_v + 2e_v^2 \lambda_a p^{-2v}},$$

again a finite sum of univariate functions.

For $H = 0$, $C_H = S_0^{-1}$, and the moments are gamma products . If $H = tH_0$, $H_0 > 0$, then $t \to \infty$ gives the Friedrichs correction zero. A mixed singular/Friedrichs coordinate relation is obtained by retaining one zero eigenvalue and sending the other diagonal entry to infinity. Lemma justifies these regular-host limits.

At equal orders the homogeneous projector correction is $C = s_v^{-1}\Pi$. Its rank-one freedom is exactly the $\mathbb{CP}^1$ line of Theorem , not a projective line at unequal orders.

Proposition (One singular-channel longitudinal stress). *For one singular diagonal channel of order* $0 < \nu < 1$, *relative to its Friedrichs channel,*

$$\text{boxed}\, \delta T^t{}_t = \delta T^z{}_z = \frac{\mathcal{N}_\alpha \nu}{r^4}\left(8\xi - \frac{4}{3} - \frac{2}{3}\nu^2\right).$$

Proof. Equation gives

$$\mathcal{M}_{\nu\nu}(2) = \nu s_\nu, \qquad \mathcal{M}_{\nu\nu}(4) = \frac{2}{3}\nu s_\nu(1 - \nu^2).$$

An integration by parts using the Bessel equation also gives

$$\int_0^\infty x^3 K_\nu'(x)^2 \text{mathop}\, \mathrm{d}x = \frac{1}{3}\nu s_\nu(4 - \nu^2).$$

The endpoint terms vanish since $0 < \nu < 1$. Thus

$$F = \mathcal{N}_\alpha \nu r^{-2}, \quad P = \frac{\mathcal{N}_\alpha \nu}{3r^4}(1 - \nu^2), \quad B = \frac{\mathcal{N}_\alpha \nu}{3r^4}(4 - \nu^2),$$

where P is the positive Euclidean longitudinal derivative moment and B the radial derivative moment. The angular contribution is $\mathcal{N}_\alpha v^3 r^{-4}$; hence $K = 2P + B + \mathcal{N}_\alpha v^3 r^{-4} = 2\mathcal{N}_\alpha v r^{-4}$. Finally, square$_g F = 4\mathcal{N}_\alpha v r^{-4}$ and $\delta T^t_{\ t} = 2P - K + 2\xi$square$_g F$, which proves .

At $\xi = \frac{1}{6}$, is $-2\mathcal{N}_\alpha v^3/(3r^4)$, not zero. More generally, conformal coupling changes the point-splitting operator, not the spectral matrix C_H. The parent's continuous-Stieltjes obstruction to rational and radius-independent Bernoulli closure remains in force [1, Theorem thm:obstruction]. We do not reprove that obstruction here.

Why an unrestricted exceptional list is false

Write

$$H = \begin{pmatrix} h_0 & h \\ \overline{h} & h_1 \end{pmatrix}, \quad a = \frac{2e_u^2 h_0}{s_u}, \quad b = \frac{2e_v^2 h_1}{s_v}, \quad c = \frac{2e_u e_v h}{\sqrt{s_u s_v}}, \quad d = ab - |c|^2.$$

Direct inversion gives

$$D(p) = 1 + ap^{-2u} + bp^{-2v} + dp^{-2S},$$

$$C_{00}(p) = \frac{1 + bp^{-2v}}{s_u D(p)}, \qquad C_{11}(p) = \frac{1 + ap^{-2u}}{s_v D(p)},$$

$$C_{01}(p) = -\frac{cp^{-S}}{\sqrt{s_u s_v}\, D(p)}.$$

For $H > 0$, $d > 0$.

Proposition (Two additional mechanisms).

1. *A mixed rank-one $H \geq 0$ has $d = 0$. Its master denominator has only two nonconstant monomials, whether or not u/v is rational.*

2. *If $u/v \in \mathbb{Q}$ and $H > 0$, every radial moment*

$$\int_0^\infty p^{m-1} C_{jk}(p) K_\eta(pr) K_\chi(pr) \mathrm{mathop\,d}p, \qquad m > |\eta| + |\chi|,$$

is a finite sum of gamma moments and absolutely convergent one-variable Mellin–Barnes integrals. This includes

all undifferentiated and shifted-order moments needed by –. Neither mechanism requires structural decoupling.

Proof. The first assertion follows from $d = 4e_u^2 e_v^2 \det H/(s_u s_v)$. The usual Mellin inversion of $(1 + A + B)^{-1}$ then uses two Barnes variables rather than three.

For the second assertion, choose $\delta > 0$ and positive integers m_0, m_1 with $u = m_0\delta$, $v = m_1\delta$, and put $t = p^{\delta}$. Multiplying by $t^{2m_0 + 2m_1}$ makes every C_{jk} a rational function of t, with common denominator

$$Q(t) = t^{2m_0 + 2m_1} + at^{2m_1} + bt^{2m_0} + d.$$

It has no zero at 0 and no positive real zero. The rational functions are bounded at infinity. Partial fractions therefore express each entry as a constant plus finitely many terms $(t - \zeta)^{-k}$, with $\zeta \notin [0, \infty)$.

After $p = x/r$, such a term is a constant multiple of $(1 + wx^{\delta})^{-k}$, where $w = -1/(\zeta r^{\delta})$ and $|\arg w| < \pi$. For

$$0 < c_0 < \min\left\{k, \frac{m - |\eta| - |\chi|}{\delta}\right\},$$

its moment is

$$\frac{1}{2\pi \mathrm{i}\,\Gamma(k)} \int_{c_0 - \mathrm{i}\infty}^{c_0 + \mathrm{i}\infty} \Gamma(z)\Gamma(k - z) w^{-z} \mathscr{M}_{\eta\chi}(m - \delta z)\mathrm{mathop}\,\mathrm{d}z.$$

Indeed, insert Mellin inversion for $(1 + wx^{\delta})^{-k}$. The Bessel integral is absolutely convergent on this line. Moreover,

$$|\mathscr{M}_{\eta\chi}(m - \delta c_0 - \mathrm{i}\delta t)| \le \mathscr{M}_{\eta\chi}(m - \delta c_0),$$

whereas the gamma pair contributes exponential decay $e^{-\pi|t|}$ up to a polynomial. The factor w^{-z} leaves exponential decay because $|\arg w| < \pi$. Fubini and absolute contour convergence follow.

For the physical moments, $m = 2, 3, 4$. Each radial derivative shifts a Bessel order by at most one and increases m by one. Thus $m > |\eta| + |\chi|$ follows from $u, v < 1$.

Example. Take

$$\alpha = \frac{4}{5}, \qquad \beta = \frac{2}{5}, \qquad (u, v) = \left(\frac{1}{2}, \frac{3}{4}\right), \qquad H_* = \begin{pmatrix} 2 & \frac{1}{2}e^{\mathrm{i}\pi/3} \\ \frac{1}{2}e^{-\mathrm{i}\pi/3} & 3 \end{pmatrix}.$$

The matrix is positive definite and mixed, with determinant 23/4. Its orders are unequal, and it is not homogeneous. Nevertheless, $\delta = \frac{1}{4}$ makes every entry of C_{H_*} rational in $p^{1/4}$, so Proposition gives univariate Barnes sums. No numerical evaluation is involved.

The structural list is therefore complete in its stated category, while the unrestricted special-function list is not. We do not claim a minimal-contour theorem for irrational orders, nor a classification of every arithmetic or hypergeometric identity.

Ledger 2: contacts under declared hypotheses

The unconditional support statement

Proposition (Regular data leave supported ambiguity). *Suppose two distributional completions of the same one-point quantity agree for $r > 0$. Their difference is supported on $\mathscr{W}$. A bound on normal derivative order alone does not make this ambiguity finite-dimensional.*

Proof. The support assertion is the definition of equality on the complement of $\mathscr{W}$. Locally, in smooth completion coordinates (y, x), with $y = (t, z)$ tangent and $x = (x^1, x^2)$ normal, a supported distribution of finite normal order has the form

$$\sum_{|\gamma| \le N} a_\gamma(y)\, \partial_x^\gamma \delta^{(2)}(x).$$

This follows by subtracting the finite normal Taylor polynomial of a test function; a supported distribution of order N annihilates the remainder. The coefficient distributions a_γ are arbitrary unless further restrictions are imposed. Already $a(y)\delta^{(2)}(x)$ gives an infinite-dimensional normal-order-zero space. This is the standard supported-distribution mechanism [22].

This proposition does not assert that every displayed contact is physically admissible. It says that exterior agreement alone cannot decide admissibility or normalization.

A complete conditional normal-jet count

A cone does not carry a canonical smooth metric at its apex. Consequently, products of its singular metric coefficients with delta derivatives must not be used without a completion convention. We now specify one convention in which the finite count can be proved exactly.

Specification (Homogeneous contact-jet convention). Choose a smooth tubular completion $\mathbb{R}^{1,1}_{y} \times \mathbb{R}^{2}_{x}$, its product density, longitudinal metric $\eta_{AB} = \mathrm{diag}\,(-1, 1)$, and an auxiliary Euclidean normal metric δ_{ij}. Contact differences are required to:

1. be translation invariant along y, covariant under proper longitudinal Lorentz transformations and normal rotations;

2. be real, with symmetric stress, and polynomial in finite normal delta jets;

3. have exact scaling dimensions $2, 4, 3$ for the Wick square, stress, and current, respectively;

4. obey the constant-coefficient contact identities

$$\partial^{\mu} T_{\mu\nu} = 0, \qquad \partial^{\mu} J_{\mu} = 0, \qquad T^{\mu}{}_{\mu} = (6\xi - 1)\Delta_{\perp} F;$$

5. introduce no additional dimensionful contact parameter. These are identities in the chosen smooth jet convention, not an undefined multiplication of distributions by the conical metric. The scaling condition can also describe differences of two prescriptions with the same scaling anomaly.

Theorem (Conditional three-parameter ledger). *Under Specification , all simultaneous contact differences are uniquely parametrized by three real numbers c, τ, w:*

$$F_{\mathrm{c}} = c\,\delta_{\perp},$$

$$T_{AB,\mathrm{c}} = \tau \eta_{AB} \Delta_{\perp} \delta_{\perp}, \qquad T_{Ai,\mathrm{c}} = 0,$$

$$T_{ij,\mathrm{c}} = [2\tau - (6\xi - 1)c](\partial_i \partial_j - \delta_{ij} \Delta_{\perp})\delta_{\perp},$$

$$J_{A,\mathrm{c}} = 0, \qquad J_{i,\mathrm{c}} = w \varepsilon_{ij} \partial_j \delta_{\perp}.$$

If charge conjugation preserves the complete realization and prescription and makes the current odd, then $w = 0$.

For a real neutral scalar there is no charged-current slot, and the simultaneous ledger has two parameters.

Proof. Tangential translation invariance makes the tangential coefficient distributions constant. Since $\delta_\perp$ has scaling dimension two, homogeneity permits normal derivative orders zero, two, and one in the three respective slots.

Rotation covariance makes the Wick-square term $c\delta_\perp$. For the symmetric stress, longitudinal covariance gives $T_{AB} = A\eta_{AB}\Delta_\perp\delta_\perp$ and $T_{Ai} = 0$. The most general normal block is

$$B\delta_{ij}\Delta_\perp\delta_\perp + C\partial_i\partial_j\delta_\perp + D(\varepsilon_{ik}\partial_k\partial_j + \varepsilon_{jk}\partial_k\partial_i)\delta_\perp.$$

Its divergence is

$$(B + C)\partial_j\Delta_\perp\delta_\perp + D\varepsilon_{jk}\partial_k\Delta_\perp\delta_\perp.$$

The two vector polynomials are independent, so $C = -B$ and $D = 0$. Put $A = \tau$. The trace is $(2\tau + B)\Delta_\perp\delta_\perp$, hence $B = (6\xi - 1)c - 2\tau$, proving –.

The general normal current is $a\partial_i\delta_\perp + w\varepsilon_{ij}\partial_j\delta_\perp$. Conservation forces $a = 0$. There is no invariant longitudinal vector. This proves .

The three displayed families are independent and satisfy all the identities, proving existence and uniqueness. If the entire prescription is charge-conjugation invariant, its odd current difference equals its own negative, so $w = 0$.

At conformal coupling, c and τ are independent even though the contact stress is traceless. A trace condition alone does not determine the Wick-square contact.

The theorem explains a three-parameter count under explicit hypotheses, but does not identify our normalization of τ with an unpublished convention without a conversion. The contact classification described in Cheung's I-B-VP-1 is the documentary comparison [9]; its count is not assumed here.

Rotation and charge conjugation are separate tests

Lemma (Symmetries of the selected relation). *For a finite graph, rotations preserve the individual realization if and only if* $H_{01} = 0$. *At* $\beta = \frac{1}{2}$, *let*

$$J = \begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix}.$$

Charge conjugation preserves the graph if and only if

$$H = J\overline{H}J.$$

For a homogeneous equal-order projector relation, the corresponding condition is $\Pi = J\overline{\Pi}J$.

Proof. Rotations act on boundary coefficients by $U_\theta = \operatorname{diag}(e^{iq_0\theta}, e^{iq_1\theta})$. They preserve the graph exactly when $U_\theta H U_\theta^\dagger = H$ for all θ. Because $q_0 - q_1 = 1$, this is equivalent to $H_{01} = 0$.

At half holonomy, conjugation exchanges the two angular channels and conjugates their coefficients: $a \mapsto J\bar{a}$, $b \mapsto J\bar{b}$. Here $N = vI$, so applying this map to the boundary relation gives the stated condition. The projector case is identical.

A rank-one projector can be written

$$\Pi = \begin{pmatrix} p & z \\ \bar{z} & 1-p \end{pmatrix}, \qquad |z|^2 = p(1-p).$$

Rotation invariance requires $z = 0$, whereas charge-conjugation invariance requires $p = \frac{1}{2}$. No rank-one projector satisfies both conditions. Thus the general equal-order projector line does not inherit the rotation-invariant two-parameter half-holonomy ledger.

This failure is observable already on the regular host. For the coordinate projector onto the $q_0 = \frac{1}{2}$ channel,

$$F = \mathscr{N}_\alpha v r^{-2}, \qquad \delta J_\varphi = -e\mathscr{N}_\alpha v r^{-2}.$$

Charge conjugation sends this realization to the other coordinate projector; it does not force its current to vanish.

The sector ledger

The singular graph H = 0. The operator is dilation invariant and rotation invariant. Its product spectral prescription inherits those symmetries. At half holonomy it is also charge-conjugation invariant. Nevertheless, a completion must still satisfy the chosen jet Ward identities and the prescribed scaling law before Theorem applies. Under those additional hypotheses the ledger is (c, τ, w), reduced to (c, τ) when the full charge-conjugation condition holds.

The equal-order projector line. Scale invariance holds, but a non-coordinate projector breaks rotation invariance. Coordinate projectors preserve rotations but break charge conjugation at half holonomy. The general line is therefore not assigned the inherited rotation-invariant contact basis. A smaller symmetry group can be used, but its contact space requires a new enumeration.

Diagonal positive finite H. Rotations are preserved. Dilations are not: finite positive entries define crossover lengths. No dilation no-go may be imported without the missing fixed-prescription scaling hypothesis. Even if a family transforms covariantly when H is scaled, an individual fixed member is not a scale-invariant vacuum sector.

Mixed finite H. An individual realization breaks rotations. Joint rotation of coordinates and the phase of H_{01} is covariance of a family, not invariance of a member. Its contacts must therefore retain the rotation-breaking data rather than being forced into –.

Resonant general U(2) relations. The present nonnegative finite graph excludes negative transverse eigenvalues and does not classify resonant boundary strata outside it. The open resonant set in the supplied I-B-VP-1 description is not closed by citation, by this contact theorem, or by the core construction.

Proposition (Mixing is visible without contact completion). *If $H_{01} \neq 0$, the regular Wick-square difference is not rotation invariant. If also $u \neq v$ and $e \neq 0$, its radial current is not identically zero. At equal orders the radial current vanishes even for mixed matrices.*

Proof. Equation shows that $C_{01}(p)$ has a constant complex phase and a strictly positive scalar magnitude for $p > 0$. Consequently its Wick-square moment is nonzero and appears as $2\mathrm{Re}\,(e^{i\varphi}R_{01}(r))$.

For the radial current, set $f(r) = K_u(pr)$, $g(r) = K_v(pr)$. Their equations give

$$\frac{\mathrm{d}}{\mathrm{d}r}[r(f'g - fg')] = \frac{u^2 - v^2}{r}fg.$$

The bracket vanishes at infinity, so for $u \neq v$ it has a fixed nonzero sign at every finite r. In the radial current integral this Wronskian is multiplied by a scalar of fixed sign and the constant phase of C_{01}. Its sinusoidal angular coefficient cannot cancel. If $u = v$, the Wronskian is identically zero.

Stress shear is obtained by the corresponding mixed radial-angular derivatives in ; it must be retained even when a particular coefficient or leading asymptotic term vanishes.

Finally, finite normal order and finite-dimensionality must not be confused. Without tangential translation invariance, Proposition already supplies infinitely many coefficients. Even with translations and rotations, removing the scaling restriction and allowing arbitrary normal order permits the tower

$$\ell^{2k}\Delta_\perp^k(\partial_i\partial_j - \delta_{ij}\Delta_\perp)\delta_\perp, \qquad k = 0, 1, \ldots,$$

with matching longitudinal terms if tracelessness is imposed. A separately imposed finite normal-order bound would truncate such a tower, but its enumeration and normalization would be a new problem.

Ledger 3: an explicit engineered smooth-core theorem

The declared class

We now construct a genuine family of nonnegative operators on smooth caps. The interaction is deliberately engineered to realize the specified boundary relation. It is not a claim that pure curvature, a nonnegative local scalar potential, or ordinary flux alone selects that relation.

Use the periodic gauge in which angular modes are $e^{im\varphi}$, $m \in \mathbb{Z}$, and the exterior angular operator has eigenvalues

$(m+\beta)^2$. The deficient modes are $m_0 = 0$, $m_1 = -1$.

Choose smooth functions A, B, χ on $[0,\infty)$ such that

$$A(s) = s, \quad B(s) = 0, \quad \chi(s) = 0 \quad (s \le \tfrac{1}{3}),$$

and

$$A(s) = \alpha s, \quad B(s) = 1, \quad \chi(s) = 1 \quad (s \ge \tfrac{2}{3}),$$

with $A(s) > 0$ for $s > 0$, $0 \le B, \chi \le 1$, and $A(s)/s$ bounded above and below by positive constants. Set

$$a_\varepsilon(r) = \varepsilon A(r/\varepsilon).$$

The transverse metric and connection are

$$\mathrm{mathop\,d}r^2 + a_\varepsilon(r)^2 \mathrm{mathop\,d}\varphi^2, \qquad D_\varphi = \partial_\varphi + \mathrm{i}\beta B(r/\varepsilon).$$

They are smooth at the origin because they are Euclidean with trivial connection near it, and exactly conical with the prescribed holonomy outside $r \ge \varepsilon$.

Definition (The matrix factor). For the fixed $H \ge 0$, define on $r > 0$

$$\mathrm{F}_H(r) = r^{-N} + r^N N^{-1} H, \qquad \mathrm{P}_H(r) = \mathrm{F}_H'(r)\mathrm{F}_H(r)^{-1}.$$

Let $M_0 = \mathrm{diag}\,(0,1)$, and define

$$\mathrm{P}_\varepsilon(r) = [1 - \chi(r/\varepsilon)]\frac{M_0}{r} + \chi(r/\varepsilon)\mathrm{P}_H(r).$$

Lemma (Matrix ground-state identities). *The matrix* $\mathrm{F}_H(r)$ *is invertible,* $\mathrm{P}_H(r)$ *is Hermitian, and*

$$\mathrm{P}_H' + r^{-1}\mathrm{P}_H + \mathrm{P}_H^2 = N^2 r^{-2}.$$

As $r\mathrm{downarrow}\,0$,

$$\|\mathrm{F}_H(r)\| \le C r^{-\nu_{\max}}, \qquad \left\|\mathrm{F}_H(r)^{-1}\right\| \le C r^{\nu_{\min}}, \qquad \mathrm{lVertr}\mathrm{P}_H(r)\| \le C,$$

where $v_{\min} = \min(u, v)$, $v_{\max} = \max(u, v) < 1$.

Proof. Factor

$$\mathrm{F}_H = r^{-N}(I + r^{2N}N^{-1}H).$$

The positive diagonal matrix $r^{2N}N^{-1}$ times $H \geq 0$ is similar to a nonnegative Hermitian matrix, so -1 is not an eigenvalue. This proves invertibility.

Each column of F_H solves

$$-F'' - r^{-1}F' + N^2 r^{-2}F = 0.$$

Direct calculation gives

$$\mathrm{F}_H^{\dagger} r \mathrm{F}_H' - \mathrm{F}_H' \wedge \dagger r \mathrm{F}_H = 0,$$

because its two constant cross terms cancel by $H = H^{\dagger}$. Multiplication by the inverses of F_H and $\mathrm{F}_H^{\dagger}$ shows $\mathrm{P}_H^{\dagger} = \mathrm{P}_H$. Differentiating $\mathrm{P}_H = \mathrm{F}_H' \mathrm{F}_H^{-1}$ proves .

The first two bounds follow from the factorization and $I + r^{2N}N^{-1}H = I + o(1)$. For the final bound, use

$$\mathrm{F}_H' = -\frac{N}{r}\mathrm{F}_H + 2r^{N-1}H.$$

The additional term in $r\mathrm{P}_H$ is bounded, and in fact tends to zero.

On the two selected angular modes define

$$\mathfrak{q}_{\varepsilon,H}^{\mathrm{def}}[f] = \int_0^{\infty} \mathrm{lVert} f'(r) - \mathrm{P}_{\varepsilon}(r) f(r) \|_{\mathbb{C}^2}^2 a_{\varepsilon}(r) \mathrm{mathop\,d}r.$$

The initial domain consists of compactly supported smooth functions on the cap in those modes. Thus near zero their radial components have the ordinary smooth $m = 0, -1$ behavior. Take the closure in form norm.

For every other angular mode use the ordinary nonnegative magnetic form

$$\mathfrak{q}_{\varepsilon,m}[f] = \int_0^\infty \left(|f'|^2 + \frac{(m + \beta B(r/\varepsilon))^2}{a_\varepsilon(r)^2} |f|^2 \right) a_\varepsilon(r) \mathrm{mathop\,d}r.$$

Their orthogonal sum defines $L_{\varepsilon,H} \geq 0$.

Integration by parts in shows that the selected block is the magnetic cap operator plus the matrix interaction

$$V_{\varepsilon,H} = \mathrm{P}_\varepsilon' + \frac{a_\varepsilon'}{a_\varepsilon} \mathrm{P}_\varepsilon + \mathrm{P}_\varepsilon^2 - \mathrm{diag}\left(\frac{(\beta B)^2}{a_\varepsilon^2}, \frac{(-1 + \beta B)^2}{a_\varepsilon^2} \right).$$

It vanishes near the center and outside the cap. It is a smooth Hermitian matrix function on its supporting annulus. Installed between the two angular projectors, it is finite-rank in angular variables and local in r, z, but generally *not* multiplication by a scalar function of angle. This is the exact declared nonlocality.

Warning (Meaning of smooth core). The geometry, connection, and radial coefficients of are smooth for each $\varepsilon > 0$. The interaction may grow as the core shrinks and is angularly nonlocal. If ``smooth core" is restricted to a local scalar metric–curvature–flux operator without this finite-mode interaction, the theorem below is not a realization theorem in that smaller class. Such a realization for general mixed H is not established here.

Identification of the limiting form

Define

$$\mathfrak{q}_H^{\mathrm{def}}[f] = \alpha \int_0^\infty \mathrm{lVert f' - P}_H f \|^2 r \mathrm{mathop\,d}r,$$

with domain consisting of $f \in L^2(\alpha r \mathrm{mathop\,d}r; \mathbb{C}^2)$ for which $f = \mathrm{F}_H v$, v is locally absolutely continuous, and the integral is finite. No zero boundary value is imposed on v.

Lemma (Trace, density, and the selected relation). *The form is densely defined, closed, and nonnegative. Its associated operator is precisely the deficient block of L_H. Functions $f = \mathrm{F}_H v$ with compact support and v constant near zero form a form core.*

Proof. Put $W(r) = \alpha r \mathrm{F}_H^\dagger \mathrm{F}_H$. Then $\mathfrak{q}_H^{\mathrm{def}}[f] = \int v' \wedge \dagger W v' \mathrm{mathop\,d}r$. Lemma gives

$$\mathrm{lVert W}(r)^{-1} \| \leq C r^{2\nu_{\min} - 1}.$$

Consequently, for $0 < s < r$ sufficiently small,

$$\lVert v(r) - v(s)\rVert \le \left(\int_s^r \lVert W(t)^{-1}\rVert \mathrm{d}t\right)^{1/2} \left(\int_s^r v' \dagger W v' \mathrm{d}t\right)^{1/2}.$$

Thus $v(0)$ exists and v is bounded near zero.

Closedness follows from completeness in L^2 and local weighted Sobolev completeness on exhausted annuli, followed by Fatou's lemma for the energy. Compactly supported functions away from zero are included, proving density.

Replace $v(r)$, for $r < \delta$, by $v(\delta)$. This removes the energy on $(0,\delta)$, and the L^2 difference tends to zero because v is bounded and

$$\int_0^\delta r\lVert F_H(r)\rVert^2 \mathrm{d}r \le C\delta^{2-2\nu_{\max}} \to 0.$$

The resulting corner at δ can be smoothed in weighted Sobolev norm on an annulus. Cutoffs at infinity have vanishing form error: the extra derivative term is bounded by $CR^{-2}\int_R^{2R} |f|^2 r \mathrm{d}r$. This proves the asserted form-core density.

Let T be the operator associated with the closed form. Testing away from zero and using shows $Tf = L_N f$, so $f \in \operatorname{Dom} L_{\max}$. For a test function equal to $F_H c$ near zero, integration by parts gives the boundary term

$$\lim_{r \downarrow 0} c^\dagger \alpha [F_H^\dagger r f' - F_H' \dagger r f].$$

Using the maximal-domain traces, this limit is $2\alpha c^\dagger (Nb - Ha)$. The form operator requires it to vanish for every c; hence $Nb = Ha$. Thus $T \subseteq L_H$. Both are self-adjoint, so they are equal.

Mosco convergence

The cap Hilbert space is

$$\mathscr{H}_\varepsilon = L^2((0,\infty) \times S^1, a_\varepsilon(r)\mathrm{d}r\mathrm{d}\varphi).$$

Identify it with the conical Hilbert space $\mathscr{H} = L^2(\alpha r\mathrm{d}r\mathrm{d}\varphi)$ by

$$(U_\varepsilon f)(r,\varphi) = \left(\frac{a_\varepsilon(r)}{\alpha r}\right)^{1/2} f(r,\varphi).$$

This map is exactly the identity outside $r < \varepsilon$, and its density ratio is bounded above and below uniformly in ε.

Theorem (Engineered smooth matrix-core limit). *For every fixed finite $H \geq 0$, the transported forms of and Mosco-converge to the form of L_H, with all unselected modes Friedrichs. Consequently,*

$$U_\varepsilon L_{\varepsilon,H} U_\varepsilon^{-1} \to L_H \quad \text{in strong resolvent sense.}$$

After tensoring with the free longitudinal operator,

$$(U_\varepsilon \otimes I)(L_{\varepsilon,H} - \partial_z^2)(U_\varepsilon^{-1} \otimes I) \to A_H \quad \text{in strong resolvent sense.}$$

In particular, this realizes the explicit mixed matrix H_* in on the declared cap class.

Proof. We prove both Mosco conditions.

Weak lower bound in the deficient block. Suppose $u_\varepsilon \mathrm{rightharpoonup} u$ in the fixed Hilbert space and the transported energies are bounded. Write $f_\varepsilon = U_\varepsilon^{-1} u_\varepsilon$. On every annulus $[\delta, R]$, for sufficiently small ε, the transport is the identity, $a_\varepsilon = \alpha r$, and $\mathrm{P}_\varepsilon = \mathrm{P}_H$. Bounded L^2 norm and bounded factored derivative imply bounded ordinary H^1 norm there, because P_H is bounded on that annulus. Weak compactness identifies the local derivative of u. Weak lower semicontinuity gives

$$\alpha \int_\delta^R \|\mathrm{V}\mathrm{e}\mathrm{r}\mathrm{t}u' - \mathrm{P}_H u\|^2 r\mathrm{d}r \leq \liminf_{\varepsilon \to 0} \mathfrak{q}_{\varepsilon,H}^{\mathrm{def}}[f_\varepsilon].$$

Exhausting $0 < r < \infty$ proves the lower bound and places u in the limiting form domain.

Recovery in the deficient block. By Lemma , first take $f = \mathrm{F}_H v$ with compact support and $v = a$ constant near zero. Let F_ε solve

$$\mathrm{F}_\varepsilon' = \mathrm{P}_\varepsilon \mathrm{F}_\varepsilon, \qquad \mathrm{F}_\varepsilon(\varepsilon) = \mathrm{F}_H(\varepsilon),$$

and define $f_\varepsilon = f$ for $r \geq \varepsilon$ and $f_\varepsilon = \mathrm{F}_\varepsilon a$ for $r < \varepsilon$. The factored derivative is zero inside the cap. Since $\mathrm{P}_\varepsilon = M_0/r$ near zero, F_ε there is r^{M_0} times a constant matrix; the resulting two angular components are smooth cap functions.

On $r/\varepsilon \in [1/3, 1]$, the scaled coefficient $\varepsilon \mathrm{P}_\varepsilon(\varepsilon s)$ is uniformly bounded. Propagation across this fixed interval gives

$$\|\mathrm{F}_\varepsilon(\varepsilon s)\| \leq C\|\mathrm{F}_H(\varepsilon)\| \leq C\varepsilon^{-\nu_{\max}}.$$

For $s \leq 1/3$, inward propagation by s^{M_0} does not increase the norm, up to a fixed constant. Hence

$$\int_0^\varepsilon \|\mathrm{F}_\varepsilon(r)a\|^2 a_\varepsilon(r)\mathrm{mathop\,d}r \leq C|a|^2\varepsilon^{2-2\nu_{\max}} \to 0.$$

The same vanishing estimate holds for f on the limiting interval. Therefore $U_\varepsilon f_\varepsilon \to f$ strongly. For small ε the exterior factored derivative of f also vanishes near the matching point, and

$$\mathfrak{q}^{\mathrm{def}}_{\varepsilon,H}[f_\varepsilon] = \mathfrak{q}^{\mathrm{def}}_H[f].$$

A diagonal argument using form-core density supplies recovery for every f in the limiting form domain.

The unselected modes. For each fixed unselected m, the annular lower-bound argument applies to . Its limiting energy is

$$\alpha \int_0^\infty \left(|f'|^2 + \frac{(m+\beta)^2}{\alpha^2 r^2}|f|^2 \right) r\mathrm{mathop\,d}r.$$

Its finite-energy domain is the Friedrichs form domain. Recovery first uses functions supported away from zero, for which the cap and cone forms eventually agree, and then Friedrichs form-core density.

For the orthogonal sum, apply the lower bound to every finite set of modes and then increase that set. For recovery, first truncate the angular sum in limiting form norm and then use a second diagonal argument. This proves Mosco convergence of the full transverse forms.

The equivalence between Mosco convergence of nonnegative closed quadratic forms and strong-resolvent convergence of their operators is the standard form-convergence theorem [20, 21]. It gives .

Finally Fourier transform in z. For every k, the transverse resolvents shifted by $k^2 + 1$ converge strongly, and their

norms are bounded by $(k^2 + 1)^{-1} \le 1$. Dominated convergence in the direct integral proves .

Exterior boundary matching

The construction does not merely name a desired limiting operator. Its regular zero-energy core solutions match the selected relation exactly at the interface.

Proposition (Boundary relation of the engineered cores). *At zero energy, the regular two-dimensional solution space inside the cap continues outside as* $\mathrm{F}_H(r)a$*, and therefore has* $Nb = Ha$*. For fixed spectral parameter* ζ*, its interface relation, expressed in zero-energy exterior trace coordinates, converges to* $Nb = Ha$*. On compact* ζ*-sets the error is* $O(\varepsilon^{2-2\nu_{\max}})$ *in fixed boundary units.*

Proof. At zero energy the regular solutions are $\mathrm{F}_\varepsilon a$. They match $\mathrm{F}_H a$ in both value and derivative at $r = \varepsilon$; exterior uniqueness proves the first assertion.

For the second, use $s = r/\varepsilon$. The regular scaled equation differs from its zero-energy equation by the parameter $\varepsilon^2\zeta$. Normalize the zero-energy fundamental matrix as

$$G_{\varepsilon,0}(s) = \mathrm{F}_\varepsilon(\varepsilon s)\varepsilon^N.$$

Its value and scaled derivative at $s = 1$ are uniformly bounded, and its value is uniformly invertible, because

$$\mathrm{F}_H(\varepsilon)\varepsilon^N = I + \varepsilon^N N^{-1} H \varepsilon^N.$$

On $s \le 1/3$ the equations are the ordinary regular $m = 0, 1$ radial equations; their regular solutions and first derivatives depend analytically on $\varepsilon^2\zeta$. On the remaining fixed interval, the coefficients are uniformly bounded. The integral equation and Gronwall's inequality therefore give

$$G_{\varepsilon,\zeta}(1) - G_{\varepsilon,0}(1) = O(\varepsilon^2), \quad G'_{\varepsilon,\zeta}(1) - G'_{\varepsilon,0}(1) = O(\varepsilon^2),$$

uniformly on compact ζ-sets.

Define interface trace coordinates by the invertible equations

$$f(\varepsilon) = \varepsilon^{-N}a + \varepsilon^{N}b, \qquad \varepsilon f'(\varepsilon) = -N\varepsilon^{-N}a + N\varepsilon^{N}b.$$

For the zero-energy normalized columns, $a = \varepsilon^{N}$, $b = N^{-1}H\varepsilon^{N}$. For the perturbed normalized columns, extraction of a changes its matrix by $O(\varepsilon^{N}\varepsilon^{2})$, while extraction of $Nb - Ha$ changes it by $O(\varepsilon^{-N}\varepsilon^{2})$. Multiplying on the right by the inverse a-matrix costs at most $C\varepsilon^{-\nu_{\max}}$. Thus the graph error is $O(\varepsilon^{2-2\nu_{\max}})$, which tends to zero.

The exponent is positive precisely because both selected orders are below one. This is another reason not to include an endpoint of Assumption by continuity of notation.

What operator convergence does not normalize

Theorem is an engineered realization theorem. It does not select H from geometry or curvature coupling. The core factor was constructed from H itself. In particular, it is not the untwisted Friedrichs-selection statement described in I-B-VP-5 [10].

Nor does strong-resolvent convergence imply convergence of ground covariances, differentiated coincidence limits, or contacts. The following elementary example isolates one obstruction.

Proposition (Strong resolvents do not control inverse moments). *There are positive operators $B_n \to I$ in strong resolvent sense and a fixed $f \in \ell^2$ for which* $\langle f, B_n^{-1/2} f\rangle \to \infty$.

Proof. On the standard basis let B_n be the identity except $B_n e_n = 2^{-8n} e_n$, and set $f = \sum_{j \geq 1} 2^{-j} e_j$. The resolvent difference is a uniformly bounded multiple of the projection onto e_n, which converges strongly to zero. But its n-th inverse-half-power contribution is $2^{-2n}2^{4n} = 2^{2n}$.

This example is not a claim that the cap covariances diverge. It proves that an additional spectral and local-renormalization argument is necessary. No such state or one-point limit is released here. In particular, neither c, τ, w nor a rotation-breaking replacement for them is normalized by .

Causal localization of the limiting operator

Theorem (Apex arrival and supported transfer). *For the constant product relation $H \geq 0$, the retarded*

equation-inverse difference from Friedrichs is supported in

$$t - t^{'} \geq \sqrt{(r + r^{'})^{2} + (z - z^{'})^{2}}.$$

The advanced statement is its time reflection. Consequently, any supported Friedrichs Green-operation comparison whose source–target support lies strictly before this threshold is unchanged for the limiting operator in Theorem . Equality of these Green operations does not identify symmetric covariances or states.

Proof. Laplace transform in time, with variable σ in a right half-plane, and Fourier transform in z, with variable k. Set $p = (\sigma^2 + k^2)^{1/2}$, choosing $\operatorname{Re} p > 0$. The transformed correction has the factors $K_{\nu_j}(pr)C_{jk}(p)K_{\nu_k}(pr^{'})$.

The identity

$$C_H(p) = \frac{1}{2}E(p)^{-1}\left(H + \operatorname{diag}\left(\frac{s_u}{2e_u^2}p^{2u}, \frac{s_v}{2e_v^2}p^{2v}\right)\right)^{-1}E(p)^{-1}$$

proves analyticity in $\operatorname{Re} p > 0$. Indeed, if $\operatorname{Im} p > 0$, both displayed diagonal powers have strictly positive imaginary part; the signs reverse below the real axis, and the positive real axis is positive definite. There are no zeros. On closed right half-planes the entries are polynomially bounded. The distributional Laplace support theorem therefore represents $C_{jk}(p)$ as transforms of distributions supported on nonnegative lengths [22].

Also,

$$K_\nu(pr) = \int_r^\infty e^{-pL}\frac{\cosh(\nu \operatorname{arcosh}(L/r))}{\sqrt{L^2 - r^2}}\operatorname{mathop}\mathrm{d}L.$$

Multiplying transforms convolves their length variables. The full radial correction is therefore supported at total length $L \geq r + r^{'}$.

Finally, the inverse Laplace–Fourier transform of $e^{-L\sqrt{\sigma^2 + k^2}}/(2\sqrt{\sigma^2 + k^2})$ is the retarded wave kernel in two spatial dimensions at transverse separation L. Its support is $t - t^{'} \geq \sqrt{L^2 + (z - z^{'})^2}$. Differentiating in L supplies the same support for $e^{-L\sqrt{\sigma^2 + k^2}}$. Convolution with the nonnegative-length distributions proves . On compact time intervals only bounded lengths contribute, so the convolution is proper.

For separated source–target supports strictly before the threshold, the difference pairs to zero. This proves the supported-window claim. A retarded inverse fixes an antisymmetric causal structure, not a symmetric state covariance, so the last conclusion is a separate non-identification.

The supported-transfer statements of I-B-G are thus relevant comparisons [5], but their laboratory-containment hypotheses remain additional obligations. Pairwise arrival bounds do not ensure that every intermediate causal shadow remains in a chosen laboratory.

Strong-resolvent convergence also implies convergence of bounded smeared wave operations, such as $\sin(t\sqrt{A})/\sqrt{A}$, on fixed Hilbert-space vectors. This does not upgrade the theorem to a covariance limit: the inverse-half-power spectral function is unbounded at zero, as Proposition demonstrates.

Ledger 4: finite reports and the shredder cut

Reference-preserving classical-only recovery

A finite symbolic expression may define a continuum function exactly. A finite observed classical word is a different object. Neither the Barnes coefficients nor the core theorem automatically supplies an instrument that acquires the associated field expectation.

The relevant classical-only obstruction is the finite entanglement-breaking mechanism [23]. Its application to destructive reports is also the content of the I-C comparison [12].

Proposition (Classical-only reference bound). *Let an unknown d-dimensional quantum code pass through a channel whose complete accessible output is classical. After any deterministic decoder, its squared fidelity with a maximally entangled reference target is at most $1/d$. Its half-diamond distance from the identity is at least $1 - 1/d$.*

Proof. A classical output has the form

$$\mathcal{M}(\rho) = \sum_x \mathrm{Tr}\,(M_x \rho)|x\rangle\mathrm{langlex}\,|.$$

A decoder assigns a density operator σ_x to each x, so the composite is measure-and-prepare. On half of a

maximally entangled state, its output is separable.

For a product density operator $\rho \otimes \sigma$,

$$\langle \Phi_d | \rho \otimes \sigma | \Phi_d \rangle = \frac{1}{d} \mathrm{Tr}\,(\rho^{\mathrm{T}} \sigma) \leq \frac{1}{d}.$$

Convexity gives the same bound for every separable state. Testing the Bell projector distinguishes this output from the ideal Bell state with probability gap at least $1 - 1/d$. That is a trace-distance lower bound on one referenced input, hence a half-diamond lower bound.

The hypothesis is *complete classical-only access*. A returned quantum memory, coherent reservoir, or other message-bearing residual changes the task. Conversely, reporting coefficients fixed independently of an unknown code does not magically encode that code; the proposition is a conditional channel statement, not a claim that the analytic coefficients themselves are unknown quantum states.

A finite alphabet can have an injective probability law on a continuum parameter family. Therefore the proposition does not say that finite-outcome statistics can never identify a real parameter asymptotically. It distinguishes laws, finite realized records, and coherent code recovery.

Append-only records and endpoint underdetermination

Proposition (Record monotonicity and prefix insufficiency). *For a finite append-only classical transcript* $R_{n+1} = (R_n, X_{n+1})$,

$$H(R_{n+1}) = H(R_n) + H(X_{n+1} \mid R_n) \geq H(R_n).$$

A finite transcript prefix does not select a unique future channel or endpoint.

Proof. The entropy identity is the Shannon chain rule. For the second assertion, fix any implemented prefix. One continuation can return a surviving quantum register unchanged; another can discard it and prepare a fixed state. Both have the same earlier transcript but different terminal channels. One can likewise append different stopping rules. No endpoint is determined without a continuation specification.

These are the precise separations associated with I-C's thm:B and thm:E; they are not a Page curve or an evaporation model [12]. The entropy of an append-only classical report is not the fine-grained entropy of unmeasured radiation.

Heat kernels are not reservoir heat

For a nonnegative spatial operator, e^{-sA} is a heat semigroup in the mathematical sense. This does not provide a reservoir Hamiltonian, temperature, or heat-transfer experiment. The mathematical heat-kernel statement in I-B-VP remains in that category [7].

To see what additional inputs a thermal statement needs, suppose a finite reservoir begins in $\tau_R = e^{-\beta_T H_R}/Z$, independently of a system S, and a declared unitary produces ρ'_{SR}. Let

$$Q = \mathrm{Tr}\, H_R(\rho'_R - \tau_R).$$

Then

$$\beta_T Q = S(\rho_S) - S(\rho'_S) + I(S:R)_{\rho'} + D(\rho'_R \| \tau_R).$$

Indeed, $D(\rho'_R\|\tau_R) = \beta_T Q - [S(\rho'_R) - S(\tau_R)]$, while unitarity and initial independence give

$$I(S:R)_{\rho'} = S(\rho'_S) - S(\rho_S) + S(\rho'_R) - S(\tau_R).$$

Adding proves . This is the standard equality behind finite-reservoir Landauer bounds [24].

Equation assumes the energy and temperature; it does not derive them from a consumed-name count. A degenerate zero-Hamiltonian implementation can have zero reservoir energy change while producing a classical record. Specified reset tasks, nondegenerate Hamiltonians, and bath preparation introduce additional obligations. Thus neither readout-and-drop nor equality of reduced channels determines a universal heat cost. This is the separation recorded in En07-A and in the supplied I-C-Th description [14, 13]. No derivation of the four laws of thermodynamics is asserted.

A trace witness is not a complete kernel

The pure-trace comparison in XVIII-A is another useful separation [15]. Algebraically, a trace map has a kernel: a nonzero traceless symmetric tensor is not determined by its vanishing trace. Similarly, the vanishing regular trace at $\xi = \frac{1}{6}$ does not determine the shear, current, full bisolution, boundary relation, or apex contacts. This is a comparison of information lost under projection, not an identification of the present scalar covariance with XVIII-A's kernel.

Release, source boundaries, and remaining obligations

What is proved

The results can be summarized without combining their interfaces:

1. Theorem gives a complete structural exceptional list and the complete homogeneous relation list in the declared two-channel chart.

2. Proposition disproves the unrestricted Barnes exceptional-list proposal and supplies a genuine additional arithmetic reduction.

3. Theorem gives a finite contact count only under Specification ; Lemma identifies sectors where its symmetry hypotheses fail.

4. Theorem constructs nonnegative engineered smooth caps for a fixed H, including the mixed H_{*}, and proves operator convergence.

5. Theorem retains the apex-arrival threshold and supported Friedrichs transfer windows.

6. Propositions and separate classical reports from coherent return and endpoint selection.

The potentially useful methodological advance is that a mixed matrix relation can be approached by an explicit nonnegative factorized cap family without pretending that operator convergence normalizes quantum one-point distributions. The arithmetic counterexample is equally important: a proposed ``sharp" special-function

classification must specify its reduction category. Neither result repairs a false algebraic closure target.

Companion separations

The following comparisons do not act as premises:

• I-B's prescribed-spacetime instrument and source warning do not derive the cone or an Einstein source law.

• I-B-G's Green operations do not select a symmetric covariance.

• I-B-G-S's free spin-1 result is not this scalar identity; no physical spin-2 Hessian or core selection is supplied.

• I-B-VP and I-B-VP-0 concern their declared Friedrichs state, heat kernel, Hadamard construction, and current. They do not remove the mixed scalar radial current.

• I-B-VP-1's contact and dilation statements require their symmetry, scaling, and completion hypotheses.

• I-B-VP-5's Friedrichs core class is not the engineered finite-angular-rank class of Section .

• I-B-VP-6's non-product relations and I-B-VP-8's thermal state are not constructed by this product massless theorem. These documentary comparisons are attributed to the indicated Cheung records [4, 5, 6, 7, 8, 9, 10, 11].

The supplied OpenAI-side descriptions have only the role of rem:not-premise: Oai282-A and Oai282-B are finite Ward comparisons, not the present Barnes functions and not BMS hair. Families 282, 362, and 376 remain documentary. No mathematical claim about those catalogue families is assumed, verified, or needed here.

Open questions retained rather than renamed

A local scalar core realization of general mixed H, without the declared angular nonlocality, remains outside this theorem. A core-state limit would require infrared control and a selected state prescription. A one-point limit would additionally require uniform differentiated kernel estimates and a renormalization comparison. A contact limit would require a common completion, test space, subtraction, and worldsheet normalization.

The general rotation-breaking contact classification and the resonant $U(2)$ ledger also remain separate questions. None is answered by joint covariance of the matrix family.

At the chart boundary, an order reaching one ceases to be limit circle. At $\beta = 0$, logarithmic traces replace the present positive-order traces. Those problems require different charts; they are not included by notation or by an unproved limiting argument.

Evidence and programme status

This is an analytical source manuscript. No numerical table, quadrature run, sampled cross-check, formal proof checking, or pdfLaTeX compilation is asserted. A build instruction is not a build log. The absence of an execution claim does not replace independent mathematical review.

The free scalar quantities defined here should not be confused with the separate interacting gravitational coefficient mentioned in the supplied programme records. No such quantum coefficient is admitted. No interacting characteristic product is constructed. The documentary status retained by this sheet is

$$\text{boxed}\,\mathrm{O.1}\ \text{on this cone ledger} = \texttt{texttt}\,\mathit{NOT_CLOSED},\qquad \text{XVI-N not discharged},\qquad \text{XV-B not opened}.$$

This does not rewrite a different version-qualified bulk-sector record under a different prescription. It records only the non-transfer boundary of the present construction.

The result is not spacetime emergence, Einstein dynamics, backreaction, holographic reconstruction, a Bel–Robinson calculation, or a Page curve.

appendix

A short audit of the core construction

The following checks are part of the mathematical specification.

Self-adjointness is not inferred from a zero-energy fit. The cap operators are defined by closed nonnegative forms. The limiting operator is identified through its form boundary term in Lemma . Proposition is an additional interface check, not the definition of the operator limit.

No hidden negative spectrum. Although the expanded matrix potential need not be pointwise nonnegative, the

full selected form is the squared norm . Its operator is therefore nonnegative. Pointwise positivity of a potential and positivity of its complete realization are different conditions.

The smooth origin has ordinary angular regularity. Near the origin, $a_\varepsilon = r$, $B = 0$, and $\mathrm{P}_\varepsilon = \mathrm{diag}\,(0,1)/r$. The selected radial expressions are consequently the ordinary $m = 0$ and $|m| = 1$ Euclidean expressions. Regular solutions are constant and linear in r, respectively. The off-diagonal interaction is supported away from the center, where angular projectors have smooth coefficient kernels.

No missing transport derivative. The unitary U_ε has a nonconstant density ratio inside the cap. We do not drop its derivative from a globally expanded transported form. Instead, the lower-bound proof is performed on annuli where U_ε is exactly the identity. Recovery is performed in the cap space and transported only after the exact energy and mass estimates have been obtained.

No core mass was silently discarded. Equation proves vanishing cap mass for the constructed recovery sequence. The lower-bound argument does not assert that every bounded-energy sequence has vanishing cap energy or mass. A nonnegative concentration can remain invisible to a lower bound without invalidating Mosco convergence.

The chosen matrix is an input. For the explicit example, restore a reference length ℓ by replacing H_* with

$$\begin{pmatrix} 2\ell^{-2u} & \frac{1}{2}e^{\mathrm{i}\pi/3}\ell^{-S} \\ \frac{1}{2}e^{-\mathrm{i}\pi/3}\ell^{-S} & 3\ell^{-2v} \end{pmatrix}.$$

Its factorization, cap potential, and matching relation all depend on that declared input. No geometric selection mechanism is being inferred.

Bibliographic and source-status note

The public parent I-B-VP-9, the Enter orientation page, I-B, I-C, I-B-G, I-B-G-S, En07-A, and XVIII-A were consulted as documentary sources. Standard mathematical references below supply the established Bessel, form, distribution, channel, and thermal frameworks.

The detailed I-B-VP-1 and I-C-Th pages were not successfully recovered during source preparation. Their comparisons here are limited to the supplied source descriptions; those descriptions were insufficient to verify their complete conventions or proofs. Accordingly, the contact count, symmetry tests, thermal identity, and all new core arguments are proved locally rather than attributed to an inaccessible theorem. Likewise, the additional companion identifiers in the release ledger are not treated as independently audited results. No missing publication date, journal status, DOI, author field for the present draft, or execution record has been invented.

References

[1] N. Cheung, *I-B-VP-9: Matrix-extension vacuum stress on a prescribed conical spacetime. An obstruction to finite Bernoulli closure, exact Mellin–Barnes formulas, and conserved scalar observables*, version 1, manuscript, Enter Calculus Companion, n.d. https://enter.grok.me/browse/I-B-VP-9. Companion record; not a Scholar-indexed journal article.

[2] N. Cheung, *The Philosophy of Enter: Destructive Readout, Quantum Recovery, and the Refusal of Identification*, Cheung Ngo Medical Limited (2026). Framing essay; not a theorem premise. Programme note: https://enter.grok.me/enter.

[3] N. Cheung, ``The Enter Calculus: A Linear Quantum Lambda Calculus with an Operational Characterization of Measurement," *Frontiers in Physics* **14** (2026), article 1954943. DOI: 10.3389/fphy.2026.1954943. https://doi.org/10.3389/fphy.2026.1954943. Interface/operational background only; not a PDE premise.

[4] N. Cheung, *I-B*, manuscript, Enter Calculus Companion, n.d. Prescribed spacetime Enter and the source-law separation. https://enter.grok.me/browse/I-B. Companion record; not a Scholar-indexed journal article.

[5] N. Cheung, *I-B-G*, version 2, manuscript, Enter Calculus Companion, n.d. Green-operation and prescribed-host comparison. https://enter.grok.me/browse/I-B-G. Companion record; not a Scholar-indexed journal article.

[6] N. Cheung, *I-B-G-S*, version 2, manuscript, Enter Calculus Companion, n.d. Separate free spin and

apex-domain questions. https://enter.grok.me/browse/I-B-G-S. Companion record; not a Scholar-indexed journal article.

[7] N. Cheung, *I-B-VP*, manuscript, Enter Calculus Companion, n.d. Friedrichs scalar, heat-kernel, and regular-state comparison. https://enter.grok.me/browse/I-B-VP. Companion record; not a Scholar-indexed journal article.

[8] N. Cheung, *I-B-VP-0: The holonomy-odd vacuum current of the charged Friedrichs ground state*, version 1, manuscript, Enter Calculus Companion, n.d. https://enter.grok.me/browse/I-B-VP-0. Companion record; not a Scholar-indexed journal article.

[9] N. Cheung, *I-B-VP-1: Prescribed apex finite parts on a conical Enter spacetime*, version 3, manuscript, Enter Calculus Companion, n.d. Contact and general $U(2)$ comparisons are limited here to the supplied source descriptions; no detailed theorem is imported. https://enter.grok.me/browse/I-B-VP-1. Companion record; not a Scholar-indexed journal article.

[10] N. Cheung, *I-B-VP-5: Curvature-core selection on the prescribed conical Enter spacetime*, version 1, manuscript, Enter Calculus Companion, n.d. Documentary comparison with the untwisted Friedrichs-selection class, not a premise of the present core proof. https://enter.grok.me/browse/I-B-VP-5. Companion record; not a Scholar-indexed journal article.

[11] N. Cheung, *I-B-VP-6*, version 1, and *I-B-VP-8*, version 1, manuscripts, Enter Calculus Companion, n.d. Non-product-extension and thermal-state comparisons only. https://enter.grok.me/browse/I-B-VP-6; https://enter.grok.me/browse/I-B-VP-8. Companion record; not a Scholar-indexed journal article.

[12] N. Cheung, *I-C: The Shredder Interface. Destructive Readout, Black-Hole Analogues, and Quantum Computers*, version 2, manuscript, Enter Calculus Companion, n.d. https://enter.grok.me/browse/I-C. Companion record; not a Scholar-indexed journal article.

[13] N. Cheung, *I-C-Th*, manuscript identifier in the supplied source record, n.d. Temperature, energy, and

zero-heat comparisons only; complete source metadata and detailed text were not independently recovered. https://enter.grok.me/browse/I-C-Th. Companion record; not a Scholar-indexed journal article.

[14] N. Cheung, *En07-A: Enter, Reversible Recycling, and Thermodynamic Access: Complete Residuals Without Holographic Identification*, version 1, manuscript, Enter Calculus Companion, n.d. https://enter.grok.me/browse/En07-A. Companion record; not a Scholar-indexed journal article.

[15] N. Cheung, *XVIII-A*, version 1, manuscript, Enter Calculus Companion, n.d. Pure-trace witness and complete-kernel separation. https://enter.grok.me/browse/XVIII-A. Companion record; not a Scholar-indexed journal article.

[16] B. S. Kay and U. M. Studer, Boundary conditions for quantum mechanics on cones and fields around cosmic strings, *Communications in Mathematical Physics* **139** (1991), 103–139. doi: 10.1007/BF02102731.

[17] J. Dereziński and S. Richard, On Schrödinger operators with inverse square potentials on the half-line, *Annales Henri Poincaré* **18** (2017), 869–928. doi: 10.1007/s00023-016-0520-7. arXiv: 1604.03340.

[18] J. Behrndt, M. M. Malamud, and H. Neidhardt, Scattering matrices and Weyl functions, *Proceedings of the London Mathematical Society* **97** (2008), 568–598. doi: 10.1112/plms/pdn016. arXiv: math-ph/0604013.

[19] NIST, *Digital Library of Mathematical Functions*, Chapter 10, especially the modified-Bessel expansions, Wronskians, integral representations, and product integrals. https://dlmf.nist.gov/10.

[20] U. Mosco, Composite media and asymptotic Dirichlet forms, *Journal of Functional Analysis* **123** (1994), 368–421. doi: 10.1006/jfan.1994.1093.

[21] T. Kato, *Perturbation Theory for Linear Operators*, Springer, Berlin, 1995, reprint of the second edition. doi: 10.1007/978-3-642-66282-9.

[22] L. Hörmander, *The Analysis of Linear Partial Differential Operators I: Distribution Theory and Fourier Analysis*, second edition, Springer, Berlin, 1990. Supported distributions and distributional Fourier–Laplace support theory.

[23] M. Horodecki, P. W. Shor, and M. B. Ruskai, Entanglement breaking channels, *Reviews in Mathematical Physics* **15** (2003), 629–641. doi: 10.1142/S0129055X03001709. arXiv: quant-ph/0302031.

[24] D. Reeb and M. M. Wolf, An improved Landauer principle with finite-size corrections, *New Journal of Physics* **16** (2014), 103011. doi: 10.1088/1367-2630/16/10/103011. arXiv: 1306.4352.

[25] N. Cheung, *Black-Hole Information as an Access Problem: Enter, Quantum Recovery, and Complete Residuals*, figshare (2026). DOI: 10.6084/m9.figshare.34335024.v1. https://doi.org/10.6084/m9.figshare.34335024.v1.

[26] N. Cheung, *Operational Wormhole Certificates—Existence, Dimension-Dependent Scaling, Operational Functions, and Composition*, figshare (2026). DOI: 10.6084/m9.figshare.34549575.v1. https://doi.org/10.6084/m9.figshare.34549575.v1.

[27] N. Cheung, *Enter, Quantum Magic, and Quantum Gravity: Readout, Recovery, and the Limits of Geometric Response*, figshare (2026). DOI: 10.6084/m9.figshare.34335582.v1. https://doi.org/10.6084/m9.figshare.34335582.v1.

[28] N. Cheung, *Operational Consistency Does Not Select Lorentzian Geometry*, figshare (2026). DOI: 10.6084/m9.figshare.34441758.v1. https://doi.org/10.6084/m9.figshare.34441758.v1.

[29] N. Cheung, *Realization of Relational Time by Destructive Enter on a Clock Register*, figshare (2026). DOI: 10.6084/m9.figshare.34442907.v1. https://doi.org/10.6084/m9.figshare.34442907.v1.