Enter, I-B-VP-9-v2, 2026, pp. 1-38

I-B-VP-9: Matrix-extension vacuum stress on a prescribed conical spacetime

An obstruction to finite Bernoulli closure, exact Mellin–Barnes formulas, and conserved scalar observables

Ngo Cheung

18 October 2026

Abstract

We study the massless scalar vacuum-stress correction produced by a non-diagonal, nonnegative boundary matrix in the two-deficiency-channel sector of a prescribed four-dimensional conical product. The proposed reduction to a finite Bernoulli polynomial is false: a finite boundary matrix introduces length scales, and even a positive diagonal example has a nonconstant radial crossover. More sharply, its stress coefficient is a Stieltjes transform with a continuous, nonzero density and is therefore not rational in the boundary parameter. This obstruction persists for conformal coupling, despite the vanishing regular trace.

We replace the impossible algebraic target by an exact construction. After deriving the boundary normalization, we express every moment needed for the complete regular stress tensor and charged current as a finite combination of explicitly defined, absolutely convergent three-variable Mellin–Barnes functions. The formulas apply to unequal channel orders, retain angular interference and the radial current, and extend continuously to every finite positive-semidefinite boundary matrix. Scalar conservation and the mixed-trace identity follow analytically from the smooth bisolution correction. Friedrichs, singular scale-invariant, diagonal scale-dependent, and equal-order projector limits are recovered.

The principal contribution is a convention-complete resolution of what finite boundary rank does and does not simplify: the tensor assembly is finite, but generic radial dependence is genuinely non-algebraic. The result

supplies exact crossover functions and a controlled special-function representation rather than an unsupported polynomial closure. The geometry, extension, field normalization, and subtraction remain inputs. No finite Enter record extracts a continuum observable; no apex contact normalization, interacting gauge theory, Einstein dynamics, backreaction, or spacetime emergence is inferred.

Bibliographic record

Title	I-B-VP-9: Matrix-extension vacuum stress on a prescribed conical spacetime
Author	Cheung, Ngo
Publication date	2026/10/18
Institution	Enter
Report number	I-B-VP-9-v2
Publisher	Cheung Ngo Medical Limited
First page	1
Last page	38
Language	en

Book edition note (t425u)

Standing Enter refusals, scope vocabulary (proved here/supplied input/conditional/ not constructed/not claimed), and Scholar bibliography policy are frozen in the companion chapter *I-B-VP Book Ledger* v1. O.1 remains NOT_CLOSED; XVI-N is not discharged; XV-B is not opened; no Host C, Page/island, or spacetime-emergence claim; wired=false; SOURCE_DATE_EPOCH=1792627200. This sheet is a companion manuscript record and is *not* claimed to be Google Scholar-indexed. The published Enter Calculus article is cited only for finite-instrument / operational interface discussions, never as a PDE premise.

Book expansion: Bernoulli correction and Mellin–Barnes

Open with the correction to the proposed finite Bernoulli closure. Define the exact reduction class before exceptional cases. Keep structural channel reduction separate from arithmetic Barnes reductions. Present the Mellin–Barnes function with convergence conditions immediately adjacent to its definition; retain the Stieltjes counterexample as the reason no rational closure is available.

Question, correction of the plan, and main results

The scalar extension programme of I-B-G and I-B-VP distinguishes a local differential expression, its apex realization, its state, and its regular one-point observables [5, 6, 7]. The supplied I-B-VP-1 version-three excerpts further describe non-diagonal two-channel relations, their ultraviolet limits, and off-diagonal apex asymptotics [10]. I-B-G-S-v2 supplies a separate helicity-domain and finite-jet analysis for free spin 1 and spin 2 [11].

The proposed V9 plan asks for a generic Hermitian boundary matrix $H \geq 0$ to give a finite Bernoulli combination with coefficients rational in H and independent of radius. Two problems must be corrected before that question can become a theorem.

First, two scalar deficient channels exist only in a specified range of cone and line-holonomy parameters. They do not exist for every cone angle. Second, finite boundary rank is not scale invariance. The factor $(H + \Delta(p))^{-1}$ depends on spectral momentum, and that dependence survives the momentum integral.

This paper proves the following corrected results.

1. A normalized two-channel Krein formula, derived from explicitly defined radial boundary coefficients (Theorem).

2. An exact scaling law and a counterexample to finite radius-independent Bernoulli closure (Theorem). The same example proves nonrational dependence on a finite boundary parameter.

3. An exact finite special-function prescription for every coincidence moment needed for the scalar stress and

current (Theorem).

4. Complete regular-host stress and current formulas, including angular shear and radial current, with analytical conservation and trace identities (Theorems and).

5. Recovery of the scale-invariant and equal-order limits, together with a precise separation between regular observables, apex contacts, and causal transfer (Sections –).

The Mellin–Barnes representation is an exact evaluation of the original Bessel moment in a specified special-function class. It is not a claim that all numerical integration has disappeared, nor a claim that a triple contour is always the fastest numerical method.

Source and novelty scope

The supplied I-B-VP-SAE record is credited for the extension problem and the stated open generic-matrix stress target. Only supplied excerpts, rather than a complete independently audited SAE source, are available for its detailed conventions. For this reason the boundary normalization and the formulas used below are derived directly.

The underlying extension theory, Bessel resolvents, Krein formulas, and scalar point splitting are established subjects [15, 16, 17, 19]. The contribution claimed here is the present exact reduction, the obstruction to the proposed polynomial target, and the complete scalar observable assembly for this declared two-channel problem. No general priority claim for matrix extensions or Mellin–Barnes methods is made.

Enter, spacetime Enter, and fixed-host boundaries

Cheung's Enter calculus concerns finite operational semantics; its measurement characterization is inside an explicitly stated strategy class [1]. The introductory essay *The Philosophy of Enter* separates destructive readout, quantum recovery, physical implementation, and geometric identification [2]. The essay is interpretive background, not an extension-theory premise.

Spacetime Enter, in I-B, places finite instruments on a prescribed Lorentzian host [3]. It does not derive that host from records. In particular, let C be a finite clock, A the complete surviving register, and $\{|\tau_j\rangle\}$ an orthonormal

clock basis. Then

$$D_j = \langle \tau_j |_C \otimes I_A, \qquad \mathscr{M}(X) = \sum_j \mathrm{proj}\, j_R \otimes D_j X D_j^\dagger$$

satisfies

$$\sum_j D_j^\dagger D_j = I_{CA}, \qquad \mathrm{Tr}_R\, \mathscr{M}(X) = \mathrm{Tr}_C\, X.$$

Expansion in the clock basis proves both identities, including untouched-reference extensions. These are the complete record–residual identities of I-B and En09-A [3, thm:instrument] [4].

The finite channel returns a classical code and its specified residual. It does not select H, evaluate an unknown continuum integral, construct a covariance, or acquire the stress tensor computed below. Consumption concerns quantum names, not geometric time.

Scope boundary (Non-sourcing). The metric, deficit, line holonomy, extension, field normalization, and renormalization convention are declared inputs. A symmetric conserved traceless tensor need not source its background. For example, in a regular flat chart, $t_{\mu\nu} = k_\mu k_\nu$ for a nonzero constant null covector is conserved, traceless, and nonzero, while the chart Einstein tensor vanishes. This is the separation recorded by I-B prop:not-source and X-A-ST prop:noeinstein [3, 12].

The standing programme boundaries are unchanged:

$$\text{O.1 on the cone} = \texttt{NOT_CLOSED}, \qquad \texttt{bulk_symmetry_restored} = \texttt{false}.$$

XVI-N is not discharged. The downstream XV-B quantum-admission exit is not opened. No result below constructs the different characteristic graph products described in the supplied XIV–XV records.

Host, charged scalar, and the two-channel boundary chart

Assumption (Prescribed regular conical product). Let

$$M = \mathbb{R}_t \times (0,\infty)_r \times S^1_\varphi \times \mathbb{R}_z, \qquad g = -\mathrm{d}t^2 + \mathrm{d}r^2 + \alpha^2 r^2 \mathrm{d}\varphi^2 + \mathrm{d}z^2, \qquad 0 < \alpha < 1.$$

The missing completion worldsheet is $\mathscr{W} = \mathbb{R}_t \times \{o\} \times \mathbb{R}_z$. A massless complex scalar is a section of a flat line bundle with

$$\Phi(\varphi + 2\pi) = e^{2\pi i\beta}\Phi(\varphi), \qquad 0 < \beta < 1.$$

Work in a local parallel bundle frame; thus ordinary derivatives below are the corresponding local covariant derivatives.

The developed angle is $\theta = \alpha\varphi$. The scalar equation is $\mathrm{Box}_g \Phi = 0$ on M. The curvature-coupling parameter $\xi \in \mathbb{R}$ enters the improved stress tensor but not this regular-host equation, because the regular host is flat.

The spatial volume is

$$\mathrm{d}V = \alpha r \, \mathrm{d}r \, \mathrm{d}\varphi \, \mathrm{d}z.$$

The regular host is not globally hyperbolic; an apex realization is therefore substantive input, not data supplied by local flatness [5].

Assumption (Two deficient channels and product realization). Assume

$$\frac{1}{2} < \alpha < 1, \qquad 1 - \alpha < \beta < \alpha.$$

Label the two deficient angular channels by

$$q_0 = \beta, \qquad q_1 = \beta - 1, \qquad u = \nu_0 = \frac{\beta}{\alpha}, \qquad \nu = \nu_1 = \frac{1-\beta}{\alpha}, \qquad S = u + \nu = \alpha^{-1}.$$

Thus

$$0 < u, \nu < 1, \qquad 1 < S < 2.$$

All other angular channels are Friedrichs. The transverse two-channel relation is independent of longitudinal momentum and is multiplied by the free $-\partial_z^2$ realization.

At the endpoints of , an order reaches one and is not a second limit-circle channel. At $\beta = 0$ a logarithmic channel requires a different treatment. Neither case is silently included here.

Boundary coefficients

For $0 < \nu < 1$, the radial expression is

$$L_\nu = -\partial_r^2 - r^{-1}\partial_r + \nu^2 r^{-2} \quad \text{on} L^2((0,\infty), r\,\mathrm{d}r).$$

Write the maximal-domain boundary expansion as

$$f_j(r) = a_j r^{-\nu_j} + b_j r^{\nu_j} + \text{a minimal-domain remainder.}$$

The coefficients mean the two boundary traces, not a pointwise regularity assertion for every maximal-domain representative.

For the convention

$$\Omega(f,g) = \lim_{r \,\mathrm{downarrow}\, 0} r(f\partial_r \overline{g} - (\partial_r f)\overline{g}),$$

one has

$$\Omega(f,g) = 2\sum_j \nu_j (a_j \overline{b_j(g)} - b_j \overline{a_j(g)}).$$

Definition (Finite nonnegative matrix chart). Fix a Hermitian matrix

$$H = \begin{pmatrix} h_0 & h \\ \overline{h} & h_1 \end{pmatrix} \geq 0$$

and impose

$$\mathrm{diag}\,(u,v)b = Ha.$$

Friedrichs is the boundary condition $a = 0$ and is not a finite matrix in this chart. The finite matrix $H = 0$ is the singular scale-invariant realization in both deficient channels.

The boundary form proves self-adjointness of the graph relation . Conversely, a finite Hermitian graph gives such a self-adjoint boundary relation. After choosing units, the Cayley transform of a boundary graph is the familiar finite chart in the $U(2)$ parameterization. Relations with a Friedrichs direction lie on its boundary.

The dimensions are

$$[H_{jk}] = \text{length}^{-(\nu_j + \nu_k)}.$$

A reference length can make the boundary coordinates and H dimensionless. It does not remove the physical crossover scales of a fixed finite relation.

Normalized Krein correction and regularity

Put

$$e_\nu = 2^{\nu-1}\Gamma(\nu), \qquad s_\nu = \frac{\pi}{2\sin(\pi\nu)}, \qquad E(p) = \operatorname{diag}(e_u p^{-u}, e_v p^{-v}), \qquad S_0 = \operatorname{diag}(s_u, s_v).$$

The subscript on S_0 distinguishes this matrix from the number $S = u + v$.

Theorem (Boundary-normalized matrix resolvent). *For $p > 0$, the correction to the transverse Friedrichs resolvent has kernel*

$$\delta R_H(p; x_\perp, x_\perp') = \frac{1}{2\pi\alpha} \sum_{j,k=0}^{1} e^{iq_j\varphi} K_{\nu_j}(pr) C_{jk}(p) K_{\nu_k}(pr') e^{-iq_k\varphi'},$$

where

$$C(p) = (S_0 + 2E(p)HE(p))^{-1}.$$

For $H \geq 0$,

$$0 \leq C(p) \leq S_0^{-1}.$$

The transverse realization is nonnegative.

Let $y = (\tau, z)$ be the Euclidean longitudinal pair and set $\mathcal{N}_\alpha = (4\pi^2\alpha)^{-1}$. The Euclidean covariance correction is

$$\delta W_{E,H}(x, x') = \mathcal{N}_\alpha \int_0^\infty p\, dp\, J_0(p|y - y'|) \sum_{j,k=0}^{1} e^{iq_j\varphi} K_{\nu_j}(pr) C_{jk}(p) K_{\nu_k}(pr') e^{-iq_k\varphi'}.$$

Proof. The Friedrichs radial Green kernel relative to $r'\,dr'$ is $I_\nu(pr_<)K_\nu(pr_>)$. The endpoint expansions are

$$K_\nu(pr) = e_\nu p^{-\nu} r^{-\nu} + d_\nu p^\nu r^\nu + \cdots, \qquad d_\nu = 2^{-\nu-1}\Gamma(-\nu),$$

$$I_\nu(pr) = \frac{p^\nu r^\nu}{2\nu e_\nu} + \cdots, \qquad 2\nu e_\nu d_\nu = -s_\nu.$$

Adding $\sum K_{\nu_j} C_{jk} K_{\nu_k}$ and imposing gives

$$I - S_0 C = 2EHEC.$$

This proves . The Bessel Wronskian fixes the unchanged Green jump [18, Chapter 10].

Positivity gives . The matrix defining the resolvent is invertible at every $p > 0$. The resulting resolvent exists on the negative real spectral axis, so the self-adjoint transverse operator has no negative spectrum. Equivalently, a decaying negative-energy eigenfunction would require $(S_0 + 2EHE)c = 0$, which is impossible. The finite-rank resolvent formula also leaves the essential spectrum unchanged. These are the standard Bessel and boundary-relation mechanisms [16, 17].

Normalized angular modes give $1/(2\pi\alpha)$. Fourier inversion in two Euclidean longitudinal variables gives $(2\pi)^{-1}\int_0^\infty pJ_0(p|y - y'|)\mathrm{d}p$. Their product is .

Proposition (Smooth regular correction and local subtraction). *The kernel and every finite derivative are well defined on compact sets with $r, r' > 0$. Near a regular diagonal it is an analytic Euclidean bisolution.*

The massless product ground covariance is finite on compact regular test functions. Its local Lorentzian continuation has the same direct Hadamard singularity as the Friedrichs covariance. Consequently the stress and current differences below require no new regular-host subtraction.

Proof. At large p, every differentiated radial factor is bounded by a polynomial in p times $e^{-p(r+r')}$ on regular compact sets. At small p, radial differentiation does not worsen the momentum power $p^{-\nu_j}$ of $K_{\nu_j}(pr)$. Equation therefore bounds the worst undifferentiated integrand by a constant times

$$p^{1-\nu_j-\nu_k}.$$

This is integrable because $\nu_j + \nu_k < 2$. Longitudinal derivatives add nonnegative momentum powers.

Differentiation under the integral follows.

The same estimates hold in sufficiently small complex neighborhoods of regular diagonal points and establish analyticity. The resolvent difference solves the homogeneous equation at both regular endpoints, since the two Green jumps cancel.

For a compact spatial test f, the spectral identity

$$\frac{1}{2\sqrt{A}} = \frac{1}{\pi}\int_0^\infty (A + \omega^2)^{-1} \mathrm{d}\omega$$

and the same two-dimensional low-momentum bound prove finiteness of the correction to its ground covariance. The Friedrichs compact-test construction is the scalar input of I-B-VP [6]. Positivity is supplied by the nonnegative self-adjoint spatial realization. The local analytic continuation of the smooth difference changes no direct Hadamard singularity.

Remark (Rank and field type). The correction has rank at most two in the transverse resolvent fiber. It is not a rank-two operator on the full longitudinal product space. Moreover, this is a scalar line-bundle problem. The two angular channels are not the helicity and ghost channels of I-B-G-S-v2. Scalar conservation below is not a transferred Maxwell or de Donder Ward theorem.

Why the finite Bernoulli target fails

Introduce

$$a = \frac{2e_u^2 h_0}{s_u}, \qquad b = \frac{2e_v^2 h_1}{s_v}, \qquad c = \frac{2e_u e_v h}{\sqrt{s_u s_v}}, \qquad d = ab - |c|^2 \geq 0.$$

Then

$$\begin{aligned} D(p) &= 1 + ap^{-2u} + bp^{-2v} + dp^{-2S}, \\ C_{00}(p) &= \frac{1 + bp^{-2v}}{s_u D(p)}, \qquad C_{11}(p) = \frac{1 + ap^{-2u}}{s_v D(p)}, \\ C_{01}(p) &= -\frac{cp^{-S}}{\sqrt{s_u s_v} D(p)}, \qquad C_{10}(p) = \overline{C_{01}(p)}. \end{aligned}$$

Proposition (Exact scaling law). *Let* $D_\lambda = \mathrm{diag}(\lambda^u, \lambda^v)$. *Simultaneous scaling of all dimensional coordinates gives*

$$\delta W_{E,H}(\lambda x, \lambda x') = \lambda^{-2} \delta W_{E, D_\lambda H D_\lambda}(x, x').$$

In particular, for the Wick square and orthonormal-frame stress,

$$\delta F_H(\lambda r, \varphi) = \lambda^{-2} \delta F_{D_\lambda H D_\lambda}(r, \varphi),$$

$$\delta T^{\wedge\wedge}_{ab,H}(\lambda r, \varphi) = \lambda^{-4} \delta T^{\wedge\wedge}_{ab, D_\lambda H D_\lambda}(r, \varphi).$$

The dimensionless crossover arguments are

$$X = a r^{2u}, \qquad Y = b r^{2v}, \qquad Z = d r^{2S}, \qquad c_r = c r^S.$$

Proof. In , replace p by p/λ. Then $E(p/\lambda) = D_\lambda E(p)$. Every derivative in an orthonormal frame contributes one additional inverse length.

Theorem (Obstruction to the proposed algebraic closure). *Fix* $\beta = 1/2$, *so* $u = v = \nu = 1/(2\alpha) \in (1/2, 1)$, *and take* $H = hI$, $h > 0$. *Let*

$$a = \frac{2 e_\nu^2 h}{s_\nu}, \qquad z = a r^{2\nu}.$$

For minimal scalar coupling,

$$\delta T^t_{\ t}(r) = -\frac{2\mathcal{N}_\alpha}{s_\nu r^4} \mathcal{G}_\nu(z), \qquad \mathcal{G}_\nu(z) = \int_0^\infty \frac{x^3 K_\nu'(x)^2 + \nu^2 x K_\nu(x)^2}{1 + z x^{-2\nu}} \, \mathrm{d}x.$$

The function $\mathcal{G}_\nu$ *is strictly decreasing, has a positive finite value at zero, and tends to zero at infinity. It is not rational in z.*

Consequently:

1. *the generic correction cannot be a finite combination of the scale-invariant Bernoulli branches with radius-independent coefficients;*

2. *it cannot be a sum of* r^{-2} *and* r^{-4} *with radius-independent coefficients;*

3. *it is not rational in the entries or eigenvalues of H, even at fixed radius;*

4. *the radial obstruction also holds at conformal coupling* $\xi = 1/6$*. This excludes the stated finite algebraic target, not exact special-function representations.*

Proof. The stress formula is derived independently in Theorem ; specializing it to the two equal diagonal channels and $\xi = 0$ gives . Its weight

$$w_\nu(x) = x^3 K_\nu'(x)^2 + \nu^2 x K_\nu(x)^2$$

is positive. It is integrable at zero because it is $O(x^{1-2\nu})$, and at infinity by exponential decay. Dominated convergence proves the two limits, while differentiation at $z > 0$ gives

$$\mathcal{G}_\nu'(z) = -\int_0^\infty \frac{w_\nu(x) x^{-2\nu}}{(1 + z x^{-2\nu})^2} \mathrm{d}x < 0.$$

Set $t = x^{2\nu}$. Then

$$\mathcal{G}_\nu(z) = \int_0^\infty \frac{\rho_\nu(t)}{t + z} \mathrm{d}t, \qquad \rho_\nu(t) = \frac{1}{2\nu} t^{1/(2\nu)} w_\nu(t^{1/(2\nu)}) > 0.$$

This density is continuous on $(0, \infty)$ and nonzero on every open subinterval. The boundary values of its Stieltjes transform satisfy, for $t > 0$,

$$\mathcal{G}_\nu(-t + \mathrm{i}0) - \mathcal{G}_\nu(-t - \mathrm{i}0) = -2\pi \mathrm{i}\, \rho_\nu(t).$$

A rational function has no such continuous jump on an interval away from its finitely many poles. Hence $\mathcal{G}_\nu$ is not rational. Restricting a claimed rational matrix formula to $H = hI$ would give the excluded rational function of h.

Every scale-invariant stress branch is homogeneous of degree -4. Its finite linear span with radius-independent coefficients cannot have the two different limits of $r^4 \delta T^t_{\ t}$. A proposed $Ar^{-2} + Br^{-4}$ would instead give $Ar^2 + B$, which cannot have those bounded, distinct endpoint limits.

For conformal coupling, the same diagonal calculation has

$$\lim_{r\,\mathrm{downarrow}\,0} r^4 \delta T^t{}_t = -\frac{4}{3}\mathcal{N}_\alpha v^3 \neq 0, \qquad \lim_{r\,\to\,\infty} r^4 \delta T^t{}_t = 0.$$

The gamma-moment evaluation establishing the first value is given in Section ; the second follows from dominated convergence. Thus tracelessness does not restore the proposed homogeneous formula.

Remark (Why diagonalizing *H* is insufficient). For $u \neq v$, the matrix $\Delta(p)$ in a normalized Weyl description has unequal powers p^{2u} and p^{2v}. A constant unitary diagonalizing H does not generally diagonalize $H + \Delta(p)$ for all p. Simultaneous diagonalization requires the corresponding commutation condition. In the unequal-order two-channel case this forces $h = 0$. Only the equal-order case permits unrestricted constant eigenvector diagonalization.

Exact Mellin–Barnes replacement

The Bessel product Mellin transform

Lemma (Product moment). *For real η, χ and* $\mathrm{Re}\,\lambda > |\eta| + |\chi|$,

$$\mathcal{M}_{\eta\chi}(\lambda) := \int_0^\infty x^{\lambda-1} K_\eta(x) K_\chi(x)\,\mathrm{d}x = \frac{2^{\lambda-3}}{\Gamma(\lambda)} \prod_{\varepsilon,\,\delta\,=\,\pm 1} \Gamma\Big(\frac{\lambda + \varepsilon\eta + \delta\chi}{2}\Big).$$

Proof. Insert the Schwinger representations of the two K functions. The x integral is a gamma integral. In the two remaining positive variables use their sum and their ratio; the ratio integral is a beta integral and the sum integral is a gamma integral. Collecting the powers of two gives . The stated inequality justifies absolute integration at zero; infinity is exponentially convergent. The Bessel integral representations used here are those of [18, Sections 10.32 and 10.43].

An explicitly specified multivariate Barnes function

Definition (Master Barnes function). For $X, Y, Z > 0$, define

$$\begin{aligned}\mathcal{B}^{\lambda}_{\eta\chi}(X,Y,Z;u,v) = \frac{1}{(2\pi\mathrm{i})^3}\int_{\mathscr{L}_1}\int_{\mathscr{L}_2}\int_{\mathscr{L}_3} \quad & \Gamma(z_1)\Gamma(z_2)\Gamma(z_3)\Gamma(1-z_1-z_2-z_3) \\ \times \quad & X^{-z_1}Y^{-z_2}Z^{-z_3} \\ \times \quad & \mathcal{M}_{\eta\chi}(\lambda + 2uz_1 + 2vz_2 + 2Sz_3)\,\mathrm{d}z_3\mathrm{d}z_2\mathrm{d}z_1.\end{aligned}$$

The upward vertical contours have real parts $c_j > 0$, with

$$c_1 + c_2 + c_3 < 1, \qquad \lambda + 2uc_1 + 2vc_2 + 2Sc_3 > |\eta| + |\chi|.$$

Positive real arguments use real logarithms.

This is a gamma-product multivariate Mellin–Barnes function, of the generalized multivariate H-function type.

Equation , rather than an ambiguous naming convention for that class, is its definition here.

Lemma (Convergence and evaluation of the master function). *Under , is absolutely convergent and*

$$\mathscr{B}^{\lambda}_{\eta\chi}(X,Y,Z;u,v) = \int_0^{\infty} \frac{x^{\lambda-1}K_{\eta}(x)K_{\chi}(x)}{1 + Xx^{-2u} + Yx^{-2v} + Zx^{-2S}}\,\mathrm{d}x.$$

The right side is an equality theorem for the Barnes function, not its definition.

Proof. The multivariate Barnes identity

$$\frac{1}{1+A+B+C} = \frac{1}{(2\pi\mathrm{i})^3}\int \Gamma(z_1)\Gamma(z_2)\Gamma(z_3)\Gamma(1-z_1-z_2-z_3)A^{-z_1}B^{-z_2}C^{-z_3}\,\mathrm{d}z$$

follows by applying Mellin inversion to the three exponential factors in $\int_0^{\infty} e^{-t}e^{-tA}e^{-tB}e^{-tC}\mathrm{d}t$. Its contour conditions are $c_j > 0$ and $\sum c_j < 1$.

Substitute $A = Xx^{-2u}$, $B = Yx^{-2v}$, and $C = Zx^{-2S}$. Absolute values of the x powers depend only on c_j. Condition gives an integrable Bessel product. Stirling bounds give exponential integrability of the Barnes gamma factors in all contour variables. Fubini's theorem is therefore applicable. The x integral is , proving the result.

All required matrix moments

For real η,χ, define the normalized radial moments

$$\mathscr{I}^{[m]}_{jk}(\eta,\chi;r) = \mathscr{N}_{\alpha}\int_0^{\infty} p^{m-1}C_{jk}(p)K_{\eta}(pr)K_{\chi}(pr)\mathrm{d}p.$$

Theorem (Finite exact special-function formula). *Suppose $H > 0$. With the arguments ,*

$$\mathscr{I}^{[m]}_{00}(\eta,\chi;r) = \frac{\mathscr{N}_{\alpha}r^{-m}}{s_u}\Big[\mathscr{B}^{m}_{\eta\chi}(X,Y,Z) + Y\mathscr{B}^{m-2v}_{\eta\chi}(X,Y,Z)\Big],$$

$$\mathscr{I}_{11}^{[m]}(\eta,\chi;r)=\frac{\mathscr{N}_{\alpha}r^{-m}}{s_{v}}\Big[\mathscr{B}_{\eta\chi}^{m}(X,Y,Z)+X\mathscr{B}_{\eta\chi}^{m-2u}(X,Y,Z)\Big],$$

$$\mathscr{I}_{01}^{[m]}(\eta,\chi;r)=-\frac{\mathscr{N}_{\alpha}r^{-m}c_{r}}{\sqrt{s_{u}s_{v}}}\mathscr{B}_{\eta\chi}^{m-S}(X,Y,Z),$$

$$\mathscr{I}_{10}^{[m]}(\eta,\chi;r)=-\frac{\mathscr{N}_{\alpha}r^{-m}\overline{c_{r}}}{\sqrt{s_{u}s_{v}}}\mathscr{B}_{\eta\chi}^{m-S}(X,Y,Z).$$

The suppressed final parameters of every $\mathscr{B}$ are (u,v).

For all moments appearing in below, the common contours

$$c_{1}=c_{2}=c_{3}=\frac{1}{4}$$

satisfy . Thus the complete regular stress and current use only finitely many evaluations of an absolutely convergent, explicitly specified special function.

For every finite $H\ge 0$, the formulas extend by a locally smooth limit from positive-definite matrices.

Proof. Replace p by x/r in and insert . Lemma gives –.

On , the added real Mellin exponent is

$$2u/4+2v/4+2S/4=S.$$

For the worst numerator shifts, the excess over the largest Bessel-order sum needed for the corresponding derivative moment is $2-S>0$. For example,

$$(2-2v)+S-2u=2-S,$$

and for two radial derivatives,

$$(4-2v)+S-(2u+2)=2-S.$$

The off-diagonal worst case is $4-S+S-(u+v+2)=2-S$. The remaining cases have the same or a larger margin.

For semidefinite H, fix a reference length $\ell>0$ and set

$$H_\varepsilon = H + \varepsilon \mathrm{diag}\,(\ell^{-2u}, \ell^{-2v}).$$

Then $H_\varepsilon > 0$, $C_{H_\varepsilon}(p) \to C_H(p)$, and is uniform. The regular-compact estimates in Proposition prove convergence of every required differentiated moment. The limit is independent of ℓ. This supplies the meaning at $Z = 0$ or at vanishing arguments; one must not insert zero directly into the powers in .

Remark (What has been simplified). The spectral Bessel integral is replaced by a finite gamma-product special-function prescription with fixed admissible contours. The matrix dependence occurs through X, Y, Z, c_r, not through radius-independent rational coefficients. This is exact analytic reduction, not finite algebraic closure.

Complete stress and current from finitely many moments

Normalization of the complex scalar

Use

$$\Phi = \frac{\varphi_1 + \mathrm{i}\varphi_2}{\sqrt{2}}$$

with two canonically normalized real scalars. The covariance $W(x, x') = \langle \Phi(x)\Phi^*(x')\rangle$ has the usual real-scalar singular normalization. The complex stress is the sum of the two real-scalar stresses. Set $\hbar = 1$; it multiplies every one-loop quantity below.

For a smooth Hermitian bisolution correction, define

$$F(x) = \delta W(x, x), \qquad A_{ab} = \mathrm{Re}\,[\nabla_a \nabla_{b'} \delta W(x, x')]_{x' = x},$$

with symmetric tensor interpretation. Let $\mathscr{K} = g^{ab} A_{ab}$. The stress and charge-$e$ current conventions are

$$\delta T_{ab} = 2A_{ab} - g_{ab}\mathscr{K} + 2\xi(g_{ab}\mathrm{Box}_g - \nabla_a \nabla_b)F,$$

$$\delta J_a = \mathrm{i}e[(\nabla_a - \nabla_{a'})\delta W(x, x')]_{x' = x}.$$

Hermiticity makes the current real. These are regular differences of the charged point-splitting prescriptions; state-independent local subtraction terms cancel [19, 20]. The sign of e is part of .

The finite matrix list

Define

$$R_{jk} = \mathscr{I}^{[2]}_{jk}(v_j, v_k; r),$$

$$L_{jk} = -\frac{1}{2}\sum_{\varepsilon=\pm1} \mathscr{I}^{[3]}_{jk}(v_j + \varepsilon, v_k; r),$$

$$\widetilde{L}_{jk} = -\frac{1}{2}\sum_{\varepsilon=\pm1} \mathscr{I}^{[3]}_{jk}(v_j, v_k + \varepsilon; r),$$

$$B_{jk} = \frac{1}{4}\sum_{\varepsilon,\delta=\pm1} \mathscr{I}^{[4]}_{jk}(v_j + \varepsilon, v_k + \delta; r),$$

$$P_{jk} = \frac{1}{2}\mathscr{I}^{[4]}_{jk}(v_j, v_k; r).$$

The identities $K_{-\lambda} = K_\lambda$ and $K'_\lambda = -(K_{\lambda-1} + K_{\lambda+1})/2$ fix all order-shift conventions [18, Sections 10.27 and 10.29].

For any matrix M, write

$$\langle M\rangle_\varphi := \sum_{j,k=0}^{1} e^{\mathrm{i}(q_j - q_k)\varphi} M_{jk}.$$

Set

$$F = \mathrm{Re}\,\langle R\rangle_\varphi, \qquad P = \mathrm{Re}\,\langle P\rangle_\varphi, \qquad B = \mathrm{Re}\,\langle B\rangle_\varphi,$$

$$Q = \mathrm{Re}\sum_{j,k} q_j q_k e^{\mathrm{i}(q_j - q_k)\varphi} R_{jk}, \quad U = \mathrm{Re}\sum_{j,k} (-\mathrm{i}q_k) e^{\mathrm{i}(q_j - q_k)\varphi} L_{jk},$$

$$\mathscr{K} = 2P + B + \frac{Q}{\alpha^2 r^2}, \qquad \mathscr{L} = 2\mathscr{K}.$$

Using the same letter for a matrix and its displayed contracted scalar is confined to ; component subscripts always indicate the matrix.

The derivatives needed for improvement are also finite:

$$F_r = \mathrm{Re}\,\langle L + \widetilde{L}\rangle_\varphi,$$

$$F_\varphi = \mathrm{Re}\sum_{j,k} \mathrm{i}(q_j - q_k) e^{\mathrm{i}(q_j - q_k)\varphi} R_{jk},$$

$$F_{r\varphi} = \mathrm{Re}\sum_{j,k} \mathrm{i}(q_j - q_k) e^{\mathrm{i}(q_j - q_k)\varphi} (L_{jk} + \widetilde{L}_{jk}),$$

$$F_{\varphi\varphi} = -\mathrm{Re}\sum_{j,k} (q_j - q_k)^2 e^{\mathrm{i}(q_j - q_k)\varphi} R_{jk},$$

$$F_{rr} = \mathrm{Re}\sum_{j,k} e^{\mathrm{i}(q_j - q_k)\varphi} \left[4P_{jk} + 2B_{jk} + \frac{v_j^2 + v_k^2}{r^2} R_{jk} - \frac{L_{jk} + \widetilde{L}_{jk}}{r} \right].$$

Theorem (All regular stress components and the current). *For every finite $H \geq 0$, the correction to the renormalized complex-scalar stress tensor is*

$$\delta T^t{}_t = \delta T^z{}_z = 2P - \mathscr{K} + 2\xi\mathscr{L},$$

$$\delta T^r{}_r = 2B - \mathscr{K} + 2\xi(\mathscr{L} - F_{rr}),$$

$$\delta T^\varphi{}_\varphi = \frac{2Q}{\alpha^2 r^2} - \mathscr{K} + 2\xi\left(\mathscr{L} - \frac{F_r}{r} - \frac{F_{\varphi\varphi}}{\alpha^2 r^2} \right),$$

$$\delta T^r{}_\varphi = 2U - 2\xi\left(F_{r\varphi} - \frac{F_\varphi}{r} \right), \qquad \delta T^\varphi{}_r = \frac{\delta T^r{}_\varphi}{\alpha^2 r^2}.$$

The remaining components vanish.

The current is

$$\delta J_t = \delta J_z = 0,$$

$$\delta J_r = \mathrm{i}e\,\langle L - \widetilde{L} \rangle_\varphi,$$

$$\delta J_\varphi = -e\sum_{j,k} (q_j + q_k) e^{\mathrm{i}(q_j - q_k)\varphi} R_{jk}.$$

The last two displayed expressions are real by Hermiticity.

Equations – and – are a finite special-function formula for the full regular correction. No differentiation of a numerically subtracted singular two-point kernel is required.

Proof. At longitudinal coincidence,

$$\partial_\tau \partial_{\tau'} J_0(p|y - y'|) = \partial_z \partial_{z'} J_0(p|y - y'|) = p^2/2.$$

Wick rotation therefore gives

$$A_{tt} = -P, \qquad A_{zz} = P, \qquad A_{rr} = B, \qquad A_{\varphi\varphi} = Q, \qquad A_{r\varphi} = U.$$

The angular and radial derivatives give –. The modified Bessel equation gives ; in particular,

$$F_{rr} + r^{-1}F_r + (\alpha^2 r^2)^{-1}F_{\varphi\varphi} = 2\left(2P + B + \frac{Q}{\alpha^2 r^2}\right) = \mathscr{L}.$$

The nonzero transverse Christoffel symbols are

$$\Gamma^{r}_{\varphi\varphi} = -\alpha^2 r, \qquad \Gamma^{\varphi}_{r\varphi} = r^{-1}.$$

Insert these and the listed A components into . Stationarity and longitudinal translation invariance remove the other improvement terms. This proves –.

Finally, $\partial_\varphi - \partial_{\varphi'}$ acts by $\mathrm{i}(q_j + q_k)$, while the radial difference acts by $L - \widehat{L}$. Equation gives –. All interchanges are justified by Proposition .

Remark (Angular structure and radial current). The only angular harmonics are $0, \pm 1$, because $q_0 - q_1 = 1$. This finite angular content does not make their radial coefficients homogeneous.

If $h = 0$, the radial current vanishes. If $\beta = 1/2$, the two Bessel orders agree, so $L_{jk} = \widehat{L}_{jk}$ and $\delta J_r = 0$ exactly, including non-diagonal matrices. For unequal orders and nonzero mixing, a radial current is generally present. It must not be removed to simplify the stress. Its angular average is zero.

Conservation, trace, and the gauge non-transfer

Theorem (Scalar regular-host identities). *For the formulas of Theorem ,*

$$\nabla_\mu \delta T^{\mu}{}_{\nu} = 0, \qquad \nabla_\mu \delta J^{\mu} = 0, \qquad \delta T^{\mu}{}_{\mu} = (6\xi - 1)\mathrm{Box}_g F.$$

In particular the regular correction is traceless for $\xi = 1/6$, even when the fixed extension introduces a scale.

The two nontrivial stress-conservation equations are

$$\partial_r \delta T^r{}_r + \frac{\delta T^r{}_r - \delta T^\varphi{}_\varphi}{r} + \frac{1}{\alpha^2 r^2}\partial_\varphi \delta T^r{}_\varphi = 0,$$

$$\partial_r \delta T^r{}_\varphi + \frac{1}{r}\delta T^r{}_\varphi + \partial_\varphi \delta T^\varphi{}_\varphi = 0.$$

The current equation is

$$\partial_r \delta J_r + \frac{\delta J_r}{r} + \frac{1}{\alpha^2 r^2}\partial_\varphi \delta J_\varphi = 0.$$

These are identities on M, not apex distributional normalizations.

Proof. For a smooth bisolution, differentiating the diagonal gives $\mathrm{Box}_g F = 2\mathscr{K}$. The canonical complex-scalar stress has divergence zero by the two field equations. The improvement has divergence zero on the regular flat host because derivatives commute there. The same bisolution argument applied to gives current conservation. Tracing gives

$$2\mathscr{K} - 4\mathscr{K} + 2\xi(4\mathrm{Box}_g F - \mathrm{Box}_g F) = (6\xi - 1)\mathrm{Box}_g F.$$

This establishes without a finite numerical test.

For a direct radial check of the current, the Bessel equations give

$$\partial_r(L_{jk} - \widetilde{L}_{jk}) + r^{-1}(L_{jk} - \widetilde{L}_{jk}) = \frac{v_j^2 - v_k^2}{r^2} R_{jk}.$$

Since $v_j^2 - v_k^2 = (q_j^2 - q_k^2)/\alpha^2$, the angular derivative of cancels this term exactly. The displayed stress equations are the coordinate expansion of the already proved divergence identity.

Scope boundary (What the spin results do not supply). The Maxwell and gravitational Ward statements in I-B-G-S-v2 concern the compatibility of different tensor and ghost apex domains [11]. They do not prove gauge compatibility of an arbitrary scalar $U(2)$ matrix and do not imply that the same matrix can be installed independently in photon, graviton, and ghost channels.

Here conservation is proved from a scalar bisolution and a specified scalar stress operator. No spin-1 or spin-2 gauge-domain theorem is imported.

Scope boundary (A vanishing trace is not full scale invariance). At conformal coupling the regular local trace vanishes. Nevertheless, the domain changes under dilation according to . A fixed finite H is generally not dilation invariant. Possible worldsheet-supported trace or scaling terms require an independently specified apex distributional prescription.

Special cases, fixed points, and asymptotic checks

Diagonal scale-dependent relations

If $h = 0$, the determinant factorizes and

$$C_{jj}(p) = \frac{1}{s_{\nu_j}(1 + a_j p^{-2\nu_j})}, \qquad a_0 = a, \quad a_1 = b.$$

The corresponding master function is one-dimensional:

$$\mathscr{U}^m_{\eta\chi}(X;\nu) = \frac{1}{2\pi \mathrm{i}} \int_{c-\mathrm{i}\infty}^{c+\mathrm{i}\infty} \Gamma(z)\Gamma(1-z) X^{-z} \mathscr{M}_{\eta\chi}(m + 2\nu z)\mathrm{d}z,$$

where $0 < c < 1$ and $m + 2\nu c > |\eta| + |\chi|$. Then

$$\mathscr{I}^{[m]}_{jj}(\eta,\chi;r) = \frac{\mathscr{N}_\alpha r^{-m}}{s_{\nu_j}} \mathscr{U}^m_{\eta\chi}(a_j r^{2\nu_j}; \nu_j).$$

This is the exact diagonal scale-dependent recovery. It is not, in general, a Bernoulli polynomial.

Friedrichs and singular fixed points

For $H = tH_0$ with $H_0 > 0$, $t \to \infty$ gives $C(p) \to 0$ and hence the Friedrichs correction zero. For $H = 0$,

$$C = S_0^{-1}, \qquad \mathscr{I}^{[m]}_{jj}(\eta,\chi;r) = \frac{\mathscr{N}_\alpha r^{-m}}{s_{\nu_j}} \mathscr{M}_{\eta\chi}(m), \qquad \mathscr{I}^{[m]}_{01} = 0.$$

Friedrichs in one channel and the singular realization in the other are obtained by sending the corresponding diagonal boundary eigenvalue to infinity or zero. Dominated convergence justifies these regular-host limits.

The useful equal-order identities are

$$\mathscr{M}_{\nu\nu}(2) = \nu s_\nu,$$

$$\mathscr{M}_{\nu\nu}(4) = \frac{2}{3}\nu s_\nu(1-\nu^2),$$

$$\int_0^\infty x^3 K_\nu'(x)^2 \mathrm{d}x = \frac{4-\nu^2}{3}\nu s_\nu.$$

The first two follow from . For the third, integrate the derivative of $x^3 K_\nu K_\nu'$ and use the Bessel equation; the endpoint terms vanish for $0 < \nu < 1$.

For one singular diagonal channel the resulting longitudinal stress is

$$\delta T^t{}_t = \frac{\mathscr{N}_\alpha \nu}{r^4}\left(8\xi - \frac{4}{3} - \frac{2}{3}\nu^2\right).$$

Two equal channels give twice this value. At $\xi = 1/6$ this is $-\frac{4}{3}\mathscr{N}_\alpha \nu^3 r^{-4}$ for the pair, as used in Theorem .

Where the Bernoulli formulas really apply

For a scale-invariant scalar label b, the Euclidean kernel is

$$W_b = \frac{f_b(\eta/\alpha - \mathrm{i}\psi) + f_{1-b}(\eta/\alpha + \mathrm{i}\psi)}{8\pi^2 \alpha r r' \sinh\eta}, \qquad f_b(z) = \frac{e^{-bz}}{1-e^{-z}},$$

where

$$\cosh\eta = \frac{r^2 + r'^2 + |y-y'|^2}{2rr'}, \qquad \psi = \varphi - \varphi'.$$

Starting from the Friedrichs label β,

$$W_{\beta+1} - W_\beta = \frac{e^{\mathrm{i}\beta\psi}\sinh(\nu_0\eta)}{4\pi^2 \alpha r r' \sinh\eta},$$

flips the $n = 0$ channel, while

$$W_{\beta-1} - W_\beta = \frac{e^{\mathrm{i}(\beta-1)\psi}\sinh(\nu_1\eta)}{4\pi^2 \alpha r r' \sinh\eta}$$

flips the $n = -1$ channel. Flipping both gives $W_{\beta+1} + W_{\beta-1} - W_\beta$.

The regular part has the local Bernoulli expansion

$$W_b - W_{\mathrm{flat}} \circ \mathrm{dev} = \frac{\eta/\sinh\eta}{8\pi^2 rr'} \sum_{k\geq 2} \frac{B_k(b)\alpha^{-k} - B_k(0)}{k!} \frac{(-1)^k(\eta - \mathrm{i}\Theta)^{k-1} + (\eta + \mathrm{i}\Theta)^{k-1}}{\eta}, \qquad \Theta = \alpha\psi.$$

For a scalar stress difference only the total second jet, hence B_2, B_3, B_4, is needed. The odd complex first jet must be retained for the current [6, 9, 11]. The fourth-jet requirement through B_6 belongs to the separate curvature-square calculation, not to this scalar stress.

These identities recover the homogeneous cases of the present moment prescription. They do not remove the crossover arguments of a finite nonzero H.

Equal orders and the CP1 line

When $\beta = 1/2$, write $u = v = \nu$. A constant unitary now diagonalizes H and $C(p)$ simultaneously:

$$H = \sum_a \lambda_a \Pi_a, \qquad C(p) = \sum_a \frac{\Pi_a}{s_\nu + 2e_\nu^2 \lambda_a p^{-2\nu}}.$$

This gives a finite spectral sum of the univariate functions .

Let Π be an orthogonal projector and send H to zero on $\operatorname{ran}\Pi$ and to infinity on its orthogonal complement. Then $C = s_\nu^{-1}\Pi$. For a rank-one projector, the singular direction is a point of $\mathbb{CP}^1$. In this limit,

$$\delta W_{E,\Pi} = \mathcal{N}_\alpha \frac{\sinh(\nu\eta)}{rr'\sinh\eta} \sum_{j,k=0}^{1} e^{\mathrm{i}q_j\varphi} \Pi_{jk} e^{-\mathrm{i}q_k\varphi'}.$$

Indeed,

$$\int_0^\infty pJ_0(p|y - y'|)K_\nu(pr)K_\nu(pr')\mathrm{d}p = s_\nu \frac{\sinh(\nu\eta)}{rr'\sinh\eta};$$

this is also the single-channel flip identity. All stress and current limits follow by the same finite derivatives. In particular the radial current is exactly zero.

For unequal orders, dilation-invariant boundary Lagrangians are the coordinate choices of regular or singular branch. The continuous projector freedom occurs only when the dilation weights coincide. Thus a general $\mathbb{CP}^1$ line

must not be asserted at unequal orders.

Ultraviolet and infrared limits

For finite H, $C(p) \to S_0^{-1}$ as $p \to \infty$. For $H > 0$, $C(p) \to 0$ as $p$downarrow 0. After $p = x/r$, dominated convergence in the differentiated moments gives:

• the leading rotation-averaged r^{-4} apex stress is that of the two singular fixed-point channels;

• the dimensionless orthonormal stress $r^4 \delta T_{\hat{a}\hat{b}}$ tends to zero at large r when $H > 0$. These are limits of a crossover function, not a formula for the whole function.

Leading off-diagonal data

For $H > 0$, define

$$\kappa_{01}(H) = \int_0^\infty p^{1-S} C_{01}(p)\mathrm{d}p, \qquad \Lambda = \mathcal{N}_\alpha e_u e_v \kappa_{01}(H).$$

The integral converges: $C_{01} = O(p^S)$ at zero and $O(p^{-S})$ at infinity, and $S > 1$.

The small-argument Bessel asymptotic and dominated convergence give

$$R_{01}(r) = \Lambda r^{-S} + o(r^{-S}).$$

Let

$$X_{\mathrm{off}} = \Lambda e^{\mathrm{i}\varphi} r^{-S}, \qquad k_\xi = (1+\alpha)\xi - \beta(1-\beta).$$

Applying the point-split operators gives the leading terms

$$F_{\mathrm{off}} = 2\mathrm{Re}\, X_{\mathrm{off}},$$

$$\delta J_{\varphi,\mathrm{off}} = 2e(1-2\beta)\mathrm{Re}\, X_{\mathrm{off}},$$

$$\delta J_{r,\mathrm{off}} = -\frac{2e(1-2\beta)}{\alpha r}\mathrm{Im}\, X_{\mathrm{off}},$$

$$\delta T^t{}_{t,\mathrm{off}} = \delta T^z{}_{z,\mathrm{off}} = o(r^{-2-S}),$$

$$\delta T^{r}_{r,\mathrm{off}} = -\delta T^{\varphi}_{\varphi,\mathrm{off}} = -\frac{4k_{\xi}}{\alpha^{2} r^{2}} \mathrm{Re}\, X_{\mathrm{off}},$$

$$\delta T^{r}_{\varphi,\mathrm{off}} = -\frac{4k_{\xi}}{\alpha r} \mathrm{Im}\, X_{\mathrm{off}}.$$

The omitted terms are lower order in the corresponding asymptotic sense.

To justify differentiation of the leading transverse powers, use boundedness of $x^{\nu+a}K_{\nu}^{(a)}(x)$ for fixed $a \le 2$ and split the longitudinal derivative integrals at $p = 1/r$. The longitudinal terms are smaller because $2S - 2 > 0$.

These formulas recover the supplied leading off-diagonal coefficient conventions of I-B-VP-1 [10, thm:u2lead].

The coefficient Λ is a global spectral functional, not a rational local function of H. Moreover, $r^{-S}e^{i\varphi}$ is harmonic on the transverse cone because $S = \alpha^{-1}$; this explains the vanishing leading off-diagonal trace without removing the shear or current.

Causal localization and supported transfer

The covariance correction is not a retarded signal. Nevertheless, the same boundary resolvent determines a retarded equation-inverse difference.

Proposition (The matrix correction starts at an apex arrival). *For a constant product relation $H \ge 0$, the retarded inverse difference from Friedrichs is supported in*

$$t - t' \ge \sqrt{(r + r')^{2} + (z - z')^{2}}.$$

The advanced statement is its time reflection. Angular mixing changes coefficients, not the earliest broken-path arrival.

Consequently a supported direct-flat Green-operation window proved for the Friedrichs operator is unchanged by this matrix correction. Equality of those Green operations does not identify their symmetric covariances or ground-state choices.

Proof. Use the Laplace variable ζ for time and the Fourier variable k for z. The transformed radial parameter is

$p = (\zeta^2 + k^2)^{1/2}$, with positive real part. The correction has factors $K_{\nu_j}(pr)C_{jk}(p)K_{\nu_k}(pr')$.

The identity

$$C(p) = \frac{1}{2}E(p)^{-1}\left(H + \operatorname{diag}\left(\frac{s_u}{2e_u^2}p^{2u}, \frac{s_v}{2e_v^2}p^{2v}\right)\right)^{-1}E(p)^{-1}$$

shows analyticity for $\operatorname{Re} p > 0$. If $\operatorname{Im} p > 0$, the two diagonal powers inside the inverse have strictly positive imaginary parts; if $\operatorname{Im} p < 0$, both have negative imaginary parts. On positive real p, the matrix is positive definite. There are therefore no zeros in that half-plane. For every fixed right half-plane the entries of $C(p)$ are polynomially bounded; at large $|p|$ they tend uniformly to S_0^{-1}.

The distributional Laplace support theorem consequently represents each entry of $C(p)$ as a Laplace transform of a distribution supported in $[0, \infty)$. Also,

$$K_\nu(pr) = \int_r^\infty e^{-pL}\frac{\cosh(\nu \operatorname{arccosh}(L/r))}{\sqrt{L^2 - r^2}}\mathrm{d}L.$$

Multiplication of these transforms is convolution in L. Thus the complete radial correction is a Laplace transform of a distribution supported in $L \geq r + r'$.

Finally, $e^{-L(\zeta^2 + k^2)^{1/2}}/(2(\zeta^2 + k^2)^{1/2})$ is the transformed retarded wave kernel in two spatial dimensions, evaluated at transverse separation L. It has support $t - t' \geq \sqrt{L^2 + (z - z')^2}$. Differentiation in L gives the same support statement for $e^{-L(\zeta^2 + k^2)^{1/2}}$. The L convolution is proper on compact time intervals, since only $L \leq t - t'$ can contribute. This proves . The distributional Laplace operations are standard [22].

The final assertion combines this support with the explicitly separate scalar arrival-window theorem of I-B-G [5, thm:transfer].

For use in a later supported construction, all intermediate causal shadows must still remain in its declared regular laboratory. A pairwise arrival bound does not supply that additional containment condition.

Apex contacts and limits of the inherited classification

All formulas released above are on M, with $r > 0$. They determine no values or field algebra on $\mathscr{W}$.

Proposition (Regular data cannot determine apex contacts). *If two distributional completions of a one-point function agree on M, their difference is supported on $\mathscr{W}$. No regular-host evaluation in this paper can distinguish them.*

A finite normal derivative-order bound does not by itself make the supported ambiguity finite dimensional.

Proof. The support assertion is the definition of equality on the complement of $\mathscr{W}$. Locally, supported distributions of finite normal order are finite sums of transverse delta derivatives with coefficient distributions along $\mathscr{W}$ [22]. Those coefficients need not belong to a finite-dimensional space. For example, $a(t,z)\delta_{\mathscr{W}}$ has normal order zero for arbitrary coefficient distribution a.

The proposed V9 plan asks that every remaining contact be recorded in the three-parameter space of I-B-VP-1. That cannot be made unconditional. The supplied classification has specific assumptions on symmetries, scaling, simultaneous observable identities, and the allowed completion prescription [10]. A finite generic H introduces dimensions and, if $h \neq 0$, breaks rotation invariance at fixed H. Joint rotation of coordinates and the phase of h is covariance of a family, not invariance of an individual member.

Accordingly:

• if the exact inherited hypotheses of the three-parameter classification are separately verified for a selected completion sector, its parameters remain normalization inputs;

• the present regular formulas do not verify those hypotheses for every generic matrix;

• the supplied off-diagonal nonresonance qualifications are not replaced by an all-parameter contact theorem;

• no resonant apex classification or distributional Ward-compatible completion is selected here.

The original finite Bernoulli target cannot be repaired by a worldsheet contact: the detecting regular points in

Theorem are disjoint from its support.

Verification protocol and evidence discipline

Analytical checks completed in the text

The following are written identities, not execution reports:

1. boundary normalization: $I - S_0 C = 2EHEC$;

2. Hermiticity and the positivity bound $0 \le C \le S_0^{-1}$;

3. exact scaling ;

4. the Bessel-to-Barnes identity ;

5. all regular stress/current formulas and their differential identities;

6. the gamma-moment benchmarks ;

7. the diagonal, fixed-point, and equal-order projector limits;

8. the nonrational Stieltjes obstruction.

No numerical result is needed for these conclusions.

A Barnes contour tail bound

The common contours give a useful analytical truncation estimate. Let

$$C_\Gamma = \sup_{t \in \mathbb{R}} e^{\pi|t|/2} |\Gamma(1/4 + it)| < \infty, \qquad a_* = \pi/2,$$

where finiteness follows from Stirling's formula and continuity. Let σ be the real part of the Mellin argument in .

Since $K_\eta K_\chi$ is positive for real orders,

$$|\mathcal{M}_{\eta\chi}(\sigma + it)| \le \mathcal{M}_{\eta\chi}(\sigma).$$

Truncating each contour imaginary part to $[-T, T]$ therefore leaves an absolute tail at most

$$\frac{C_{\Gamma}^{4}}{(2\pi)^{3}}(XYZ)^{-1/4}\mathscr{M}_{\eta\chi}(\sigma)\frac{24}{a_{*}^{3}}e^{-a_{*}T}.$$

This follows by bounding the gamma product by $C_{\Gamma}^{4}e^{-a_{*}(|t_{1}|+|t_{2}|+|t_{3}|)}$ and using the union bound outside the cube.

Equation is an analytical tail estimate. A numerical certificate must still provide a justified numerical bound for C_{Γ}, finite-contour quadrature error, input roundoff, and conditioning of the final tensor combination. The estimate can be inefficient near a singular matrix limit; the limiting diagonal or projector representation is then preferable.

Proposed finite cross-checks, not reported runs

Following the supplied XVI-O1-ST integrity discipline [13], use:

\texttt{ANALYTIC} A displayed proof or an explicitly identified analytical input.

\texttt{SUPPLIED} An inherited computational report not independently executed here.

\texttt{UNCERTIFIED} An approximation without a complete justified total-error bound.

A future numerical audit should compare:

1. direct Bessel quadrature using against –;

2. all stress components, not only one diagonal entry;

3. the radial current and angular shear at nontrivial angular phases;

4. the exact diagonal and equal-order limiting formulas;

5. the directly evaluated conservation residuals, without using the same residual equation to define a missing component. Suitable declared test inputs include

$$(\alpha,\beta) = (4/5, 3/10), \quad (9/10, 9/20), \quad (1/\sqrt{2}, 1/2),$$

and, in a fixed reference-length convention,

$$H = \begin{pmatrix} 1 & \frac{1}{2}e^{3\mathrm{i}/10} \\ \frac{1}{2}e^{-3\mathrm{i}/10} & 7/10 \end{pmatrix} > 0.$$

For each input retain several radii on both sides of the crossover, the raw paired values before rounding, exact input formulas, working precision, quadrature partitions and tolerances, Barnes cutoff, and every low- and high-momentum tail bound. A requested integrator tolerance is not a total-error theorem.

The complete error ledger must distinguish

$$\varepsilon_{\text{input}},\quad \varepsilon_{\text{finite contour}},\quad \varepsilon_{\text{Barnes tail}},\quad \varepsilon_{\text{Bessel quadrature}},\quad \varepsilon_{\text{spectral tails}},\quad \varepsilon_{\text{roundoff}}.$$

Failed comparisons must remain in the report.

No such numerical run is asserted here. The supplied predecessor's numerical asymptotics are not relabelled as a cross-check of the new full Barnes stress.

Scope boundary (Finite-test coverage). Finite samples do not establish an identity of distributions. An independently specified finite-dimensional residual space and an evaluation map injective on that space can make a finite test complete within that class; both hypotheses need proof [14, prop:coverage, prop:invisible]. The finite number of matrix entries is not a proof that the allowed radial residual functions form such a space.

Consequences, remaining questions, and exact release

Plan audit

toprule Proposed step Corrected outcome

midrule endfirsthead toprule Proposed step Corrected outcome

midrule endhead bottomrule endfoot

Generic H for every cone parameter A two-channel scalar chart requires $\alpha > 1/2$ and $1 - \alpha < \beta < \alpha$. Other deficiency counts are different problems.

Finite boundary rank implies finitely many radial monomials False. Finite nonzero H introduces the crossover arguments X, Y, Z, c_r.

Coefficients rational in H False even on $H = hI$. The stress coefficient has a continuous Stieltjes jump.

Diagonalize H and integrate fixed scalar branches Valid without further qualification only when the two orders coincide, or when H already respects the unequal-order decomposition.

Use Bernoulli jets for generic finite H Use the finite moment list and Barnes evaluations. Bernoulli jets recover the scale-invariant limits only.

Transfer spin gauge Ward identities Not valid. The scalar conservation and trace identities are proved directly from its scalar bisolution.

Discard the off-diagonal radial current Not valid. Equations – retain it and prove its conservation.

Every apex residual belongs to three parameters Only under the original classification hypotheses. No all-H or resonant completion theorem is inferred.

Finite numerical checks complete the closed form They are supplementary checks. The Barnes identity and point-split derivation are the proof.

Companion interfaces

I-B supplies the prescribed geometry and finite operational separation, not the spectral calculation. I-B-G supplies the scalar extension and supported-transfer background. I-B-VP and I-B-VP-0 supply the regular normalization and complex-first-jet comparisons. The supplied I-B-VP-SAE and I-B-VP-1 records identify the matrix target and its asymptotic predecessors. I-B-G-S-v2 supplies a useful methodological lesson—finite local jets require an independently justified kernel and domain—but no scalar matrix Ward identity is imported.

The proposed ledger sentence is: I-B-VP-9 supplies exact Mellin–Barnes formulas and complete regular scalar stress/current assembly for the specified two-channel nonnegative product relations; it proves that generic finite Bernoulli/rational closure fails. No interacting product or dynamics is supplied. This is suggested metadata, not an assertion that any external registry has been modified.

Remaining questions

The following are not completed by this note:

1. a classification of exceptional parameter sets or special matrices permitting a lower-dimensional or elementary special-function reduction;

2. an independently executed, total-error-controlled numerical comparison of the new generic formulas;

3. a full resonant, scale-dependent, rotation-breaking worldsheet-contact classification;

4. a physical core-limit selection and convergence theorem for the chosen matrix relation;

5. an extension to longitudinally non-product relations, massive fields, or a thermal prescription with its own infrared analysis.

The full spin gauge form-domain theorem, physical spin-2 core selection, and complete Bel–Robinson tensor remain the separate open questions of I-B-G-S-v2. No conclusion about them follows from a scalar Barnes reduction.

Theorem (I-B-VP-9 mathematical release). *For Assumptions and , the boundary normalization , finite $H \geq 0$, and the complex-scalar prescription –:*

1. *the matrix correction is the normalized Krein kernel , smooth at regular diagonal points;*

2. *a generic finite Bernoulli combination with radius-independent rational matrix coefficients is excluded;*

3. *the exact finite special-function prescription – gives every regular stress and current component;*

4. *the scalar conservation, current, and mixed-trace identities hold analytically;*

5. *the diagonal, Friedrichs, singular fixed-point, and equal-order projector limits are recovered;*

6. *the retarded branch correction begins at the apex-arrival threshold, without a covariance-transfer claim. No all-H apex-contact normalization, interacting differential, Einstein source equation, backreaction, or emergence statement is released.*

Proof. Item (i) is Theorem and Proposition . Item (ii) is Theorem . Item (iii) is Theorem followed by the finite assembly of Theorem . Item (iv) is Theorem . Item (v) is Section , with limits justified by . Item (vi) is Proposition

. The excluded conclusions require additional objects or hypotheses not supplied by these results.

$$\text{boxed}\begin{array}{c}\text{generic finite Bernoulli/rational closure: disproved,}\\ \text{exact regular scalar Barnes formula: constructed,}\\ \text{apex normalization and interacting dynamics: not constructed,}\\ \text{O.1 on the cone = texttt}\,\textit{NOT_CLOSED},\\ \text{XVI-N not discharged; XV-B not opened,}\\ \text{texttt}\,\textit{bulk_symmetry_restored} = \text{texttt}\,\textit{false}.\end{array}$$

appendix

A compact derivation of the Barnes tail estimate

On the quarter contours write $z_j = 1/4 + \mathrm{i}t_j$. The fourth Barnes gamma factor is $\Gamma(1/4 - \mathrm{i}(t_1 + t_2 + t_3))$. By the definition of C_Γ,

$$\prod_{j=1}^{3} |\Gamma(z_j)||\Gamma(1 - z_1 - z_2 - z_3)| \le C_\Gamma^4 e^{-\frac{\pi}{2}(|t_1| + |t_2| + |t_3| + |t_1 + t_2 + t_3|)}.$$

Dropping the final nonpositive exponent gives an integrable product bound. The modulus of $X^{-z_1}Y^{-z_2}Z^{-z_3}$ is $(XYZ)^{-1/4}$, and the Mellin transform is bounded by its value at the real part of its argument.

For $a > 0$,

$$\int_{\mathbb{R}^3 \setminus [-T,T]^3} e^{-a(|t_1| + |t_2| + |t_3|)} \mathrm{d}t \le 3\left(\frac{2}{a}e^{-aT}\right)\left(\frac{2}{a}\right)^2 = \frac{24}{a^3}e^{-aT}.$$

The contour measure contributes $(2\pi)^{-3}$. These facts prove . No numerical value of C_Γ or finite-contour quadrature certificate is inferred from this symbolic estimate.

Declarations

Authorship and source attribution. The author name follows the supplied manuscript record. Cheung companion sources are cited by identifier and supplied version. An introductory web page, a supplied excerpt, and an independently verified complete manuscript are not treated as the same evidence class.

Analytical dependence. The normalized boundary resolvent, obstruction theorem, Barnes reduction, and complete regular scalar observable formulas are proved here. The Friedrichs compact-test covariance and

supported flat arrival-window interfaces are attributed explicitly. No unavailable SAE theorem is silently used to fix a normalization that is not displayed.

Novelty. The claimed contribution is the exact generic two-channel regular observable prescription and the obstruction to the proposed finite algebraic closure. No novelty is claimed for Krein theory, Bessel extensions, scalar cosmic-string point splitting, Mellin transforms, or the general existence of special-function representations.

Physical and operational scope. The cone, deficit, bundle, realization, normalization, coupling ξ, and vacuum prescription remain inputs. A finite reporting channel is not a field detector or an acquisition theorem. A conserved traceless tensor is not thereby the source of the prescribed geometry.

Apex and quantum scope. The observables are ordinary regular-host fields. No apex field algebra or contact normalization is selected. The free scalar one-point calculation is not an admitted interacting characteristic graph coefficient, a quantum master equation, or an ultraviolet completion.

Bibliographic status. The Enter journal record is distinguished from unpublished companion manuscripts. The publisher record identifies acceptance on 28 September 2026 and indicates that final formatting is forthcoming. No publication status, DOI, or complete-proof audit is invented for an internal companion record.

Execution and submission scope. This is one conventional pdfLaTeX source with an internal bibliography and no external figure, code, or data dependency. Two pdfLaTeX runs are intended for references and contents. No successful compiler run, numerical quadrature, symbolic program execution, proof-assistant verification, independent rebuild, registry installation, physical implementation, independent referee review, or journal acceptance of this manuscript is asserted.

References

[1] N. Cheung, ``The Enter Calculus: A Linear Quantum Lambda Calculus with an Operational Characterization of Measurement," *Frontiers in Physics* **14** (2026), article 1954943. DOI: 10.3389/fphy.2026.1954943. https://doi.org/10.3389/fphy.2026.1954943. Interface/operational background only; not a PDE premise.

[2] N. Cheung, *The Philosophy of Enter: Destructive Readout, Quantum Recovery, and the Refusal of Identification*, Cheung Ngo Medical Limited (2026). Framing essay; not a theorem premise. Programme note: https://enter.grok.me/enter.

[3] N. Cheung, *I-B-v1: A Finite Enter Spacetime: Calibrated Destructive Time Readout on a Prescribed Conical Product*, version 1, Enter Calculus Companion. Prescribed host, complete clock instrument, Einstein comparator, and prop:not-source. https://enter.grok.me/browse/I-B. Companion record; not a Scholar-indexed journal article.

[4] N. Cheung, *En09-A: Realization of Relational Time by Destructive Enter on a Clock Register*, version 1, supplied companion excerpts. Complete record–residual interface and its additional covariance/stationarity boundaries. https://enter.grok.me/browse/En09-A. Companion record; not a Scholar-indexed journal article.

[5] N. Cheung, *I-B-G: Intrinsic and holonomy-twisted Green operators on a prescribed conical spacetime: Self-adjoint extensions, diffraction, finite propagation, and supported transfer from flat-completion laboratories*, version 2, supplied source record and excerpts, Enter Calculus Companion. Deficiency count, product realizations, non-global-hyperbolicity, and thm:transfer. The matrix causal support argument used here is written out separately. https://enter.grok.me/browse/I-B-G. Companion record; not a Scholar-indexed journal article.

[6] N. Cheung, *I-B-VP: Vacuum polarization of the Friedrichs ground state on a prescribed conical spacetime: Local Hadamard subtraction, holonomy, and the renormalized stress tensor*, version 3, supplied version-qualified excerpts, Enter Calculus Companion. Relevant comparison items: lem:infrared, thm:hadamard, thm:friedrichs, and the product extension/state ledger. https://enter.grok.me/browse/I-B-VP. Companion record; not a Scholar-indexed journal article.

[7] N. Cheung, *I-B-VP-SAE: Closing the self-adjoint extension ledger*, version 3, supplied companion description and formula-level excerpts, Enter Calculus Companion. Source of the generic matrix-relation target and the cited Krein/integral interfaces thm:krein, thm:Elam, prop:u2. A complete independent source audit is not asserted. The boundary normalization needed here is derived in Theorem . Companion record; not a Scholar-indexed journal article.

[8] N. Cheung, *I-B-VP-AB: Vacuum polarization and induced current of the Friedrichs ground state on the conical Enter spacetime*, version 2, supplied comparison record. Holonomy and normalization context only; no generic matrix formula is imported. Companion record; not a Scholar-indexed journal article.

[9] N. Cheung, *I-B-VP-0: The holonomy-odd vacuum current of the charged Friedrichs ground state*, version 1, supplied source excerpts, Enter Calculus Companion. Complex-first-jet and current-sign comparison. The generic matrix current is derived independently here. https://enter.grok.me/browse/I-B-VP-0. Companion record; not a Scholar-indexed journal article.

[10] N. Cheung, *I-B-VP-1: Prescribed apex finite parts on a conical Enter spacetime: Wick square, stress tensor, and charged current: strong extension, Ward identities, and classified contact freedom*, version 3, supplied excerpts from I-B-VP-1-v4.tex, Enter Calculus Companion. Relevant supplied labels: thm:uv, thm:u2lead, thm:u2ext, and the hypothesis-dependent contact classification. The reported numerical logs were not rerun. The resonant and generic finite-*H* contact hypotheses are not declared discharged here. https://enter.grok.me/browse/I-B-VP-1. Companion record; not a Scholar-indexed journal article.

[11] N. Cheung, *I-B-G-S: Intrinsic spin-1 and spin-2 Green operators on the prescribed conical spacetime: Helicity domains, apex Ward obstructions, curvature regularity, and supported transfer*, version 2, supplied complete source I-B-G-S-v2.tex. Finite-jet methodology and domain/non-transfer comparison. Its spin gauge identities are not scalar matrix-extension premises. https://enter.grok.me/browse/I-B-G-S. Companion record; not a Scholar-indexed journal article.

[12] N. Cheung, *X-A-ST*, version 1, supplied excerpts, Enter Calculus Companion. prop:noeinstein and the fixed-host/non-extraction comparisons. https://enter.grok.me/browse/X-A-ST. Companion record; not a Scholar-indexed journal article.

[13] N. Cheung, *XVI-O1-ST*, version 5, supplied XVI-O1-ST-v5.tex excerpts, Enter Calculus Companion. Complete supplied-report interface, evidence grades, residual retention, and execution scope. https://enter.grok.me/browse/XVI-O1-ST. Companion record; not a Scholar-indexed journal article.

[14] N. Cheung, *Oai282-A*, version 1, supplied prop:coverage and prop:invisible excerpts. Finite-test coverage discipline only. No external catalogue proof is a premise. https://enter.grok.me/browse/Oai282-A. Companion record; not a Scholar-indexed journal article.

[15] B. S. Kay and U. M. Studer, Boundary conditions for quantum mechanics on cones and fields around cosmic strings, *Communications in Mathematical Physics* **139** (1991), 103–139. DOI: 10.1007/BF02102731. Independent apex-domain context.

[16] J. Dereziński and S. Richard, On Schrödinger operators with inverse square potentials on the half-line, *Annales Henri Poincaré* **18** (2017), 869–928. DOI: 10.1007/s00023-016-0520-7. https://arxiv.org/abs/1604.03340. Bessel realizations, homogeneous endpoints, resolvents, and spectral background.

[17] J. Behrndt, M. M. Malamud, and H. Neidhardt, Scattering matrices and Weyl functions, *Proceedings of the London Mathematical Society* **97** (2008), 568–598. https://arxiv.org/abs/math-ph/0604013. Boundary relations, Weyl functions, and Krein resolvent methods. The particular normalization is derived in the present text.

[18] F. W. J. Olver et al., editors, *NIST Digital Library of Mathematical Functions*. Chapters 5, 10, and 24: gamma and beta integrals, Bessel representations and recurrences, and Bernoulli generating functions. https://dlmf.nist.gov/.

[19] V. Balakumar and E. Winstanley, Hadamard renormalization for a charged scalar field, *Classical and Quantum Gravity* **37** (2020), 065004. DOI: 10.1088/1361-6382/ab6b6e. https://arxiv.org/abs/1910.03666. Charged scalar point splitting and local renormalization framework; not an apex-domain selection.

[20] S. Hollands and R. M. Wald, Local Wick polynomials and time ordered products of quantum fields in curved spacetime, *Communications in Mathematical Physics* **223** (2001), 289–326. DOI: 10.1007/s002200100540. https://arxiv.org/abs/gr-qc/0103074. Smooth-background local normalization context. No interacting conical product construction is imported.

[21] L. Sriramkumar, Fluctuations in the current and energy densities around a magnetic-flux-carrying cosmic

string, *Classical and Quantum Gravity* **18** (2001), 1015–1025. DOI: 10.1088/0264-9381/18/6/304. https://arxiv.org/abs/gr-qc/0011074. Established scalar current and stress context; no generic matrix coefficient or fluctuation formula is attributed as verified here.

[22] L. Hörmander, *The Analysis of Linear Partial Differential Operators I: Distribution Theory and Fourier Analysis*, second edition, Springer, 1990. Distributional Fourier–Laplace operations, support, and distributions supported on submanifolds.

[23] N. Cheung, *Black-Hole Information as an Access Problem: Enter, Quantum Recovery, and Complete Residuals*, figshare (2026). DOI: 10.6084/m9.figshare.34335024.v1. https://doi.org/10.6084/m9.figshare.34335024.v1.

[24] N. Cheung, *Operational Wormhole Certificates—Existence, Dimension-Dependent Scaling, Operational Functions, and Composition*, figshare (2026). DOI: 10.6084/m9.figshare.34549575.v1. https://doi.org/10.6084/m9.figshare.34549575.v1.

[25] N. Cheung, *Enter, Quantum Magic, and Quantum Gravity: Readout, Recovery, and the Limits of Geometric Response*, figshare (2026). DOI: 10.6084/m9.figshare.34335582.v1. https://doi.org/10.6084/m9.figshare.34335582.v1.

[26] N. Cheung, *Operational Consistency Does Not Select Lorentzian Geometry*, figshare (2026). DOI: 10.6084/m9.figshare.34441758.v1. https://doi.org/10.6084/m9.figshare.34441758.v1.

[27] N. Cheung, *Realization of Relational Time by Destructive Enter on a Clock Register*, figshare (2026). DOI: 10.6084/m9.figshare.34442907.v1. https://doi.org/10.6084/m9.figshare.34442907.v1.